Non-dominated Sorting Differential Evolution with Improved Directional Convergence and Spread for Multiobjective Optimization

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ABSTRACT

A non-dominated sorting differential evolution algorithm with improved directional convergence and spread (NSDE-IDCS) is developed. Taking advantage of differential evolution, searching direction for a dominated solution is determined by its nearest non-dominated neighbor, while searching direction for a non-dominated solution is determined by other two non-dominated solutions. A simplex local search operator with an adaptive search probability is embedded to further exploit the neighborhood of non-dominated solutions.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*

Keywords

multiobjective optimization, differential evolution, directional information, simplex local search

1. INTRODUCTION

Differential evolution (DE) is extensively used in multiobjective evolutionary algorithm (MOEA) as a population based global optimization algorithm [3]. To the best of the authors' knowledge, most of existing DE based MOEAs only employ the classical DE as a search operator. However, little improvement in the searching direction has been made to DE with respect to the characteristic of multiobjective optimization problems (MOPs). In [1], the non-dominated sorting differential evolution - directional convergence and spread (NSDE-DCS) is proposed, in which the directional information is embedded into DE to speed up the searching process and to improve the searching spread. In this paper, we study on constructing more efficient convergence direction and spread direction for DE by using the directional information contained in non-dominated solutions. The proposed MOEA is named as non-dominated sorting differential evolution - improved directional convergence and spread (NSDE-IDCS). Inspired by [2], simplex search is employed as a local search operator to refine non-dominated solutions in NSDE-IDCS.

2. ALGORITHM

2.1 DE with Improved Directional Information

First we introduce NSDE-DCS briefly. NSDE-DCS [1] is illustrated in Figure 1(a) and Figure 1(c). NSDE-DCS replaces the crossover and mutation operation in non-dominated sorting genetic algorithm-II (NSGA-II) with a DE operator incorporating directional information. The DE operator is $u_i = x_i + K(x_{r_3} - x_i) + F(x_{r_1} - x_{r_2})$ for dominated solutions and $u_i = x_i + F(x_{r_1} - x_{r_2})$ for non-dominated solutions, where x_i is the ith solution in the current generation, u_i is the offspring of x_i, solution x_{r_3} has a lower Pareto rank than x_i, solutions x_{r_1} and x_{r_2} have a same rank, K and F are control parameters. $x_{r_3} - x_i$ and $x_{r_1} - x_{r_2}$ are convergence direction and spread direction, respectively. Next we discuss how to construct a more efficient direction.

For dominated solutions, it is more important to converge to the Pareto-optimal set. So in the NSDE-IDCS, the spread direction for a dominated solution is omitted. On the other hand, in NSDE-IDCS, a dominated solution takes *the nearest non-dominated solution that dominates itself* to construct a searching direction, as illustrated in Figure 1(b), where $u_i = x_i + K(x_{r_3} - x_i)$. The searching step is relatively larger when using the nearest non-dominated solution. It implies that NSDE-IDCS has a faster convergence speed.

For non-dominated solutions, it is more important to distribute uniformly and to exploit new Pareto-optimal solutions. However, x_{r_1} and x_{r_2} in NSDE-DCS might be dominated solutions and mislead u_i to a dominated region, as illustrated in Figure 1(c). So in NSDE-IDCS, *two other non-dominated solutions* are selected to construct a searching direction. Searching along this direction may be of higher probability to generate a new solution toward the edge of Pareto-optimal set or to make the population more uniformly distributed, as shown in Figure 1(d), where $u_i = x_i + F(x_{r_1} - x_{r_2})$.

2.2 Simplex Local Search Operator

For a dominated solution in NSDE-IDCS, evolving toward its nearest non-dominated solution is enough to improve convergence. Hence, the simplex local search operator is only performed to refine non-dominated solutions. A schematic of the simplex local search is illustrated in Figure 2(a), where x_1, x_2 and x_3 are three nearby non-dominated solutions. Let $x_4 = (x_1 + x_2)/2$, then $d = x_4 - x_3$ might be a potential

