

# Expressive Genetic Programming

Tutorial

Genetic and Evolutionary Computation Conference

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## Instructor



**Lee Spector** is a Professor of Computer Science in the School of Cognitive Science at Hampshire College in Amherst, Massachusetts, and an adjunct professor in the Department of Computer Science at the University of Massachusetts, Amherst. He received a B.A. in Philosophy from Oberlin College in 1984 and a Ph.D. from the Department of Computer Science at the University of Maryland in 1992. His areas of teaching and research include genetic and evolutionary computation, quantum computation, and a variety of intersections between computer science, cognitive science, evolutionary biology, and the arts. He is the Editor-in-Chief of the journal *Genetic Programming and Evolvable Machines* (published by Springer) and a member of the editorial board of *Evolutionary Computation* (published by MIT Press). He is also a member of the SIGEVO executive committee and he was named a Fellow of the International Society for Genetic and Evolutionary Computation.

## Tutorial Description (1)

The language in which evolving programs are expressed can have significant impacts on the problem-solving capabilities of a genetic programming system. These impacts stem both from the absolute computational power of the languages that are used, as elucidated by formal language theory, and from the ease with which various computational structures can be produced by random code generation and by the action of genetic operators. Highly expressive languages can facilitate the evolution of programs for any computable function using, when appropriate, multiple data types, evolved subroutines, evolved control structures, evolved data structures, and evolved modular program and data architectures. In some cases expressive languages can even support the evolution of programs that express methods for their own reproduction and variation (and hence for the evolution of their offspring).

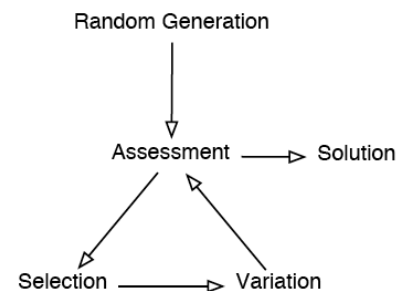
## Tutorial Description (2)

This tutorial will begin with a comparative survey of approaches to the evolution of programs in expressive programming languages ranging from machine code to graphical and grammatical representations. Within this context it will then provide a detailed introduction to the Push programming language, which was designed specifically for expressiveness and specifically for use in genetic programming systems. Push programs are syntactically unconstrained but can nonetheless make use of multiple data types and express arbitrary control structures, supporting the evolution of complex, modular programs in a particularly simple and flexible way. The Push language will be described and demonstrated, and ten years of Push-based research, including the production of human-competitive results, will be briefly surveyed. The tutorial will conclude with a discussion of recent enhancements to Push that are intended to support the evolution of complex and robust software systems.

# Course Agenda

- Genetic Programming
- Evolving programs in expressive languages
- Expressivity, evolvability, and syntactic minimality
- **Push + DEMO**
- Expressing the evolution of expressive evolution

# Evolutionary Computation



# Evolution, the Designer

“Darwinian evolution is itself a designer worthy of significant respect, if not religious devotion.” *Boston Globe* OpEd, Aug 29, 2005

WHAT WOULD DARWIN SAY? | LEE SPECTOR  
**And now, digital evolution**

The Boston Globe

By Lee Spector | August 29, 2005

RECENT developments in computer science provide new perspective on "intelligent design," the view that life's complexity could only have arisen through the hand of an intelligent designer. These developments show that complex and useful designs can indeed emerge from random Darwinian processes.

# Genetic Programming (GP)

- Evolutionary computing to produce executable computer programs
- Programs are assessed by executing them
- Automatic programming; producing software

## Program Representations

- Lisp-style symbolic expressions (Koza, ...).
- Purely functional/lambda expressions (Walsh, Yu, ...).
- Linear sequences of machine/byte code (Nordin et al., ...).
- Artificial assembly-like languages (Ray, Adami, ...).
- Stack-based languages (Perkis, Spector, Stoffel, Tchernev, ...).
- Graph-structured programs (Teller, Globus, ...).
- Object hierarchies (Bruce, Abbott, Schmutter, Lucas, ...).
- Fuzzy rule systems (Tunstel, Jamshidi, ...).
- Logic programs (Osborn, Charif, Lamas, Dubossarsky, ...).
- Strings, grammar-mapped to arbitrary languages (O'Neill, Ryan, ...).

## Mutating Lisp

```
(+ (* X Y)
   (+ 4 (- Z 23)))
```

```
(+ (* X Y)
   (+ 4 (- Z 23)))
```

```
(+ (- (+ 2 2) Z)
   (+ 4 (- Z 23)))
```

## Recombining Lisp

```
Parent 1: (+ (* X Y)
              (+ 4 (- Z 23)))
Parent 2: (- (* 17 (+ 2 X))
            (* (- (* 2 Z) 1)
              (+ 14 (/ Y X))))
```

```
Child 1: (+ (- (* 2 Z) 1)
            (+ 4 (- Z 23)))
Child 2: (- (* 17 (+ 2 X))
            (* (* X Y)
              (+ 14 (/ Y X))))
```

## Symbolic Regression

A simple example

Given a set of data points, evolve a program that produces  $y$  from  $x$ .

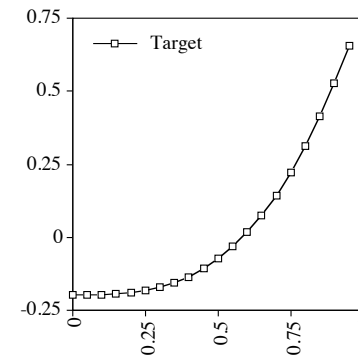
Primordial ooze: +, -, \*, %,  $x$ , 0.1

Fitness = error (smaller is better)

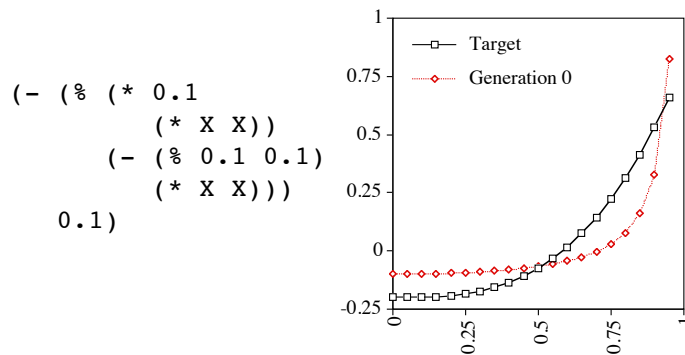
# GP Parameters

Maximum number of Generations: 51  
 Size of Population: 1000  
 Maximum depth of new individuals: 6  
 Maximum depth of new subtrees for mutants: 4  
 Maximum depth of individuals after crossover: 17  
 Fitness-proportionate reproduction fraction: 0.1  
 Crossover at any point fraction: 0.3  
 Crossover at function points fraction: 0.5  
 Selection method: FITNESS-PROPORTIONATE  
 Generation method: RAMPED-HALF-AND-HALF  
 Randomizer seed: 1.2

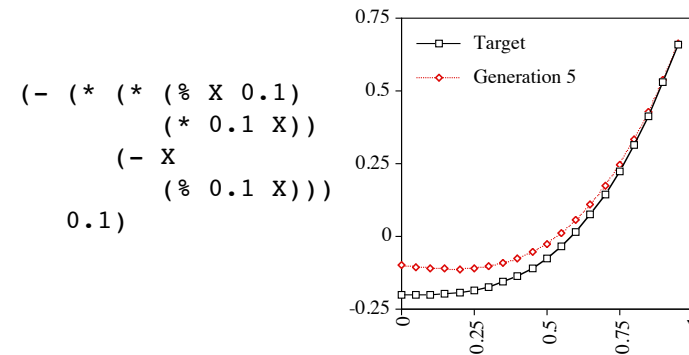
# Evolving $y = x^3 - 0.2$



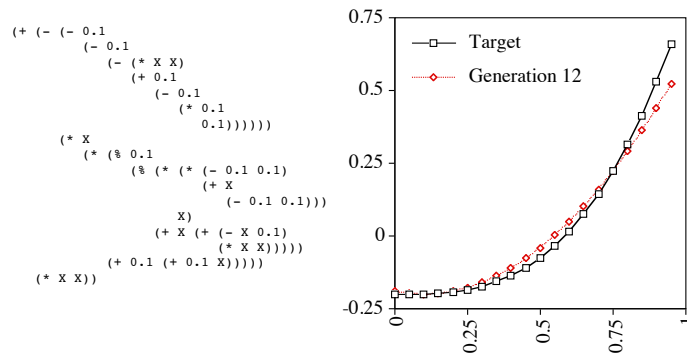
## Best Program, Gen 0



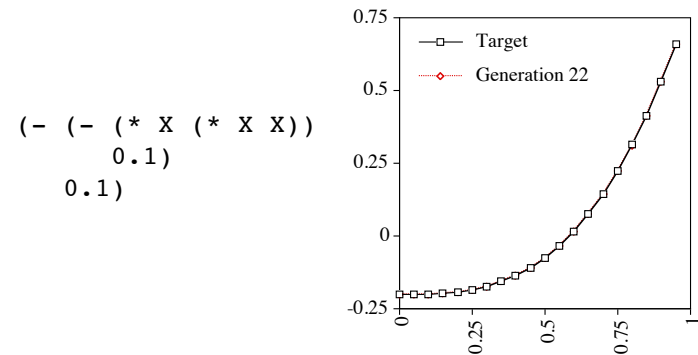
## Best Program, Gen 5



## Best Program, Gen 12



## Best Program, Gen 22



## Expressiveness

- Turing machine tables
- Lambda calculus expressions
- Partial recursive functions
- Register machine programs
- Assembly language programs
- etc.

