Evolutionary Many-Objective Optimization Using Preference on Hyperplane

Kaname Narukawa
Honda Research Institute Europe GmbH
Carl-Legien-Straße 30,
63073 Offenbach am Main,
Germany
kaname.narukawa@honda-ri.de

Yuki Tanigaki
Osaka Prefecture University
1-1 Gakuen-cho, Nakaku,
Sakai, Osaka 599-8531,
Japan
yuki.tanigaki@ci.cs.osakafu-u.ac.jp

Hisao Ishibuchi
Osaka Prefecture University
1-1 Gakuen-cho, Nakaku,
Sakai, Osaka 599-8531,
Japan
hisaoi@cs.osakafu-u.ac.jp

ABSTRACT

This paper proposes to represent the preference of a decision maker by Gaussian functions on a hyperplane. The preference is used to evaluate non-dominated solutions as a second criterion instead of the crowding distance in NSGA-II. High performance of our proposal is demonstrated for many-objective DTLZ problems.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic Methods*

General Terms

Algorithms

Keywords

Preference, hyperplane, many-objective optimization, evolutionary multi-objective optimization (EMO)

1. INTRODUCTION

As reviewed in [4], evolutionary multi-objective optimization (EMO) algorithms often degrade their search ability for optimization problems with more than three objectives which we refer to as many-objective problems. That is, selection pressure generated by the Pareto dominance relation will be lost in many-objective problems, resulting in less convergence to the Pareto front. Furthermore, it will become exponentially difficult to approximate the Pareto front at the same quality by a finite number of solutions with an increase in the number of objectives.

One of remedies to ease these difficulties is to incorporate the preference of a decision maker (DM) into EMO algorithms [1]. Whereas a number of preference-based EMO algorithms have been proposed, it is not trivial for the DM to specify his/her preference in the algorithms.

This paper proposes a preference-based EMO algorithm using Gaussian functions on a hyperplane in which the DM can easily specify his/her preference. The capability of our proposal to handle many objectives is examined through computational experiments on DTLZ problems [3].

Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage, and that copies bear this notice and the full citation on the first page. Copyrights for third-party components of this work must be honored. For all other uses, contact the owner/author(s). Copyright is held by the author/owner(s).
GECCO'14, July 12–16, 2014, Vancouver, BC, Canada.
ACM 978-1-4503-2881-4/14/07.
http://dx.doi.org/10.1145/2598394.2598420.

2. PREFERENCE REPRESENTATION

In order to calculate how much the DM prefers a solution $\mathbf{x}$, we first obtain a normalized m-objective vector $\mathbf{f}'(\mathbf{x}) = (f'_1, f'_2, ..., f'_m)$ for $\mathbf{f}(\mathbf{x}) = (f_1, f_2, ..., f_m)$ as follows:

$$f'_k = \frac{f_k - f_k^{\min}}{f_k^{\mathrm{ext}} - f_k^{\min}},\ k = 1, 2, ..., m, \tag{1}$$

where $f_k^{\min}$ is the minimum value for the k-th objective, f_k^{ext} is the k-th objective value of an extreme solution $\mathbf{x}_k^{\mathrm{ext}}$ for the k-th objective. The extreme solution $\mathbf{x}_k^{\mathrm{ext}}$ is identified by finding a solution that minimizes the following achievement scalarizing function ASF with a weight vector $\mathbf{d}_k$ indicating the k-axis direction (e.g., $\mathbf{d}_1 = (1, 0, ..., 0)$);

$$ASF(\mathbf{x}, \mathbf{d}_k) = \max_{i=1}^{m} f'_i(\mathbf{x})/d_{k,i}, \tag{2}$$

