Meta-Level Multi-Objective Formulations of Set Optimization for Multi-Objective Optimization Problems: Multi-Reference Point Approach to Hypervolume Maximization

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ABSTRACT
Hypervolume has been frequently used as an indicator to evaluate a solution set in indicator-based evolutionary algorithms (IBEAs). One important issue in such an IBEA is the choice of a reference point. A different solution set is often obtained from a different reference point since the hypervolume calculation depends on the location of the reference point. In this paper, we propose an idea of utilizing this dependency to formulate a meta-level multi-objective set optimization problem. Hypervolume maximization for a different reference point is used as a different objective.

Categories and Subject Descriptors
I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *Heuristic Methods*

General Terms
Algorithms.

Keywords
Evolutionary multiobjective optimization, hypervolume, solution set optimization, indicator-based evolutionary algorithms.

1. INTRODUCTION
Evolutionary multi-objective optimization (EMO) has been a very active research area in the last decade. Pareto dominance-based EMO algorithms such as NSGA-II [4] and SPEA [8] have been almost always the mainstream in the EMO community. Recently, indicator-based evolutionary algorithms (IBEAs) such as SMS-EMOA [3] and HypE [2] have attracted increasing attention. The main characteristic feature of IBEAs is the handling of multi-objective optimization as single-objective set optimization where an indicator is used to evaluate a solution set. Hypervolume has been frequently used in IBEAs for solution set evaluation [2], [3]. In this case, a multi-objective optimization problem is handled as a hypervolume maximization problem.

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One issue to be addressed in the use of a hypervolume indicator is the dependency of the hypervolume calculation on the choice of a reference point [1]. A different solution set is obtained from a different reference point through hypervolume maximization. In this paper, we propose an idea of utilizing this dependency to formulate a meta-level multi-objective set optimization problem. An original multi-objective problem to be solved is reformulated as a multi-objective set optimization problem. Each objective is the hypervolume maximization for a different reference point. Thus the number of objectives is the same as the number of different reference points for hypervolume maximization.

2. SINGLE-OBJECTIVE FORMULATION
Let us consider the following k-objective maximization problem:

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ ...,\ f_k(\mathbf{x}))\,, \tag{1}$$

$$\text{subject to } \mathbf{x} \in \mathbf{X}\,. \tag{2}$$

We denote a solution set by S, which is an arbitrary subset of $\mathbf{X}$ in (2). The solution set S is evaluated by an indicator $I(S)$. In this paper, we use hypervolume as $I(S)$. The maximization of $I(S)$ by an indicator-based EMO algorithm with a constraint condition on the size of S can be formulated as follows [1], [5], [6]:

[Single-Objective Hypervolume Maximization]

$$\text{Maximize } I(S)\,, \tag{3}$$

$$\text{subject to } S \subset \mathbf{X} \text{ and } |S| \leq N\,, \tag{4}$$

where $|S|$ is the number of solutions in S (i.e., the cardinality of the solution set S), and N is its upper bound. The inequality condition in (4) can be replaced with $|S| = N$.

In [5], [6], this formulation was generalized as follows:

[Hypervolume Maximization and Cardinality Minimization]

$$\text{Maximize } I(S) \text{ and minimize } |S|\,, \tag{5}$$

$$\text{subject to } S \subset \mathbf{X}\,. \tag{6}$$

3. MULTI-OBJECTIVE FORMULATION
Let us assume that we have M reference points R_p, p = 1, 2, ..., M. Our problem is hypervolume maximization for those reference points. Let us denote the hypervolume $I(S)$ for the p-th reference point R_p by $I(S, R_p)$. This is the hypervolume of the solution set S calculated for the reference point R_p. In this case, the single-objective hypervolume maximization problem in (3)-(4) can be generalized to the following M-objective problem:

[Multi-Objective Hypervolume Maximization]

Maximize $I(S, R_1), I(S, R_2), ..., I(S, R_M)$, (7)

subject to $S \subset \mathbf{X}$ and $|S| \leq N$. (8)

4. COMPUTATIONAL EXPERIMENTS

Due to page limitation, we report experimental results without explaining their settings in detail. As a test problem, we used a three-objective 500-item knapsack problem (3-500 problem). First, we tried to search for a large number of Pareto optimal solutions by applying MOEA/D [7] with an unbounded archive population to the the 3-500 problem. From ten runs of MOEA/D with the population size 10011 and the 1000 generations, we obtained 31509 non-dominated solutions in Fig. 1. All of those solutions were used as candidate solutions. That is, **X** in (8) was the set of the 31509 candidate solutions, and S was its subset. S was coded by a binary string of length 31509.