## Evolvability

The fact that a computation *can* be expressed in a formalism does not imply that a correct expression can be produced in that formalism by a human programmer or by an evolutionary process.

## Data/Control Structure

- Data abstraction and organization

Data types, variables, name spaces, data structures, ...

- Control abstraction and organization

Conditionals, loops, modules, threads, ...

## Structure via GP (1)

- Specialize GP techniques to directly support human programming language abstractions
- Strongly typed genetic programming
- Module acquisition/encapsulation systems
- Automatically defined functions
- Automatically defined macros
- Architecture altering operations

## Evolving Modular Programs

With “automatically defined functions”

- All programs in the population have the same, pre-specified architecture
- Genetic operators respect that architecture
- Significant implementation costs
- Significant pre-specification
- Architecture-altering operations: more power and higher costs

## ADMs

- Macros implement control structures
- ADMs can be implemented via small tweaks to any system that supports ADFs
- Similar pros and cons to ADFs, but provide additional expressive power

## Control Structures (1)

Multiple evaluation

```
(defmacro do-twice (code)
  `(progn ,code ,code))

(do-twice (incf x))
```

## Control Structures (2)

Conditional evaluation

```
(defmacro numeric-if (exp neg zero pos)
  `(if (< ,exp 0)
      ,neg
      (if (< 0 ,exp) ,pos ,zero)))

(numeric-if (foo) (bar) (baz) (bix))
```

## Structure via GP (2)

- Specialize GP techniques to **indirectly** support human programming language abstractions
- Map from unstructured genomes to programs in languages that support abstraction (e.g. via grammars)

## Structure via GP (3)

- Evolve programs in a minimal-syntax language that nonetheless supports a full range of data and control abstractions
- For example: orchestrate data flows via stacks, not via syntax
- Minimal syntax + maximal semantics
- **Push**

## Push

- Stack-based postfix language with one stack per type
- Types include: integer, float, Boolean, name, **code**, **exec**, vector, matrix, quantum gate, [add more as needed]
- Missing argument? NOOP
- Minimal syntax:  
program  $\rightarrow$  instruction | literal | ( program\* )

## Why Push?

- Highly expressive: data types, data structures, variables, conditionals, loops, recursion, modules, ...
- Elegant: minimal syntax and a simple, stack-based execution architecture
- Evolvable
- Extensible
- Supports uniform variation
- Supports several forms of meta-evolution

## Sample Push Instructions

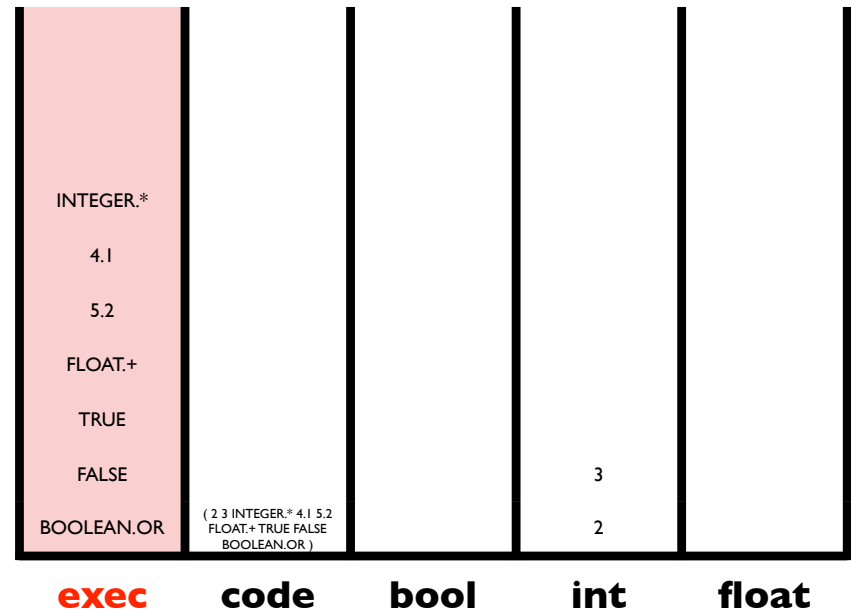
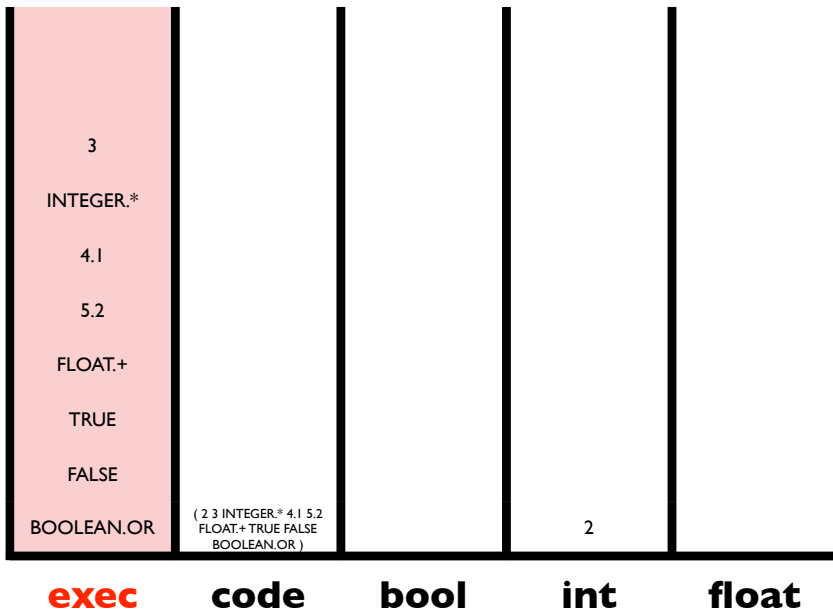
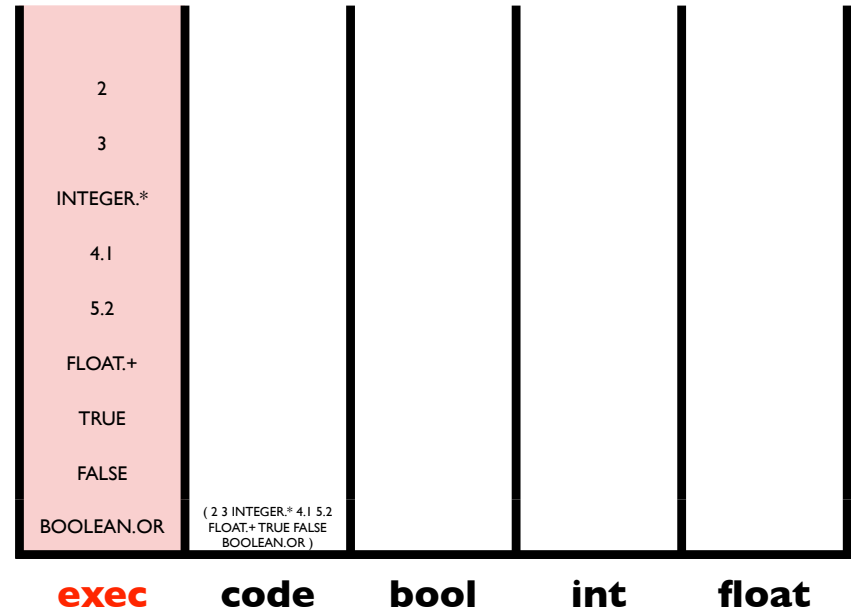
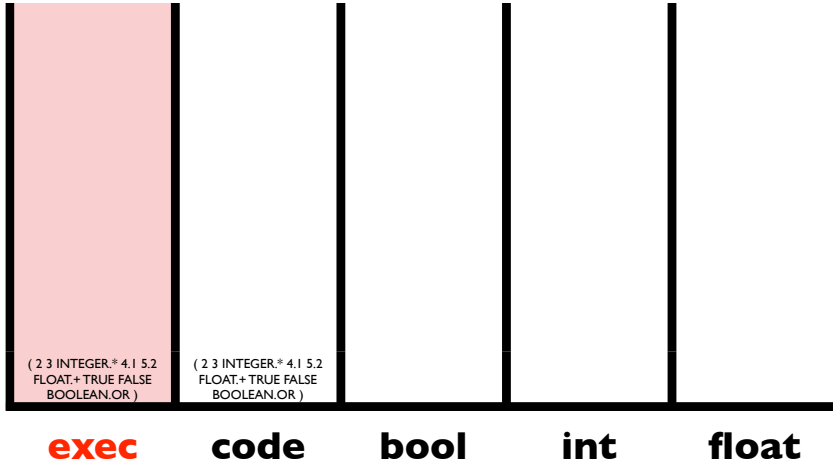
Stack manipulation instructions (all types)	POP, SWAP, YANK, DUP, STACKDEPTH, SHOVE, FLUSH, =
Math (INTEGER and FLOAT)	+, -, /, *, >, <, MIN, MAX
Logic (BOOLEAN)	AND, OR, NOT, FROMINTEGER
Code manipulation (CODE)	QUOTE, CAR, CDR, CONS, INSERT, LENGTH, LIST, MEMBER, NTH, EXTRACT
Control manipulation (CODE and EXEC)	DO*, DO*COUNT, DO*RANGE, DO*TIMES, IF

