Risk Aversion and Mobility in the Public Goods Game

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ABSTRACT

In this paper, we study the evolutionary dynamics of the public goods game where the population of mobile individuals is divided into separate groups. We extend the usual discrete strategy game, by introducing "conditional investors" who have a real-value genetic trait that determines their level of *risk aversion*, or willingness to invest into the common pool. At the end of each round of the game, each individual has an opportunity to (a) update their risk aversion trait using a form of imitation from within their current group, and (b) to switch groups if they are not satsisfied with their payoff in their current group. Detailed simulation experiments show that investment levels can be maintained within groups. The mean value of the risk aversion trait is significantly lower in smaller groups and is correlated with the underlying migration mode. In the conditional migration scenarios, levels of investment consistent with risk aversion emerge.

Categories and Subject Descriptors

I.2 [**Computing Methodologies**]: Artificial Intelligence

General Terms

Algorithms, Experimentation, Theory

Keywords

public goods game; risk aversion; mobility; social dilemma

1. INTRODUCTION

The public goods game is a well known mathematical paradigm often used to investigate the population dynamics of groups of interacting players of arbitrary size [5, 3]. Here, players must decide simultaneously whether they contribute to the common pool or not. All contributions are then multiplied by a factor $\tau > 1$ that takes into account the added value of collaborative effort, and the resulting amount is divided equally among all group members irrespective of their initial decision. From the perspective of each individual, "defection" (a decision not to invest) is clearly the rational decision to make, as it yields the highest personal income if compared to other group members.

When confronted with a social dilemma, such as that encapsulated within a multiple group implementation of the public goods game, a decision-maker must decide (a) which group to join, and (b) how much to invest – both potentially risky choices. Whereas traditional evolutionary game theory seeks to promote cooperation, i.e. high investment and high risk, countering the individual interest of maximizing personal income, the objective in this paper is to promote risk aversion, i.e. balancing investment and risk, thus aligning collective and individual benefits.

We use an evolutionary game theory framework to extend recent work focussed on the evolution of risk aversion [2] to examine evolutionary dynamics in a multiple-group public goods game. A key feature of our model is the introduction of "conditional investors" into the game. That is, rather than using the binary actions of "cooperate" (invest) or "defect" (do not invest), individuals have a real-value genetic trait that determines their level of *risk aversion*, or willingness to invest into the common pool. At the end of each game playing round, an opportunity arises for individuals to change their risk aversion trait and/or their current group.

Monte Carlo simulation experiments are used to investigate model behaviour. The results show that the average value of the risk aversion trait is significantly lower in smaller groups and in the random migration model. We find that lower levels of investment are typically maintained in the conditional migration model suggesting that individuals who make decisions based on whether that are satisfied with their local neighbourhood tend to be more risk averse.

2. MODEL

The model consists of a population composed of multiple fixed-sized interaction groups playing the public goods game. At an abstract level, the model has some silimarities with the probabilistic participation framework introduced in [4]. However, an important differentiating factor is the fact that an individual's level of investment I depends on χ as described in equation 1, where the probability p is any value between 0 and 1, including the two endpoints, thereby capturing purely deterministic behaviour; and I_{max} is a parameter of the model controlling the maximum possible investment. I is a Bernoulli random variable representing the expressed behaviour or "phenotype" of an individual given the underlying "genotype" χ. Individuals with a high trait

GECCO'14, July 12–16, 2014, Vancouver, BC, Canada.
ACM 978-1-4503-2881-4/14/07.
http://dx.doi.org/10.1145/2598394.2599988.

value (greater than 0.5) are considered to be risk averse, corresponding to a low chance of investing a small amount.

$$I = \begin{cases} (1-\chi) * I_{max} & p = 1-\chi \\ 0 & p = \chi \end{cases} \quad (1)$$

The payoff the individual receives is based on equation 2. This payoff π_i is added to an individual's assets (initialized as I_{max}), which serve as a function of the individual's fitness.

$$\pi_i = \frac{\tau}{n} \sum_{j=1}^{n} I_j - I_i \quad (2)$$

At each time step, an individual i can adopt the strategy χ_j from individual j with probability

$$W(\chi_i \leftarrow \chi_j) = \frac{1}{1 + exp[(\pi_i - \pi_j)/K]} \quad (3)$$

where K quantifies the uncertainty by strategy adoptions (without loss of generality we use $K = 0.1$). All individuals are also subject to mutation. That is, with probability μ the offspring mutates to a random strategy; otherwise its risk trait χ is identical to its parent.