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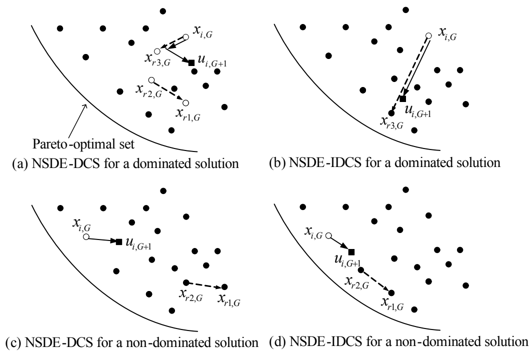

Figure 1: Evolution mechanism (in objective space)

searching direction for x_3. Let $x_5 = x_3 + 2d$, $x_6 = x_3 + 0.5d$, $x_7 = x_3 - d$, $x_8 = x_3 - 2d$, $x_9 = x_3 - 0.5d$. Then the flowchart of the simplex local search for x_3 is illustrated in Figure 2(b).An adaptive search probability for the local search is designed as $p_{SLS} = (n/N) \cdot (t/T)$, where p_{SLS} is the probability to decide whether to perform the simplex local search, n and N are the number of non-dominated solutions and the population size, respectively, and t and T are the current generation and the total generations, respectively. The probability p_{SLS} increases as n or t increases. In this way, the simplex local search can exploit non-dominated solutions with a high efficiency while avoiding prematurity.

3. NUMERICAL EXPERIMENT

Four rotated MOPs [1] are used in the experiment. NSGA-II and NSDE-DCS are introduced as comparative algorithms. Parameter settings of NSGA-II and NSDE-DCS can be found in [1]. For NSDE-IDCS, the control parameters are $F = 0.8$ and $K = 0.4$, which is the same as NSDE-DCS.

Figure 3 (a)-(d) show the convergence performance, where a smaller $GD(Q, P^*)$ indicates a better convergence [1]. It can be seen that NSDE-IDCS has a significantly faster convergence speed, which is a consequence of dominated solutions' evolving toward the nearest non-dominated solution. The spread performance is shown in Figure 3 (e)-(h), where a smaller $GD(P^*, Q)$ indicates a better spread [1]. Figure 3 (e)-(h) show that NSDE-IDCS can maintain enough diversity during searching, as a result of non-dominated solutions' evolving along the Pareto front.

4. DISCUSSION AND CONCLUSION

In single-objective optimization problems, algorithms must restrict their preference for the current best solution to avoid prematurity and to guarantee global exploration ability. In NSDE-IDCS, current best solutions, namely non-dominated solutions, are used to generate searching direction. However, the algorithm is not trapped into local solutions. The inner mechanism is that the best solution in MOP, is not a single solution, but a non-dominated solution set composed of several solutions. Since the population is initialized randomly, the non-dominated solution set has a relatively uniform distribution and a good diversity. Besides, as the evolution proceeds, the number of non-dominated solutions increases, and the diversity is improved continuously. This mechanism guarantees that NSDE-IDCS will not be trapped into local optima. Experiment results verify that NSDE-IDCS can converge fast while maintaining a good spread performance.

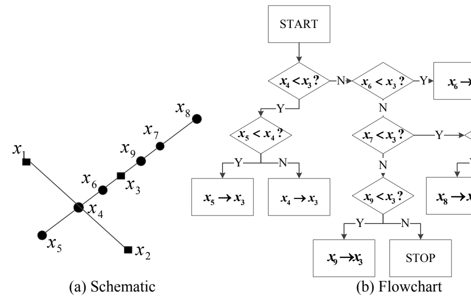

Figure 2: Simplex local search

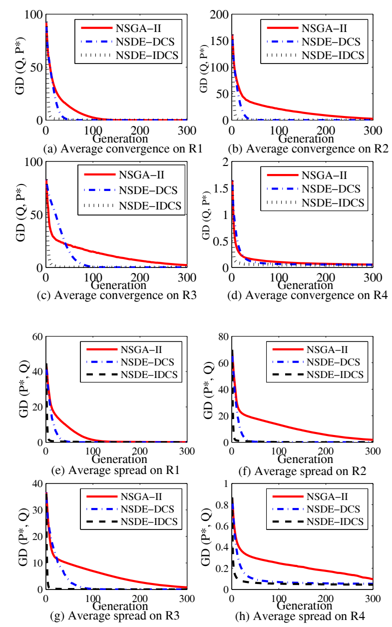

Figure 3: Convergence and spread over 50 runs

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5. REFERENCES

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