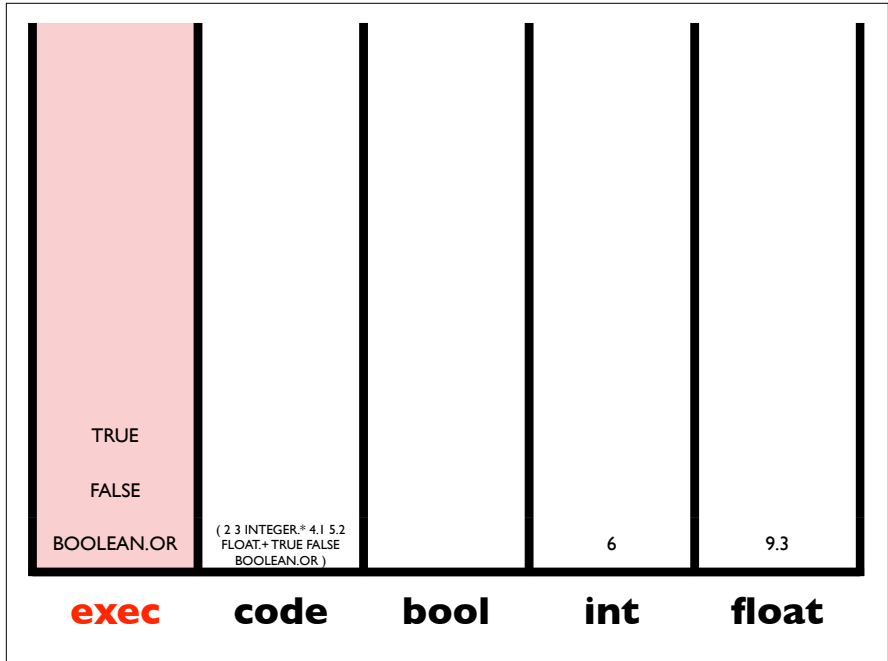
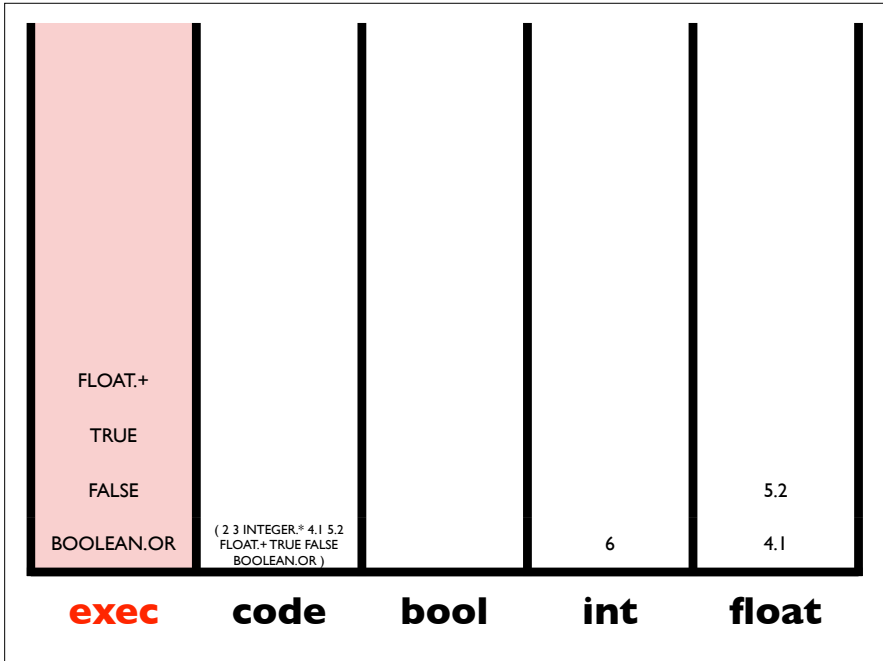
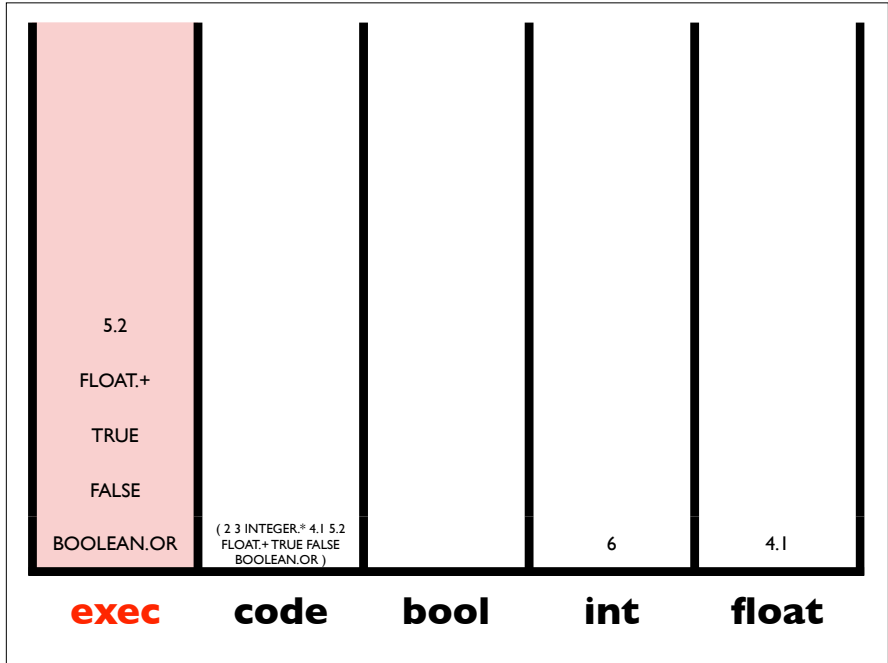
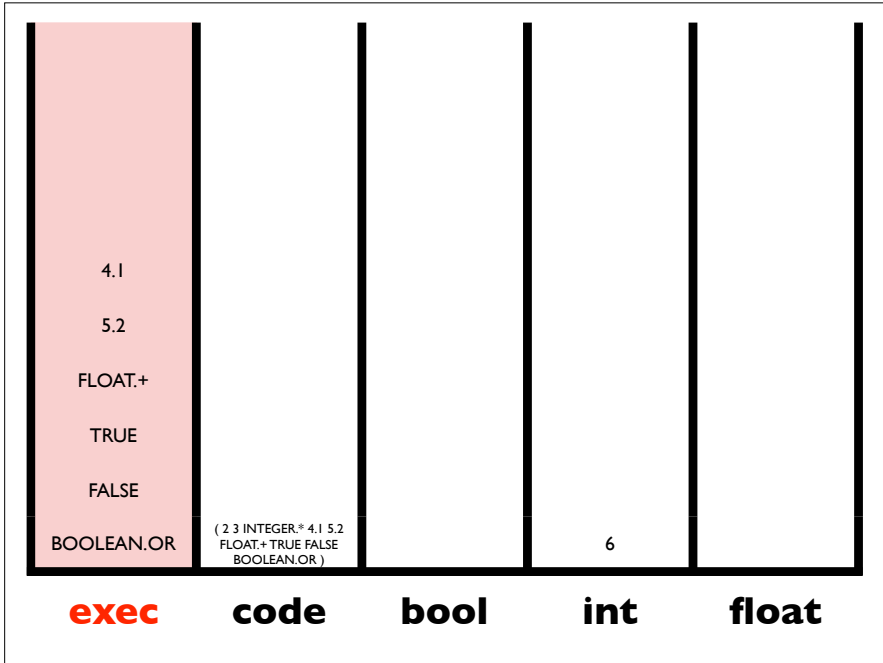
## Push(3) Semantics

- To execute program  $P$ :
  1. Push  $P$  onto the EXEC stack.
  2. While the EXEC stack is not empty, pop and process the top element of the EXEC stack,  $E$ :
    - (a) If  $E$  is an instruction: execute  $E$  (accessing whatever stacks are required).
    - (b) If  $E$  is a literal: push  $E$  onto the appropriate stack.
    - (c) If  $E$  is a list: push each element of  $E$  onto the EXEC stack, in reverse order.



