MOEA/D with A Delaunay Triangulation Based Weight Adjustment

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ABSTRACT

MOEA/D decomposes a multi-objective optimization problem (MOP) into a set of scalar sub-problems with evenly spread weight vectors. Recent studies have shown that the fixed weight vectors used in MOEA/D might not be able to cover the whole Pareto front (PF) very well. Due to this, we developed an adaptive weight adjustment method in our previous work by removing subproblems from the crowded parts of the PF and adding new ones into the sparse parts. Although it performs well, we found that the sparse measurement of a subproblem which is determined by the m-nearest (m is the dimensional of the object space) neighbors of its solution can be more appropriately defined. In this work, the neighborhood relationship between subproblems is defined by using Delaunay triangulation (DT) of the points in the population.

Categories and Subject Descriptors

G.1.6 [**Optimization**]

General Terms

Algorithms

Keywords

Evolutionary multi-objective optimization, Decomposition, Delaunay triangulation, Weight Adjustment.

1. INTRODUCTION

In recent years, the multi-objective evolutionary algorithm based on decomposition (MOEA/D) has achieved a great success and became a popular algorithmic framework of MOEA [4]. One of the important issues of MOEA/D, which is the focus of this work, is the determination of weight vectors. The original MOEA/D employs a fixed set of evenly spaced weight vectors and assumes that the uniformity of weight vectors will naturally lead to evenly spread Pareto optimal solutions. However, the basic assumption of MOEA/D might be violated when solving MOPs with irregularly shaped PFs rather than PFs close to the hyperplane $f_1 + f_2 + \quad + f_m = 1$ ($f_1, ..., f_m$ are the m objective function values) in the objective space [2].

In our previous work, we developed an adaptive weight adjust-ment method to improve the performance of MOEA/D on MOPs with complex PFs. Experimental results have shown that our previous method (AWA) [3] can significantly improve the performance of MOEA/D on MOPs with irregularly shaped or discontinuous PFs. The basic idea of the AWA method is to remove sub-problems from the crowded parts of the PF and add new ones into the sparse parts. Therefore, the sparse measurement of a sub-problem is essential to the efficiency of the newly developed AWA method. In our original AWA method, the sparse measurement of a subproblem is determined by the m-nearest neighbors of its solution, in which m is the dimension of the objective space. However, the sparse measurement may not be accurate when the solution of a sub-problem has several close neighbors within the objective space in similar directions.

In this work, the neighborhood relationship between subproblems is defined by using Delaunay triangulation (DT) of the points in the population rather than by the Euclidean distance between solutions only. Then, a new sparse measurement of a sub-problem is determined by its neighbors in the DT net.

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2. PROPOSED METHOD

In our previous work, the AWA method removes subproblems from the crowded parts and adds new ones into the sparse parts. But we took only Euclidian distance based on weight vectors into consideration, and we did not consider the direction of the solution of a sub-problem. Now we hope we can design one method which combines the neighborhood-based ideology and distance-based ideology.

The triangles established by Delaunay triangulation are closest to equilateral triangles. Assuming that all triangles in the triangulation net are equilateral triangles, then in this case we can obtain an absolutely evenly spaced PF. The reality is that we can promise the differences of edges in the Delaunay triangulation net are as small as possible. So we can obtain a relatively evenly spaced PF. Based on the Delaunay triangulation net of points, the sparse level of individual among population can be defined as:

$$V^i = (\prod_{j=1}^{n_i} L_2^{DT_j^i})^{\frac{1}{n_i}} \tag{1}$$

Where $L_2^{DT_j^i}$ is the Euclidean distance from the j-th solution to its i-th one-hop neighbor (i.e., only one edge is between the j-th and i-th solution in the Delaunay triangulation net of points in population). n_i is the number of one-hop neighbors of the j-th individual in the Delaunay triangulation net.

Then according to our previous work, now we can replace the sparse measurement with our newly developed measurement so as to describe the sparse level more accurately. The experimental study compares the proposed MOEA/D-DT with the original MOEA/D [4], our previous work MOEA/D-AWA and NSGA-II [1]. Three tri-objective problems with irregularly shaped PFs are used to verify the effectiveness of the proposed approach, they are the DTLZ7 problem, WFG1 and WFG2. Tables 1 and 2 show the mean and standard deviation of IGD and HV metrics values of the solutions obtained by each algorithm for the three testing problems. It can be seen from tables 1 and 2 that the solutions found by MOEA/D-DT are better than MOEA/D and MOEA/D-AWA on the three testing problems in terms of IGD metric.

Table 1: Statistic IGD metrics values of the founded solutions by the four compared algorithms. The numbers in parentheses indicate their standard deviation and square deviation. Bold italics mean better result.

IGD	MOEA/D-DT	MOEA/D	MOEA/D-AWA	NSGA-II
DTLZ7	**4.1839e-2**	8.7534e-2	4.8099e-2	5.1691e-2
	(8.5934e-4)	(2.6830e-3)	(2.4240e-3)	(8.0104e-3)
WFG1	**7.5727e-2**	1.1652e-1	8.1258e-2	1.3716e-1
	(2.5006e-3)	(1.4964e-3)	(7.8404e-4)	(3.3727e-3)
WFG2	**1.4643e-1**	4.1251e-1	1.5957e-1	1.2428e-1
	(1.1538e-1)	(4.3690e-3)	(1.1595e-1)	(4.1300e-3)

Table 2: Statistic HV metrics values of the founded solutions by the four compared algorithms. The numbers in parentheses indicate their standard deviation and square deviation. Bold italics mean better result.

HV	MOEA/D-DT	MOEA/D	MOEA/D-AWA	NSGA-II
DTLZ7	1.9928e-1	1.7894e-1	1.9574e-1	**2.1022e-1**
	(3.9804e-4)	(1.3030e-3)	(4.4072e-4)	(8.2618e-4)
WFG1	**9.2073e-1**	9.0589e-1	9.2029e-1	8.8553e-1
	(5.6878e-6)	(8.1252e-6)	(1.4528e-5)	(9.5698e-6)
WFG2	8.7191e-1	7.0837e-1	8.6930e-1	**8.7733e-1**
	(5.0990e-3)	(1.1705e-5)	(5.4548e-3)	(1.6772e-5)

3. CONCLUSIONS

Following our previous research idea of adjusting weight vectors in MOEA/D, a Delaunay triangulation based sparse measurement of a sub-problem has been proposed to determine which sub-problem should be removed and which part of Pareto front needs more sub-problems. By using Delaunay triangulation net of the points in the population, a new neighborhood relationship between subproblems is defined. The sparse measurement of a subproblem is then determined by its neighbors in the Delaunay triangulation net. With the advantage of considering both distance between solutions and their distribution, the Delaunay triangulation based sparse measurement was expected to be more accurate than the Euclidean distances based sparse measurement when the solution of a sub-problem has several close neighbors within the objective space in similar directions. Experimental results have indicated that the newly developed method can obtain more uniformly scattered solutions than those found by the original MOEA/D and our previous work MOEA/D-AWA on tri-objective optimization problems with irregularly shaped Pareto fronts.

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