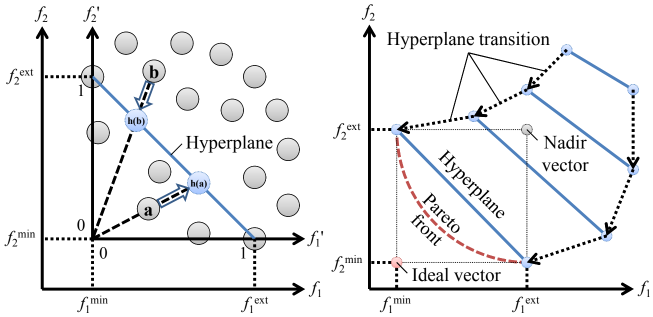
where $d_{k,i} = 0$ is replaced with $d_{k,i} = 10^{-6}$ for avoiding a division by zero. In every generation, $f_k^{\min}$ and $\mathbf{x}_k^{\mathrm{ext}}$ will be updated for $k = 1, 2, ..., m$ by a set of solutions from parent and offspring populations. The extreme values $f_1^{\mathrm{ext}}, f_2^{\mathrm{ext}}, ..., f_m^{\mathrm{ext}}$ are used to define a hyperplane, and $\mathbf{f}'$ can be mapped onto that hyperplane as $\mathbf{h}(\mathbf{f}') = (h_1, h_2, ..., h_m)$, where

$$h_i = \frac{f'_i}{f'_1 + f'_2 + ... + f'_m},\ i = 1, 2, ..., m, \tag{3}$$

and $h_1 + h_2 + ... + h_m = 1$ holds. Fig. 1(a) shows how normalized objective vectors $\mathbf{a}$ and $\mathbf{b}$ are mapped onto the hyperplane for two-objective minimization. How much the DM prefers the solution can now be calculated as follows:

$$\begin{aligned} p(\mathbf{h}) &= \max_{i=1}^{q} p_i(\mathbf{h}), \\ p_i(\mathbf{h}) &= \exp\{-\sum_{l=1}^{m} \frac{(h_l - w_l^i)^2}{(s_l^i)^2}\},\ i = 1, 2, ..., q, \end{aligned} \tag{4}$$

where w_l^i is the center (or the mean) and s_l^i is the spread (or the standard deviation) of a Gaussian function. It should be



(a) Mapping to a hyperplane (b) Hyperplane transition

Figure 1: Preference on a hyperplane

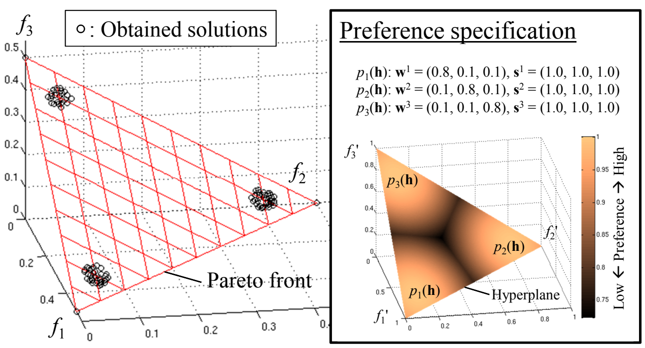


Figure 2: Solutions obtained at the 1000th generation by P-NSGA-II for three-objective DTLZ1

noted that q sets of m Gaussian functions are used to represent the preference in Eq. (4). In this paper, we assume that the center vector $\mathbf{w}^i$ and the spread vector $\mathbf{s}^i$ have already been specified by the DM under the following condition;

$$\begin{aligned}\mathbf{w}^i &= (w_1^i, w_2^i, ..., w_m^i),\ w_1^i + w_2^i + ... + w_m^i = 1,\\ \mathbf{s}^i &= (s_1^i, s_2^i, ..., s_m^i),\\ & \quad w_j^i \geq 0 \text{ and } s_j^i > 0 \text{ for } j = 1, 2, ..., m. \qquad (5)\end{aligned}$$

The preference function of Eq. (4) is used as a second criterion to compare solutions instead of the crowding distance in NSGA-II [2]. Furthermore, in order to keep diversity of solutions, we consider the minimum distance $d_{\min}$ between solutions in the objective space. In this paper, the smallest preference function value (i.e., 0) is assigned to each solution that has a distance of $d_{\min} = 0.01$ or less to its nearest neighbor in the objective space. We also modified the parent selection mechanism of NSGA-II from tournament selection to random selection.