( 2 3 INTEGER.\* 4.1 5.2 FLOAT.+  
TRUE FALSE BOOLEAN.OR )





FALSE				
BOOLEAN.OR	( 2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR )	TRUE	6	9.3
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

		FALSE		
BOOLEAN.OR	( 2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR )	TRUE	6	9.3
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

	( 2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR )	TRUE	6	9.3
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

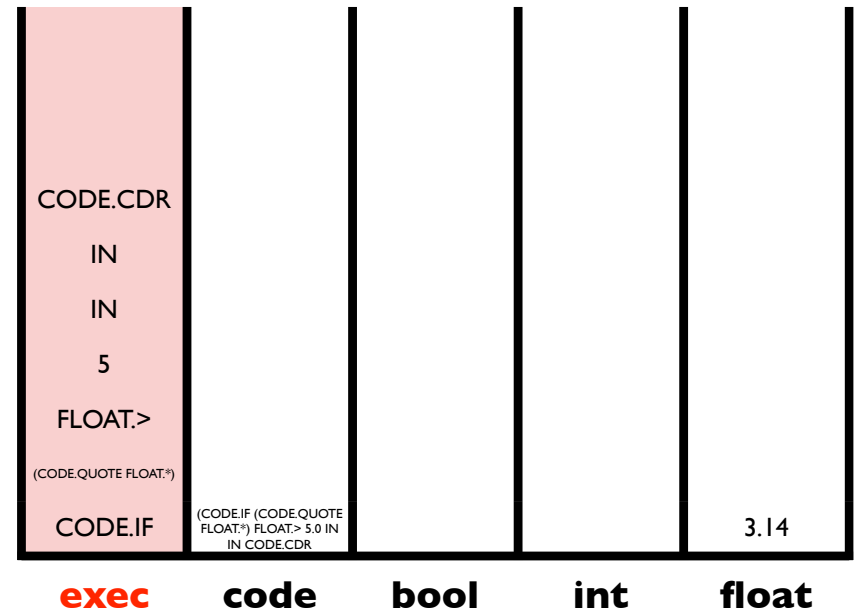
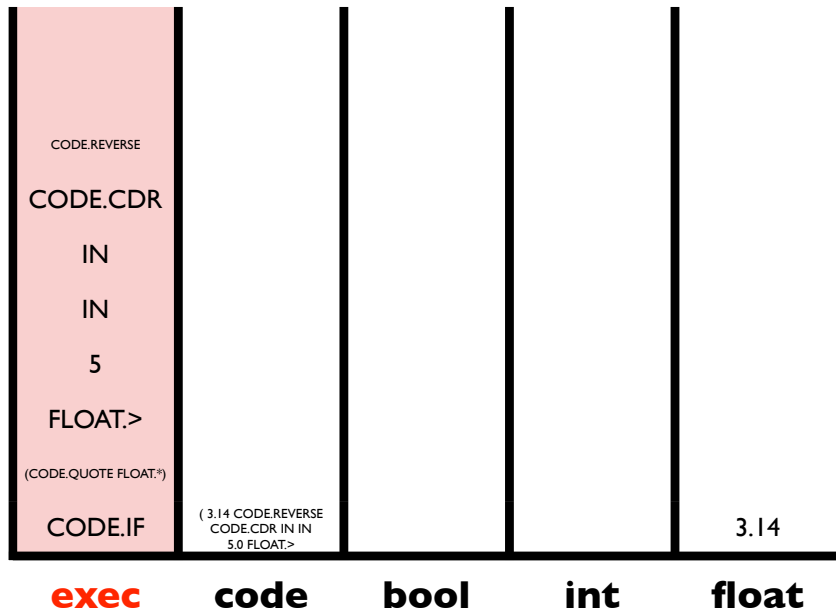
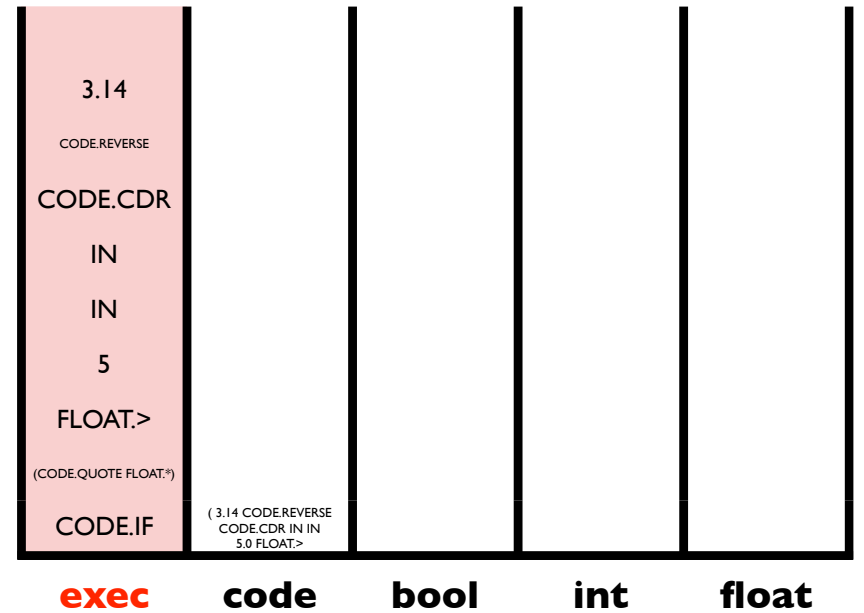
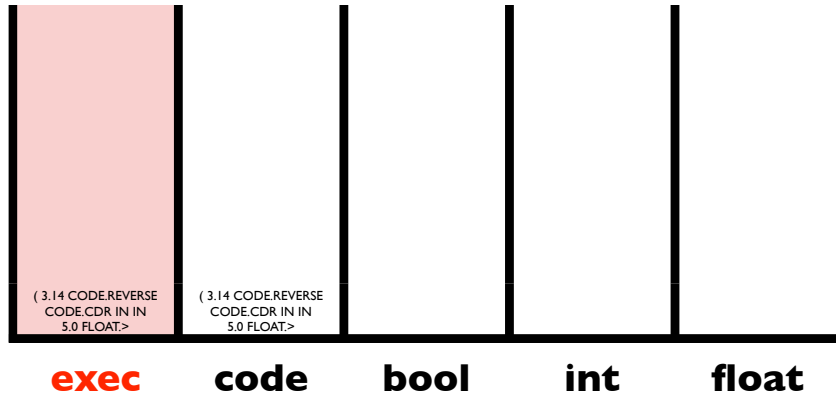
## Same Results

```
( 2 3 INTEGER.* 4.1 5.2 FLOAT.+  
  TRUE FALSE BOOLEAN.OR )
```

```
( 2 BOOLEAN.AND 4.1 TRUE INTEGER./ FALSE  
  3 5.2 BOOLEAN.OR INTEGER.* FLOAT.+ )
```

```
( 3.14 CODE.REVERSE CODE.CDR IN IN 5.0
FLOAT.> (CODE.QUOTE FLOAT.*) CODE.IF )
```

