On Dedicated Evolutionary Algorithms for Large Non-Linear Constrained Optimization Problems

Janusz Orkisz
Cracow University of Technology
Warszawska St. 24
31-155 Cracow, Poland
0048 126282560
plorkisz@cyf-kr.edu.pl

Maciej Glowacki
Cracow University of Technology
Warszawska St. 24
31-155 Cracow, Poland
0048 126282569
mglowac@gmail.com

ABSTRACT
This paper considers advances in development of dedicated Evolutionary Algorithms (EA) for efficiently solving large, non-linear, constrained optimization problems. The EA are precisely understood here as decimal-coded Genetic Algorithms consisting of three operators: selection, crossover and mutation, followed by several newly developed calculation speed-up techniques based on simple concepts. These techniques include: solution smoothing and balancing, a'posteriori solution error analysis and related techniques, non-standard use of distributed and parallel calculations, and adaptive step-by-step mesh refinement. Efficiency of the techniques proposed here has been evaluated using several benchmark problems e.g. residual stresses analysis in chosen elastic-plastic bodies under cyclic loadings. These preliminary tests indicate significant acceleration of the large optimization processes involved. The final objective of our research is development of an algorithm efficient enough for solving real, large engineering problems.

Categories and Subject Descriptors
G.1.6 [**Numerical Analysis**]: Optimization – *constrained optimization, global optimization, nonlinear programming, stochastic programming.*

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *Heuristic methods.*

I.2.m [**Artificial Intelligence**]: Miscellaneous.

General Terms
Algorithms, Measurement, Performance, Experimentation, Theory, Verification.

Keywords
Genetic Algorithms, Parallelization, Speedup technique, Empirical study, Mechanical engineering

GECCO'14, July 12–16, 2014, Vancouver, BC, Canada.
ACM 978-1-4503-2881-4/14/07.
http://dx.doi.org/10.1145/2598394.2598436

1. INTRODUCTION
In this paper we consider improvement of computational efficiency of the optimization approach based on Evolutionary Algorithms (EA). In contrast to the deterministic optimization methods, the EA may be successfully applied with similar efficiency to both the convex and non-convex problems. However, general efficiency of the standard EA is rather low. Therefore, the main objective of our research is to develop means of an essential acceleration of the EA-based solution approach. So far, we have already proposed, and preliminarily tested, several acceleration techniques [2,5]. Selected well known acceleration techniques, including parallel, and hybrid algorithms [3] are considered as well. We are presenting here an overview of the proposed techniques, with particular consideration to smoothing, and balancing techniques. Our long-term research is oriented towards development of efficient tool for numerical solution of large, non-linear, constrained optimization problems. This research involves analysis of several benchmark problems. Finally, we take into account applications of the improved EA to residual stresses analysis in railroad rails, and vehicle wheels, and to a wide class of problems involving the Physically Based Approximation of experimental and/or numerical data [4].

2. PROBLEM FORMULATION
We consider a wide class of large, non-linear, constrained optimization problems. In such problems usually a function $u(\mathbf{x})$, $\mathbf{x} \in R^N$ is sought, in the discrete form of the vector $\mathbf{u} = \{u_i\}$ consisting of nodal values u_i, $i = 1, 2, \ldots, n$. These nodal values are defined on a mesh formed by arbitrarily distributed nodes. Here N is the dimension of the physical space (1D, 2D or 3D), and n is a number of decision variables. The solution usually has also to satisfy numerous equality, and inequality constraints.

3. APPLIED ALGORITHMS
The EA are precisely understood here as decimal-coded Genetic Algorithms. The standard algorithm consists of selection, crossover and mutation [1]. Our current research is mostly concentrated on the development of new acceleration techniques, including solution smoothing and balancing, a'posteriori solution error analysis and various related techniques, as well as adaptive step-by-step mesh refinement. Some of the proposed techniques are problem-oriented, other are of more general character.

When the optimization process involves large number of decision variables e.g. nodal function values, raw results obtained from the EA approach usually present a collection of locally scattered data. In such case, available additional information about solution smoothness (at least piecewise) allows to use, e.g., an extra procedure based on the Moving Weighted Least Squares (MWLS) technique [6] for smoothing the raw EA results. Weighting function may be introduced as in [4]:

$$w_i^2 = \left(h_i^2 + \frac{g^4}{h_i^2 + g^2} \right)^{-p-1}, \tag{1}$$

where h_i is a distance between nodes, p is the local approximation order, and g is a smoothing intensity parameter.

In problems of mechanics each smoothing may result in the global equilibrium loss of a considered body. The equilibrium is restored here by an artificial linear balancing of body forces. Information about smoothness may be also used in the selection process [2]. A new criterion based on the mean solution curvature [4] may be introduced into any selection operator.

A'posteriori error estimation is based on a stochastic nature of EA [5]. Reference solutions required to estimate errors are obtained here by weighted averaging of the best solutions taken from independent populations. Information about errors is used by improved EA operators. Detailed information about a'posteriori error estimation, and related techniques may be found in [5].

