Combinatorial Optimization with Differential Evolution: a Set-Based Approach

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ABSTRACT

This work presents a differential evolution algorithm for combinatorial optimization, in which a set-based representation and operators define subproblems that are used to explore the search space. The proposed method is tested on the capacitated centered clustering problem.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*heuristic methods*; G.2.1 [**Discrete Mathematics**]: Combinatorics

General Terms

Algorithms

Keywords

Differential evolution, heuristics, combinatorial optimization

1. INTRODUCTION

Specific heuristics and metaheuristics are found in the literature to solve a wide variety of combinatorial optimization problems, as many of these problems are known to be $\mathcal{NP}$-hard. In the last two decades, the differential evolution (DE) [10] metaheuristic has drawn attention due to its simplicity and efficiency in solving continuous optimization tasks. These features have inspired attempts at adapting the DE for the solution of combinatorial optimization problems [6, 7], but in general results have not met expectations. In most cases, adaptations have been found to perform little more than a random search in the space of solutions [9].

We have recently reported preliminary results for a set-based representation and specific operators to effectively adapt the DE structure for combinatorial optimization [3]. In this work we extend those results by using specific heuristics to solve the subproblems defined by the proposed DE on instances of the capacitated centered clustering problem.

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2. PROPOSED METHOD

In combinatorial optimization one is interested in choosing a subset of elements from a given set of possibilities such that some objective is minimized, subject to possible constraints. Since combinatorial optimization consists mainly of exploring a search space that can be described by a set, if follows that sets can be regarded as a natural encoding for these problems. In the proposed method we employ set-based representations to encode candidate solutions, e.g., [8] for graph-based problems. To operate in this set-based encoding, the mutation and crossover operators are redefined as set operations:

$$\mathbb{V}_{i,g} = \mathbb{X}_r \cup \psi(\mathbb{X}_{r_1,g} \oplus \mathbb{X}_{r_2,g}) \tag{1}$$

$$\mathbb{U}_{i,g} = \mathbb{V}_{i,g} \cup \mathbb{X}_{i,g} \tag{2}$$

where $\mathbb{X}_r$ is a randomly generated candidate solution, and $r_1 \neq r_2 \neq i \in [1, n_p]$ are random integers. The sum and subtraction operations of the original DE are replaced by union and exclusive-or logical operations. The mutation operator works like the original one, that is, adding the difference between two solutions to a third one. A randomly generated base $\mathbb{X}_r$ is used to introduce diversity. The scalar weighting factor $\psi \in (0, 1]$ can be used to control the size of the resulting set by defining a probability of maintenance of a given element from $(\mathbb{X}_{r_1,g} \oplus \mathbb{X}_{r_2,g})$ in $\mathbb{V}_{i,g}$, as suggested in [7].

After mutation, the recombination step creates a subproblem by the union of $\mathbb{V}_{i,g}$ with the target solution $\mathbb{X}_{i,g}$. This subproblem is defined as a limited subset of the original search space that includes the target solution, and is much smaller than the complete search domain. Problem-specific local search methods are then used to solve this subproblem.

The proposed adaptation employs the DE structure to define subproblems containing elements that differ across the solutions. The most promising elements that compose good candidate solutions will tend to remain in the population, which will eventually converge in a manner similar to that of the original DE [3]. The remaining aspects of the original DE, such as selection and stopping criteria, are left unchanged. This approach can be adapted to any combinatorial problem, with two steps required for this adaptation: the representation of the problem domain as a set, and the definition of the local search movements to be used in the solution of the subproblems.

3. EXAMPLE OF APPLICATION

In this work we applied the proposed method to the capacitated centered clustering problem (CCCP)[1], which is an $\mathcal{NP}$-hard problem [5] with a non-linear objective function. As this problem can be seen as a bipartite graph with the edges connecting k_1 nodes to k_2 clusters, candidate solutions were represented as sets of edges (i, j), with each edge indicating the allocation of node i to cluster j. The movement used for the local search within the subproblems is node reallocation to another cluster.

Besides the proposed method, three other approaches were tested to provide a comparison baseline: a branch-and-bound approach that applies a nonlinear solver to successive subproblems (Gurobi) [1]; a heuristic based on the iterated local search (ILS) method [2]; and a multi-start local search from random initial points (MultiStart). The full description of the experimental setup can be found in [4].

The significance tests performed for comparing mean performance of the algorithms on both sets of problems yielded significant results at the 95% confidence level, suggesting differences among the average performance of the algorithms. The estimates of mean performance of the methods, both overall and for each problem, are shown in Figure 1.

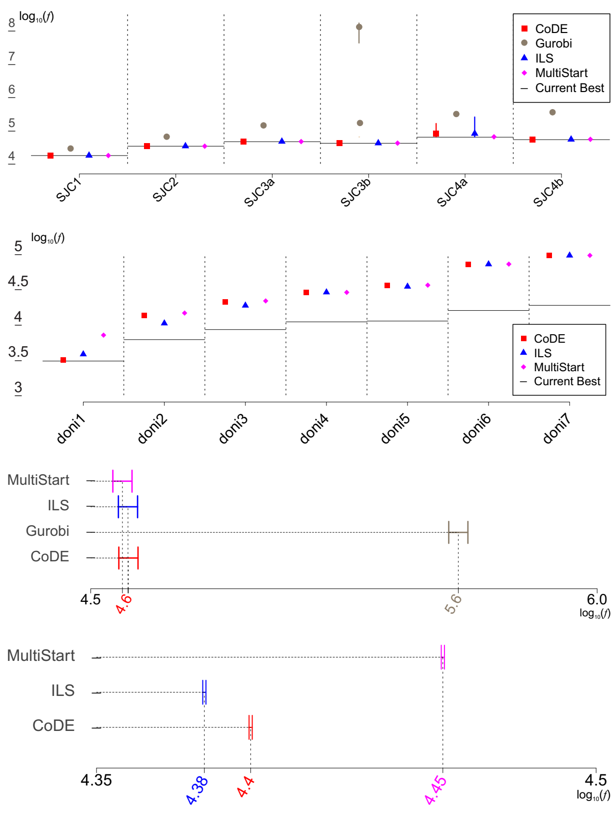

Figure 1: Top: Mean performance of the algorithms on all instances. Vertical bars represent 95% confidence intervals. Bottom: Confidence intervals on global performance of the algorithms on the SJC and Doni instances, respectively.

[1]We used six problems from the SJC family (100 - 402 nodes) and seven from the Doni family (1000 - 13221 nodes) for the tests. For more details, see [4].

4. CONCLUSIONS

A set-based version of the DE algorithm was proposed for the solution of combinatorial optimization problems. This method uses the DE structure and set-based operations to define subproblems which are then solved using local search methods, which can operate in reduced subspaces. An example of application using the Capacitated Centered Clustering Problem as a testbed was resented. The experimental comparison showed that the set-based DE was able to return competitive solutions when compared to other widely used approaches. More tests, particularly in much larger instances, are still necessary to evaluate the applicability of the proposed method.

5. ACKNOWLEDGMENTS

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