IN=4.0



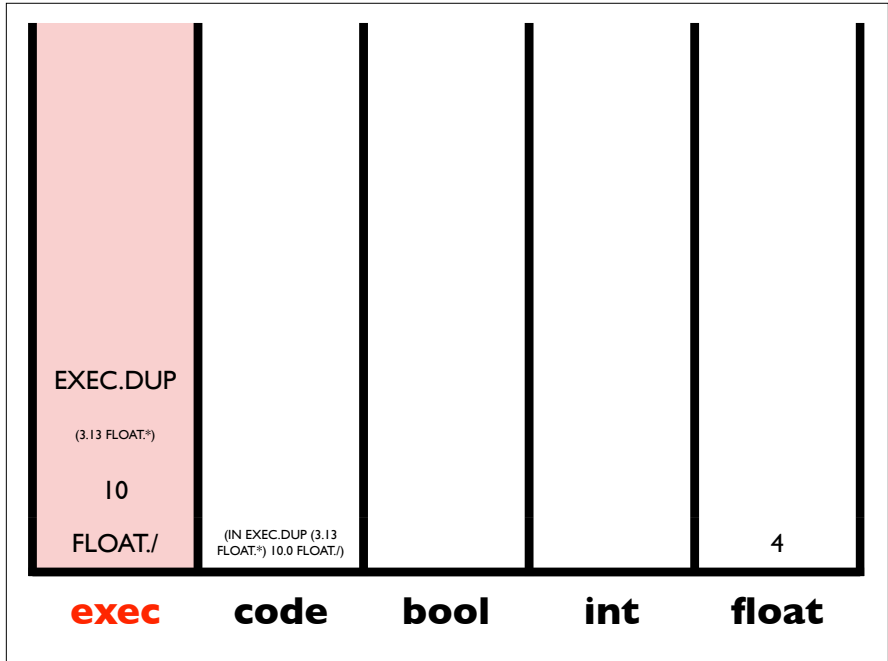
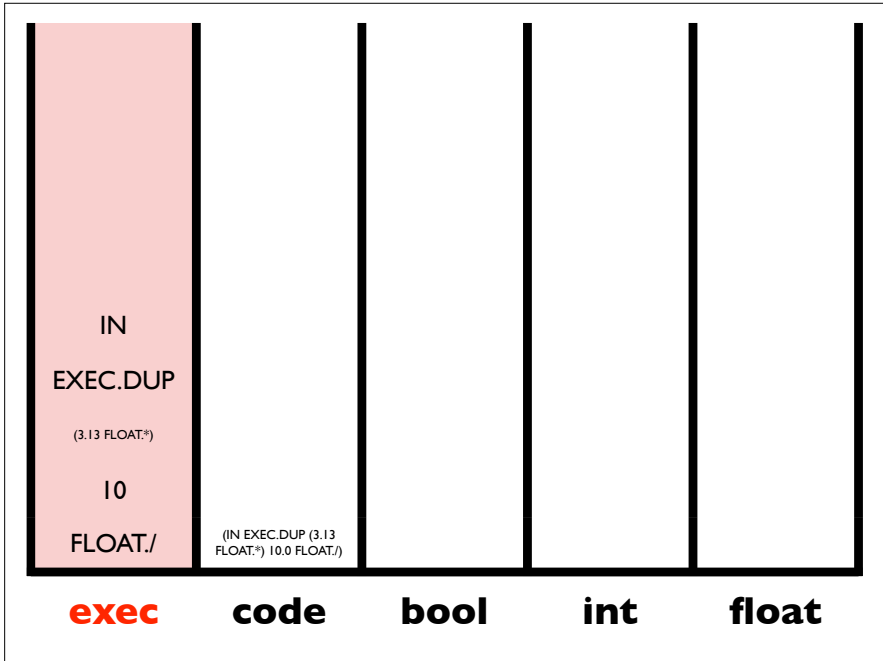
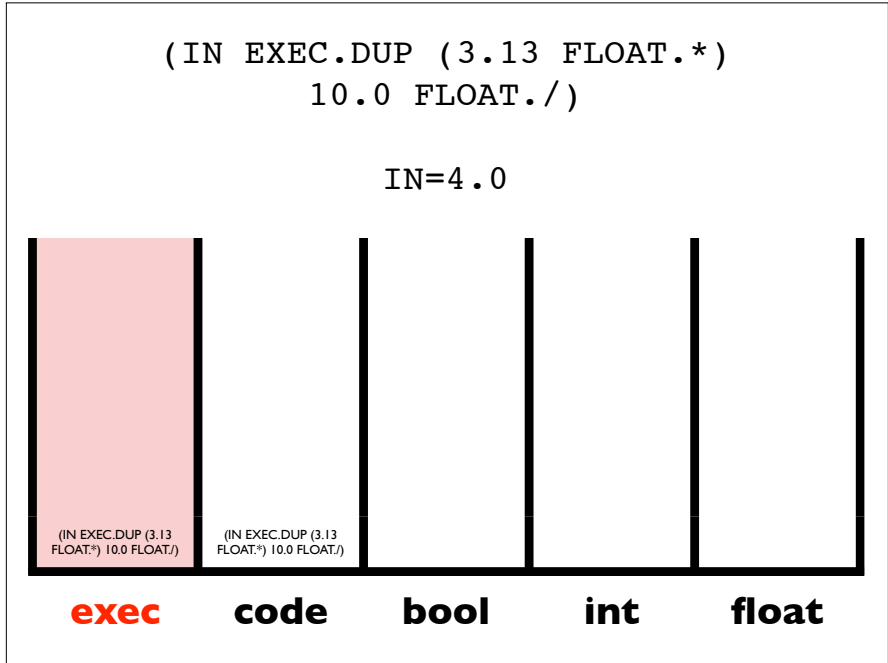
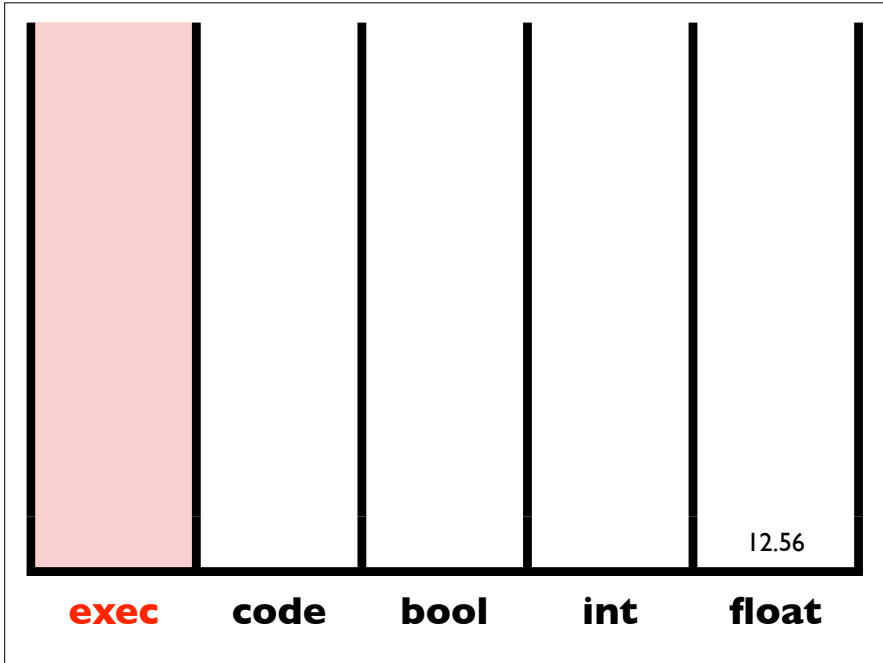
IN				
IN				
5				
Float.>				
(CODE.QUOTE FLOAT.?)				
CODE.IF	((CODE.QUOTE FLOAT.?) Float.> 5.0 IN IN CODE.CDR			3.14
exec	code	bool	int	float