Each individual is also presented with an opportunity to switch groups via (a) random migration or (b) conditional migration – a form of the so-called *walk-away-rule* based on current payoff values, where different groups are nominated (best or worse groups from the populations) [1].

3. SIMULATION EXPERIMENTS

Monte Carlo simulation experiments were performed to examine population dynamics. Key model parameters include population size, the number of groups, and mobility mode. Instead of reporting results based on the "levels of cooperation" as is the norm in many social dilemma papers, we report results illustrating the evolutionary trajectory of the risk aversion trait across the population, and indirectly the average investment level.

The value of χ was initialised randomly from a uniform distribution at the beginning of a trial. The value of the maximum investment I_{max} and payoff multiplier τ were 5.0 and 2.0 respectively. In the reproduction stage, the mutation rate μ was 0.01. The random migration rate λ was set to 0.05. In conditional migration, the threshold value used to determine whether an agent attempts to switch groups was simply the payoff value in the current round – a loss corresponding to a low-quality social environments.

Due to space constraints we limit the presentation of results to one scenario: Figure 1 plots the average value of χ, calculated over the final 2500 generations of a simulation trial, for different group sizes (n) when the number of groups was fixed ($g = 32$). Based on Wilcoxon rank sum test, there are significant differences bewteen each of the migration modes consider ($p < 0.01$) for larger group sizes. When the group size is smaller ($n = 2$ and $n = 4$) the differences are not as clear cut. This is to be expected as the game is relatively easy (event though there is still a social dilemma) under this condition. It is important to note that as the size of the group increases there is transition to higher average χ values.

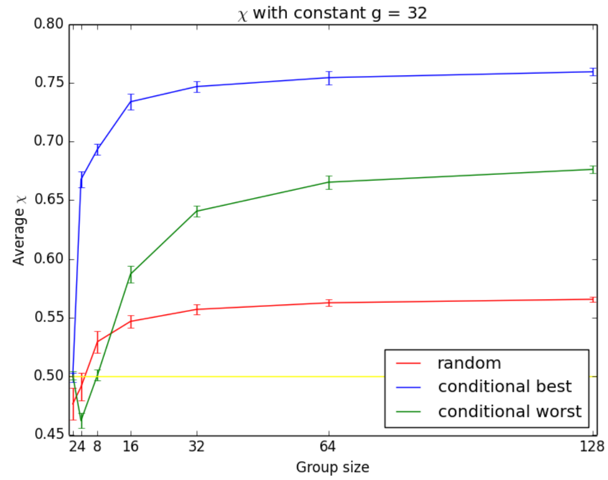

Figure 1: The average strategy χ across the population for varying groups sizes calculated over the last 2500 generations for each of the migration model. The yellow line indicates the expected value of 0.5 for unbiased evolution; i.e. no risk bias/preference.

4. CONCLUDING COMMENTS

In sum, we have studied the evolution of risk aversion in a group structured pubic goods game. An individual's investment strategy was based on a real-value genetic trait corresponding to their level of risk aversion. Although risky, the opportunity to change group membership may be the preferred action to take if a satisfactory level of performance (payoff or return on investment) was not achieved in the current location. Simulation experiments show that aligning collective and individual benefits is challenging. Conditional movement was shown to be an important driver in the evolution of risk averse traits since it enables individuals to respond to local conditions and to take advantage of potentially more beneficial social environments.

5. REFERENCES

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