3. COMPUTATIONAL EXPERIMENTS

The solutions obtained by our preference-based NSGA-II (P-NSGA-II) for three-objective DTLZ1 are illustrated in Fig. 2 with three sets of Gaussian functions. Next, we calculate relative hypervolume and convergence measures [5] to evaluate solutions obtained by NSGA-II and P-NSGA-II on DTLZ problems. In P-NSGA-II, $\mathbf{w} = (1/m, 1/m, ..., 1/m)$ and $\mathbf{s} = (1.0, 1.0, ..., 1.0)$ are used for a set of Gaussian functions where m is the number of objectives. The average results over 20 runs are summarized in Table 1. From the

Table 1: Results of computational experiments

Number of objectives	Applied problem	Relative hypervolume		Convergence	
		NSGA-II	P-NSGA-II	NSGA-II	P-NSGA-II
3	DTLZ1	**0.97321**	0.83277	0.13231	**0.00040006**
	DTLZ2	**0.87082**	0.60327	0.0086835	**0.0035105**
	DTLZ3	**0.87201**	0.5864	0.039698	**0.0061963**
	DTLZ4	**0.87349**	0.58821	0.0083337	**0.0022749**
6	DTLZ1	0	**0.88211**	398.89	**0.004947**
	DTLZ2	0.06667	**0.41874**	1.6627	**0.0082517**
	DTLZ3	0	**0.32018**	1105.5	**0.068373**
	DTLZ4	0	**0.44472**	2.2595	**0.0064341**
9	DTLZ1	0	**0.85317**	475.75	**0.0095242**
	DTLZ2	0.01095	**0.40863**	2.1514	**0.012198**
	DTLZ3	0	**0.24631**	1714.2	**0.12129**
	DTLZ4	0	**0.49332**	2.4821	**0.013926**
12	DTLZ1	0	**0.84468**	474.84	**0.010666**
	DTLZ2	0.012874	**0.39198**	2.1938	**0.01487**
	DTLZ3	0	**0.21141**	1819.7	**3.254**
	DTLZ4	0	**0.55092**	2.497	**0.024316**

results, it can be seen that the proposed modifications considerably improve the performance of NSGA-II for many-objective problems.

4. CONCLUSIONS

Preference-based NSGA-II using Gaussian functions on a hyperplane was proposed. Its performance was demonstrated for many-objective DTLZ problems. In our proposal, a DM can easily specify his/her preference by the center and spread vectors of Gaussian functions on the hyperplane. It should be noted that the hyperplane automatically moves toward the Pareto front together with the population of solutions in our modified NSGA-II (see Fig. 1(b)). The DM hopefully obtains solutions close to the Pareto front around regions preferred by him/her.

5. REFERENCES

[1] J. Branke, K. Deb, K. Miettinen, and R. Slowinski. *Multiobjective optimization: Interactive and evolutionary approaches*. Springer, 2008.

[2] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans. on Evolutionary Computation*, 6(2):182–197, 2002.

[3] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. Scalable test problems for evolutionary multi-objective optimization. *KanGAL Report*, no. 2001001. Indian Institute of Technology Kanpur, 2001.

[4] H. Ishibuchi, N. Tsukamoto, and Y. Nojima. Evolutionary many-objective optimization: A short review. *Proc. of Congress on Evolutionary Computation: CEC 2008*, pages 2419–2426. IEEE, 2008.

[5] T. Wagner, N. Beume, and B. Naujoks. Pareto-, aggregation-, and indicator-based methods in many-objective optimization. *Evolutionary Multi-Criterion Optimization: EMO 2007*, volume 4403 of *Lecture Notes in Computer Science*, pages 742–756. Springer, 2007.