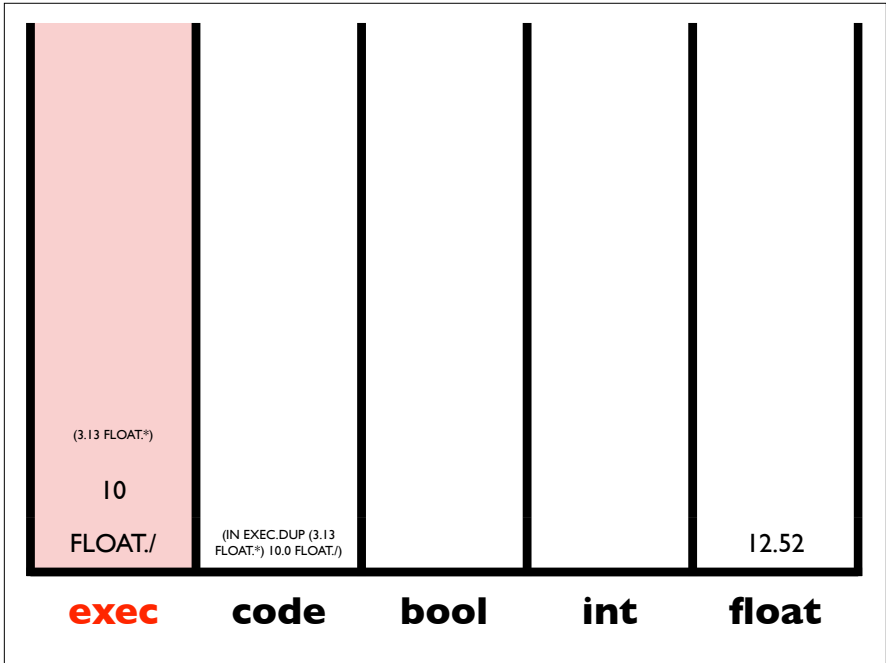
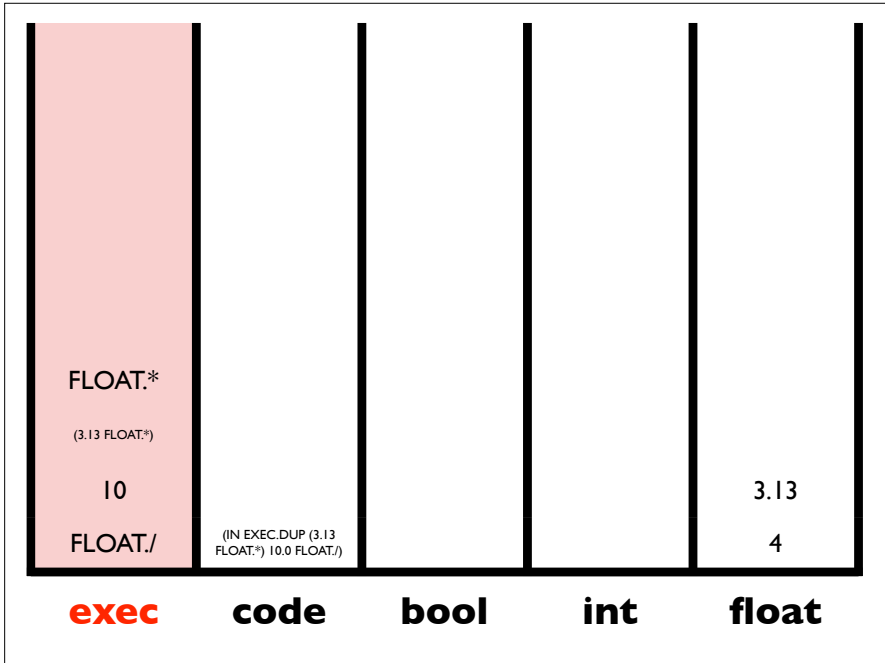
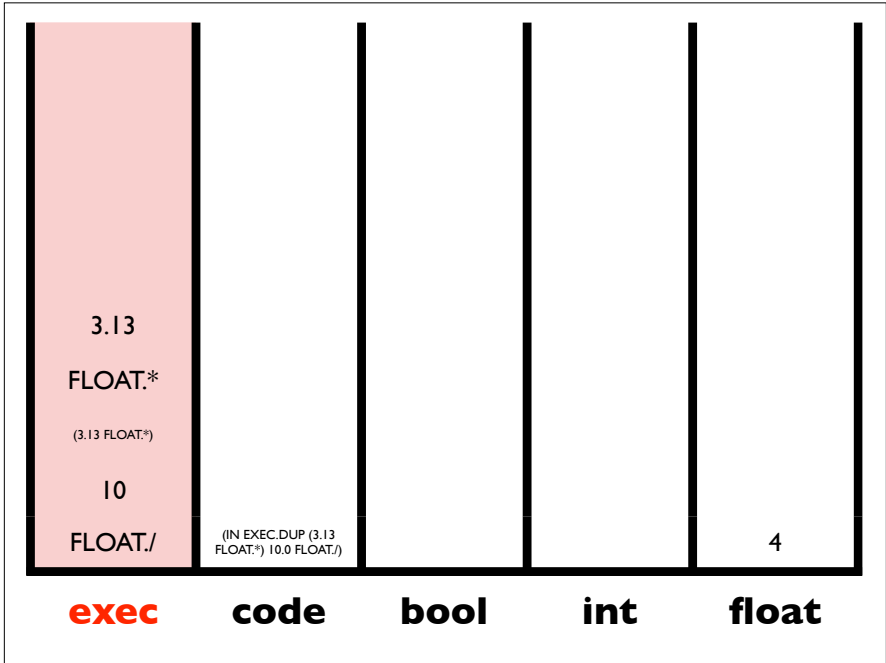
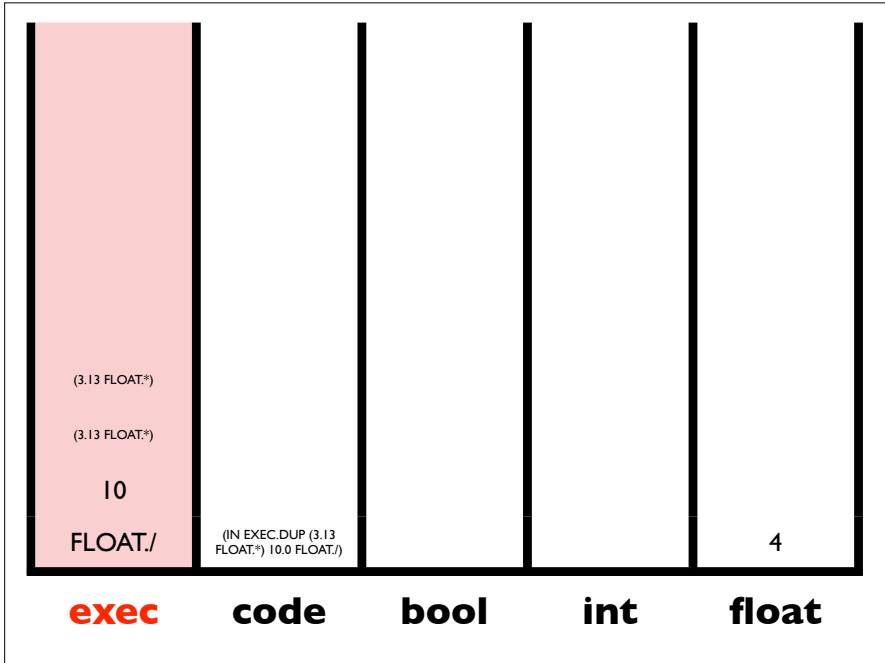
IN				
5				
Float.>				
(CODE.QUOTE FLOAT.?)				
CODE.IF	((CODE.QUOTE FLOAT.?) Float.> 5.0 IN IN CODE.CDR			4 3.14
exec	code	bool	int	float

5				
Float.>				
(CODE.QUOTE FLOAT.?)				
CODE.IF	((CODE.QUOTE FLOAT.?) Float.> 5.0 IN IN CODE.CDR			4 4 3.14
exec	code	bool	int	float