A general idea of adaptive step-by-step mesh refinement is to start analysis from a coarse mesh, allowing to obtain fast, but not sufficiently precise solution. The mesh is refined by inserting new nodes in order to increase its precision. The initial function values at these nodes are found by using the MWLS approximation [6]. A combined strategy, using together mesh refinement, a'posteriori error analysis, and smoothing was preliminarily proposed in [2].

4. NUMERICAL ANALYSIS

In order to evaluate the efficiency of the proposed acceleration techniques, various demanding benchmark problems were chosen, including residual stresses analysis in elastic-perfectly plastic bodies, such as prismatic bar and thick-walled cylinder, under various cyclic loadings [2,5]. These problems may be analyzed in either 1D or 2D, and allow selecting almost any number of decision variables. One of the simplest benchmark problems analyzed is briefly described here.

Considered is residual stresses analysis in an elastic-perfectly plastic bar subject to cyclic bending in 1D [5]. After discretization by using the Finite Difference Method approach the following formulation of optimization problem is obtained:

Find stresses $\sigma = \sigma(z)$ in the form of their discrete nodal values $\sigma_1, \sigma_2, \ldots, \sigma_n$ satisfying the minimum of the total complementary energy, as well as the global self-equilibrium equation expressed in terms of σ_n :

$$\min_{\sigma_1, \sigma_2, \ldots, \sigma_{n-1}} \left(\sum_{k=1}^{n-1} \sigma_k^2 + \frac{1}{2} \sigma_n^2 \right), \quad \sigma_n = -\frac{2}{z_n} \sum_{k=1}^{n-1} \sigma_k z_k , \tag{2}$$

and inequality constraints (yield condition for total stresses):

$$-\sigma_Y \le \sigma_k + \sigma_k^e \le \sigma_Y, \qquad k = 1, 2, \ldots, n , \tag{3}$$

where σ_Y is the yield stress (plastic limit), and σ^e is the purely elastic solution of the problem.

The main objective of numerous tests was to examine the behavior of the proposed techniques, to find their appropriate parameters, and to investigate the most efficient solution strategy. The typical results of numerical analysis are shown in Figure 1. They were obtained for the already mentioned simple benchmark problem. We are presenting here sample results for one particular speed-up technique only. One may see the influence of additional smoothing and balancing on the convergence of the mean solution error. The algorithm used rank selection, heuristic crossover (P_C = 0.9), and non-uniform mutation (P_M = 0.1). For smoothing the MWLS technique with p = 1, and g = 5 was used. All processes shown in Figure 1 were carried out for 3000 iterations, so one may find the additional time needed for all extra operations.

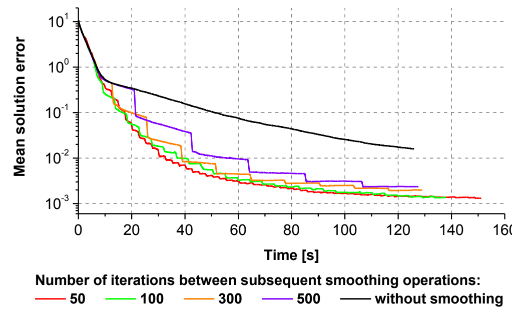

Figure 1. Results of smoothing and balancing technique

5. FINAL REMARKS

Preliminary results show, that the EA-based solution approach may be significantly accelerated using several simple concepts. Application of smoothing, and balancing procedure allowed to achieve up to about 4 times efficiency increase. However, the greatest acceleration till now, namely 120 times, was obtained for a mesh refinement combined with the other considered techniques. Future research will be mostly focused on practical application of the dedicated EA to real engineering problems.

6. REFERENCES

[1] Engelbrecht, A. P. 2007. *Computational intelligence: an introduction*. Wiley, Chichester.

[2] Glowacki, M., and Orkisz, J. 2013. Advances in Development of Dedicated Evolutionary Algorithms for Large Non-Linear Constrained Optimization Problems. *IPPT Reports on Fund. Tech. Research*. 47, 4 (Dec. 2013), 25-29.

[3] Grosan, C., Abraham, A., and Ishibuchi, H., Eds. 2007. Hybrid Evolutionary Algorithms. *Studies in Computational Intelligence*. 75, Springer.

[4] Karmowski, W., and Orkisz, J. 1993. Physically Based Method of Enhancement of Experimental Data - Concepts, Formulation and Application to Identification of Residual Stresses. In *Inv. Problems in Engng. Mech. Proc. of IUTAM Symp. on Inv. Problems in Engng. Mech.* (Tokyo, Japan, 1992), M. Tanaka, and H. D. Bui, Eds. Springer, 61-70.

[5] Orkisz, J., and Glowacki, M. 2014. On Acceleration of Evolutionary Algorithms Taking Advantage from A'posteriori Error Analysis. *Computing and Informatics*. 33, 1 (Feb. 2014), 154-174.

[6] Salkauskas, K., and Lancaster, P. 1990. *Curve and surface fitting*. Academic Press.