In the meta-level multi-objective set optimization problem in (7), we used two reference points: (0, 0, 0) and (17000, 17000, 17000). We used NSGA-II to search for Pareto optimal solution sets of the two-objective hypervolume maximization problem. Fig. 2 shows the obtained non-dominated solution sets. Each point in Fig. 2 is a set of candidate solutions as shown in Fig. 3.

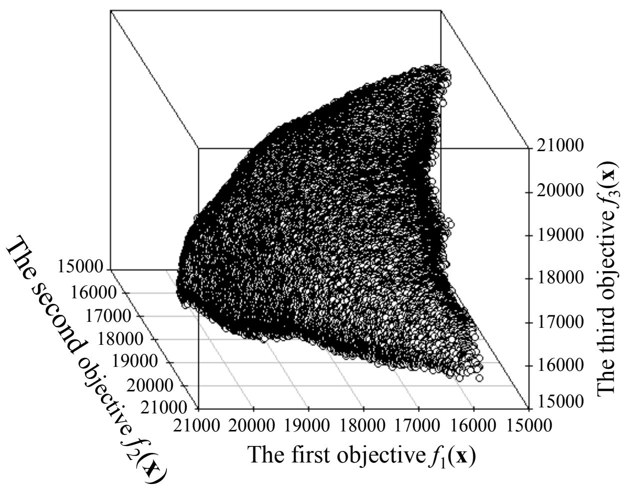

Figure 1. Obtained solutions of the 3-500 problem.

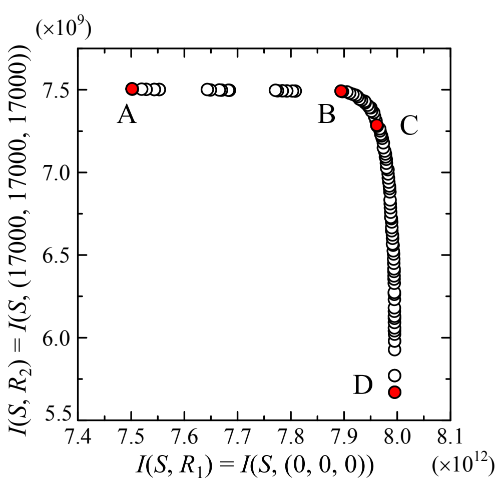

Figure 2. Obtained non-dominated solution sets.

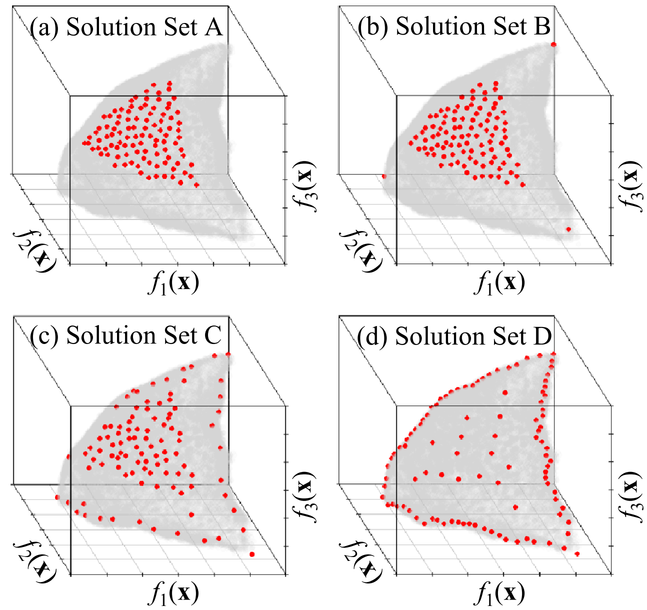

Figure 3. Solution sets corresponding to A-D in Fig. 2.

5. CONCLUDING REMARKS

We proposed an idea of a meta-level multi-objective formulation of set optimization using multiple reference points. The proposed formulation is explained through computational experiments. As shown in Fig. 3, a number of solution sets are obtained from our approach. Our approach can be used for solution selection (i.e., to choose a small number of solutions to be presented to the decision maker from a large number of obtained non-dominated solutions).

6. REFERENCES

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