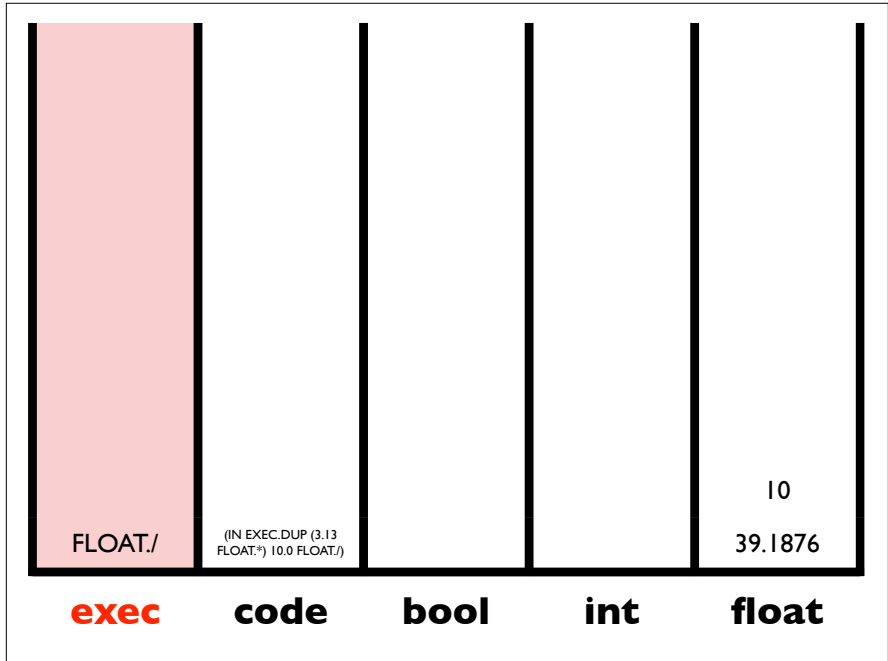
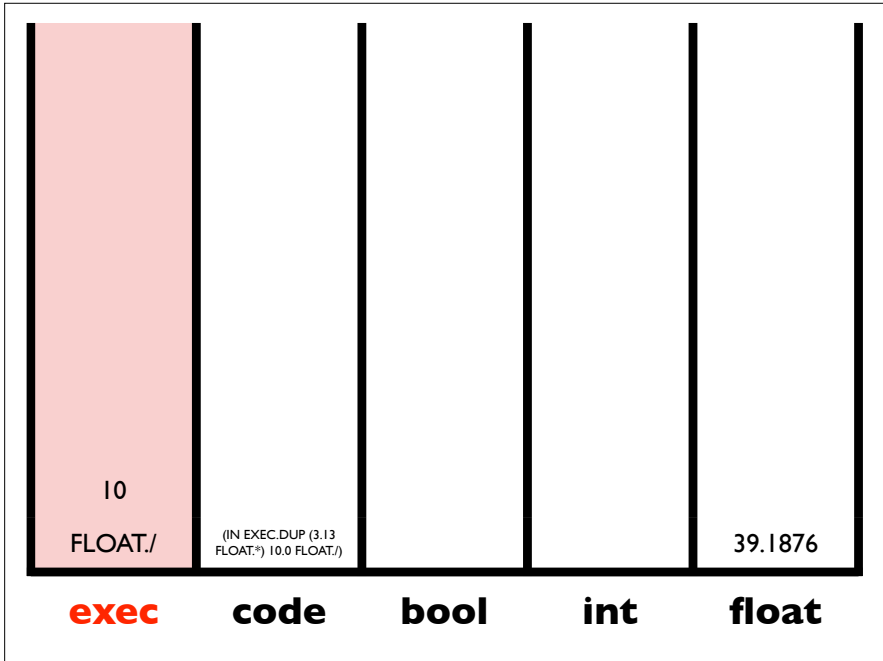
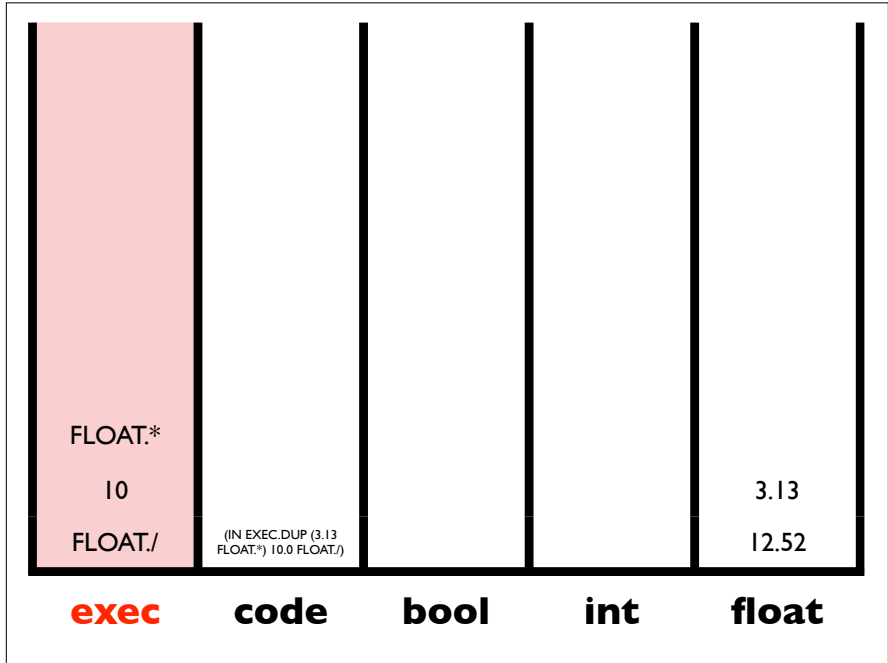
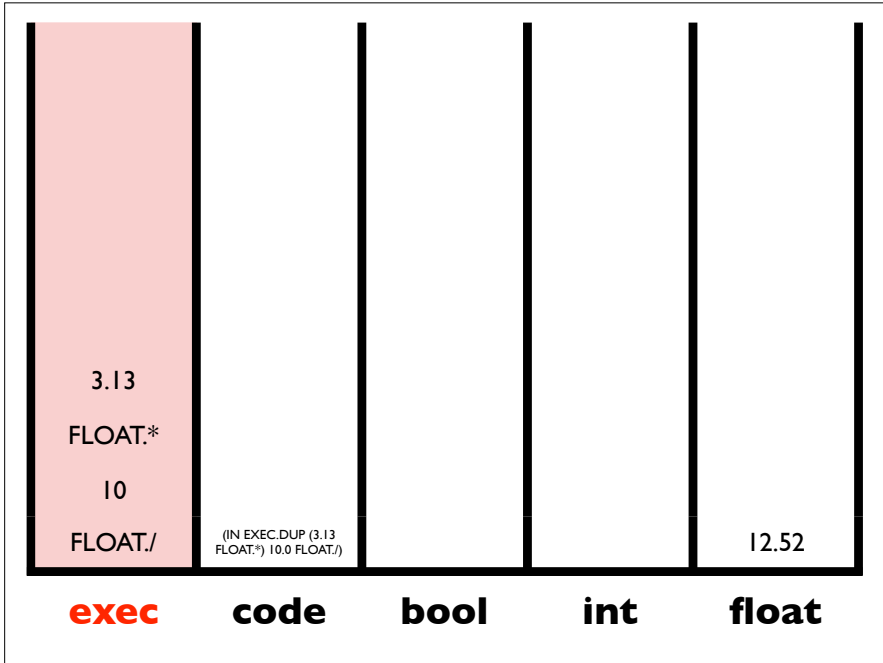
Float.>				5 4 4 3.14
(CODE.QUOTE FLOAT.?)				
CODE.IF	((CODE.QUOTE FLOAT.?) Float.> 5.0 IN IN CODE.CDR			
exec	code	bool	int	float





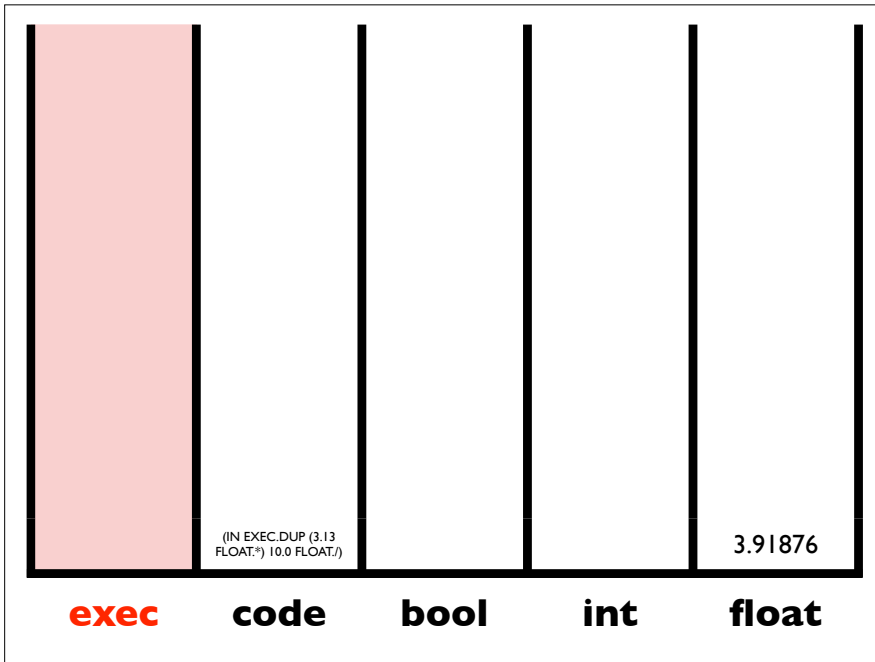






## The Odd Problem

- Integer input
- Boolean output
- Was the input odd?
- `((code.nth) code.atom)`



## Combinators

- Standard *K*, *S*, and *Y* combinators:
  - `EXEC.K` removes the second item from the `EXEC` stack.
  - `EXEC.S` pops three items (call them A, B, and C) and then pushes `(B C)`, C, and then A.
  - `EXEC.Y` inserts `(EXEC.Y T)` under the top item (*T*).
- A *Y*-based “while” loop:
 

```
( EXEC.Y
  ( <BODY/CONDITION> EXEC.IF
  ( ) EXEC.POP ) )
```

## Iterators

`CODE.DO*TIMES`, `CODE.DO*COUNT`,  
`CODE.DO*RANGE`

`EXEC.DO*TIMES`, `EXEC.DO*COUNT`,  
`EXEC.DO*RANGE`

Additional forms of iteration are supported through code manipulation (e.g. via `CODE.DUP CODE.APPEND CODE.DO`)

# Evolving Modular Programs

## With Code Manipulation

- Transform code as data on “code” stack
- Execute transformed code with `code.do`, etc.
- Simple uses of modules can be evolved easily

# Evolving Modular Programs

## With Execution Stack Manipulation

- Code queued for execution is stored on an “execution stack”
- Allow programs to duplicate and manipulate code that on the stack
- Example: `(3 exec.dup (1 integer.+))`

# Evolving Modular Programs

## With Named Modules

- Uses Push’s “name” stack
- Example:  

```
(plus1 exec.define (1 integer.+))  
...  
plus1
```
- Coordinating definitions/references is tricky

# Holland’s Tags

- Initially arbitrary identifiers that come to have meaning over time
- Matches may be inexact
- Appear to be present in some form in many different kinds of complex adaptive systems
- Examples range from immune systems to armies on a battlefield
- A general tool for the support of emergent complexity

# Evolving Modular Programs

## With tags

- Include instructions that tag code (modules)
- Include instructions that recall and execute modules by *closest matching* tag
- If a single module has been tagged then all tag references will recall modules
- The number of tagged modules can grow incrementally over evolutionary time
- **Expressive and evolvable**

# Tags in Push

- Tags are integers embedded in instruction names
- Instructions like `tag.exec.123` tag values
- Instructions like `tagged.456` recall values by *closest matching* tag
- If a single value has been tagged then all tag references will recall (and execute) values
- The number of tagged values can grow incrementally over evolutionary time

# Tags in Trees

- Example:  

```
(progn (tag.123 (+ a b))  
      (+ tagged.034 tagged.108))
```
- Must do something about endless recursion
- Must do something about return values of tagging operations and references prior to tagging
- Non-trivial to support arguments in a general way
- Utility not clear from experiments conducted to date

# Auto-simplification

Loop:    Make it randomly simpler  
          If it's as good or better: keep it  
          Otherwise: revert

GECCO-2014 poster shows that this can efficiently and reliably reduce the size of the evolved programs

GECCO-2014 student paper explores its utility in a genetic operator

## The ULTRA Operator

- Uniform Linear Transformation with Repair and Alternation
- Linearize 2 parents, treating “(” and “)” as ordinary tokens
- Start at the beginning of one parent and copy tokens to the child, switching parents stochastically (according to the *alternation rate*, and subject to an *alignment deviation*)
- Post-process with uniform mutation (according to a *mutation rate*) and repair

## ULTRA Results

- Dramatic improvements in problem solving and parsimony for:
  - Bioavailability regression problem\*
  - Pagie-1 regression problem
  - Factorial regression problem
- Negative results for 6-multiplexer
- See GTP-XI chapter for details

\* James McDermott has recently pointed out weaknesses of this problem as a GP benchmark: <http://jmmcd.net/2013/12/19/gp-needs-better-baselines.html>

# DEMO

## Problems Solved by PushGP in the GECCO-2005 Paper on Push3

- Reversing a list
- Factorial (many algorithms)
- Fibonacci (many algorithms)
- Parity (any size input)
- Exponentiation
- Sorting

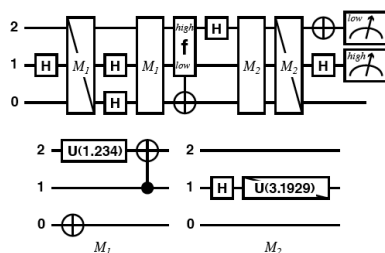
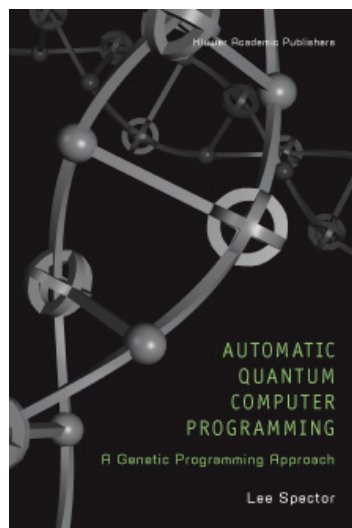


Figure 8.7. A gate array diagram for an evolved version of Grover's database search algorithm for a 4-item database. The full gate array is shown at the top, with  $M_1$  and  $M_2$  standing for the smaller gate arrays shown at the bottom. A diagonal line through a gate symbol indicates that the matrix for the gate is transposed. The "f" gate is the oracle.

**Humies 2004  
GOLD MEDAL**

## Genetic Programming for Finite Algebras

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**Humies 2008  
GOLD MEDAL**

## Autoconstructive Evolution

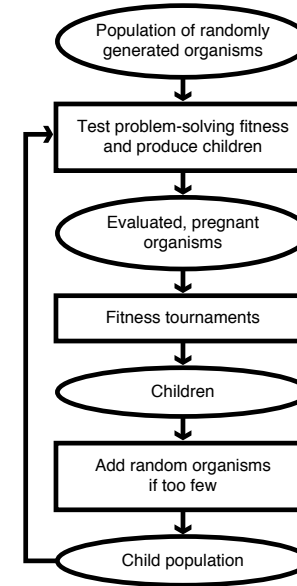
- Individuals make their own children
- Agents thereby control their own mutation rates, sexuality, and reproductive timing
- The machinery of reproduction and diversification (i.e., the machinery of evolution) evolves
- Radical self-adaptation

## Related Work

- MetaGP: but (1) programs and reproductive strategies dissociated and (2) generally restricted reproductive strategies
- ALife systems such as Tierra, Avida, SeMar: but (1) hand-crafted ancestors, (2) reliance on cosmic ray mutation, and (3) weak problem solving
- Evolved self-reproduction: but generally exact reproduction, non-improving (exception: Koza, but very limited tools for problem solving *and* for construction of offspring)

## Pushpop

- A soup of evolving Push programs
- Reproductive procedures emerge ex nihilo:
  - No hand-designed “ancestor”
  - Children constructed by any computable process
  - No externally applied mutation procedure or rate
  - Exact clones are prohibited, but near-clones are permitted.
- Selection for problem-solving performance



## Pushpop Results

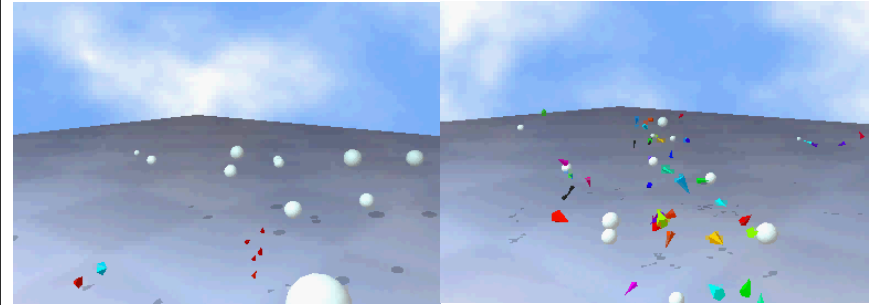
- In adaptive populations:
  - Species are more numerous
  - Diversification processes are more reliable
- Selection can promote diversity
- Provides a possible explanation for the evolution of diversifying reproductive systems

## SwarmEvolve 2.0

- Behavior (**including reproduction**) controlled by evolved Push programs
- Color, color-based agent discrimination controlled by agents
- Energy conservation
- Facilities for communication, energy sharing
- Ample user feedback (e.g. diversity metrics, agent energy determines size)

Instruction(s)	Description
DUP, POP, SWAP, REP, =, NOOP, PULL, PULLDUP, CONVERT, CAR, CDR, QUOTE, ATOM, NULL, NTH, +, *, /, >, <, NOT, AND, NAND OR, NOR, DO*, IF	Standard Push instructions (See [11])
VectorX, VectorY, VectorZ, VPlus, VMinus, VTimes, VDivide, VectorLength, Make-Vector	Vector access, construction, and manipulation
RandI, RandF, RandV, RandC	Random number, vector, and code generators
SetServoSetpoint, SetServoGain, Servo	Servo-based persistent memory
Mutate, Crossover	Stochastic list manipulation (parameters from stacks)
Spawn	Produce a child with code from code stack
ToFood	Vector to energy source
FoodIntensity	Energy of energy source
MyAge, MyEnergy, MyHue, MyVelocity, MyLocation, MyProgram	Information about self
ToFriend, FriendAge, FriendEnergy, FriendHue, FriendVelocity, FriendLocation, FriendProgram	Information about closest agent of similar hue
ToOther, OtherAge, OtherEnergy, OtherHue, OtherVelocity, OtherLocation, OtherProgram	Information about closest agent of non-similar hue
FeedFriend, FeedOther	Transfer energy to closest agent of indicated category

## SwarmEvolve 2.0



Winner, Best Paper Award, AAAA Track, GECCO-2003

## AutoPush

- Goals:
  - Superior problem-solving performance
  - Tractable analysis
- Push3
- Asexual
- Children produced on demand (not during fitness testing)
- Constraints on selection and birth
- Still work in progress

## Expressiveness and Assessment

- Expressive languages ease representation of programs that over-fit training sets
- Expressive languages ease representation of programs that work only on subsets of training sets
- Lexicase selection may help: Select parents by starting with a pool of candidates and then filtering by performance on individual fitness cases, considered one at a time



## Future Work

- Expression of variable scope and environments (implemented in Push, but not yet studied systematically)
- Expression of concurrency and parallelism
- Applications for which expressiveness is likely to be essential, e.g. complete software applications, agents in complex, dynamic environments
- Epigenetics

## Conclusions

- GP in expressive languages may allow for the evolution of complex software
- Minimal-syntax languages can be expressive, and GP systems that evolve programs in such languages can be unusually simple and powerful
- Push is expressive, evolvable, successful, and extensible

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