

# Artificial Immune Systems for Optimisation

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Introduction	Natural Immune System	AIS as Classifiers	AIS as Optimisers	Analysing AIS	Conclusion & References
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## Artificial Immune Systems are...

- a model of the natural immune system  
if you are interested in the natural immune system
- computational systems inspired by the natural immune system with natural applications in anomaly detection & classification  
if you are interested in solving a classification problem
- nature-inspired algorithms using the natural immune system as metaphor for problem-solving  
if you are interested in solving difficult problems
- nature-inspired randomised search heuristics like many others, e. g., evolutionary algorithms, ACO, SA, ...  
if you are interested in randomised search heuristics
- a fascinating area of research in any case

Good News We cover all these aspects. (↪ structure governed by this)

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## Plans for Today

- 1 Introduction
  - AIS are ...
  - Overview
- 2 AIS as Model of the Natural Immune System
- 3 AIS as Classifiers
- 4 AIS as Optimisers
- 5 Analysing AIS
- 6 Summary and Conclusions
  - AIS Tutorial Summary
  - AIS as Future Research Topics

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Introduction	Natural Immune System	AIS as Classifiers	AIS as Optimisers	Analysing AIS	Conclusion & References
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## Biological Inspiration: The Immune System of Vertebrates

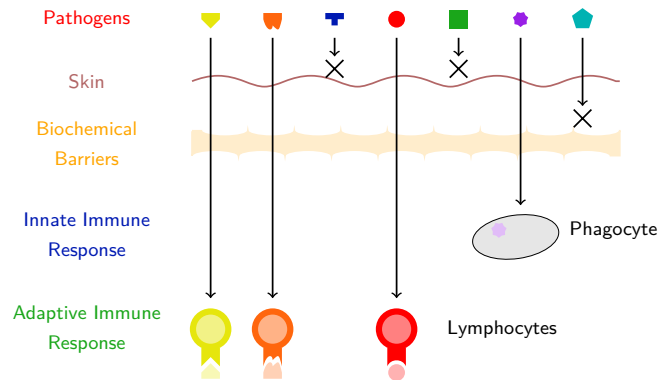
"The immune system recognizes infection and induces protective responses." [27]

### Main Tasks

- Immunological recognition
- Immune effector functions
- Immune regulation
- Immunological memory

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## Multilayer Structure of the Immune System



(reproduced from [6, 31])

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## Innate vs. Adaptive Immunity

### Innate Immunity

- **Non-specific** response against large number of bacteria
- First line of defense:  
Controls infection before adaptive immune response kicks in
- Initiates and controls adaptive immune response
- Dendritic cells form bridge between innate/adaptive immunity

### Adaptive Immunity

- **Specific** and **preventive** immune response
- Mediated by **lymphocytes** in the lymph nodes
- Two main types: **B cells** and **T cells**
- Develops immunological memory
- Described by the **Clonal Selection Theory**

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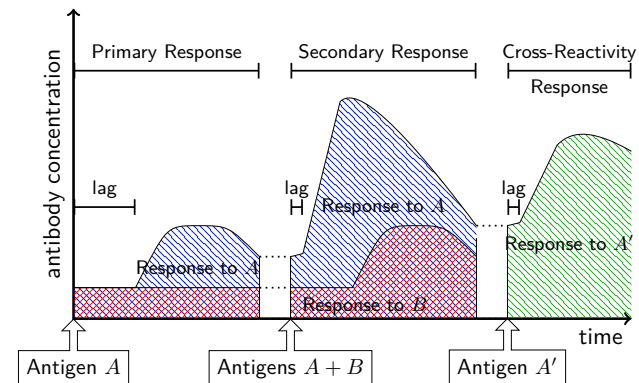
## Adaptive Immunity – Immunological Recognition

- **Lymphocytes**
  - **Naïve**: not yet involved in an immune response
  - Become **active** due to interaction with antigenic stimulus
- Carry **antigen receptors** of **single specificity**
- Receptor **diversity** due to
  - **Random recombination** of gene fragments from several libraries
  - **Somatic hypermutation** to increase antigen-antibody affinity
- Recognition based on **complementarity** between **binding region** of receptor and **epitope** of antigen on molecular level
- Antigens may present several epitopes
- Require **co-stimulatory signals**
- **B cell receptor** interacts directly while **T cell receptor** requires preprocessing and presenting by other cells



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## Immunological Memory and Cross-Reactive Response



(reproduced from [6, 27])

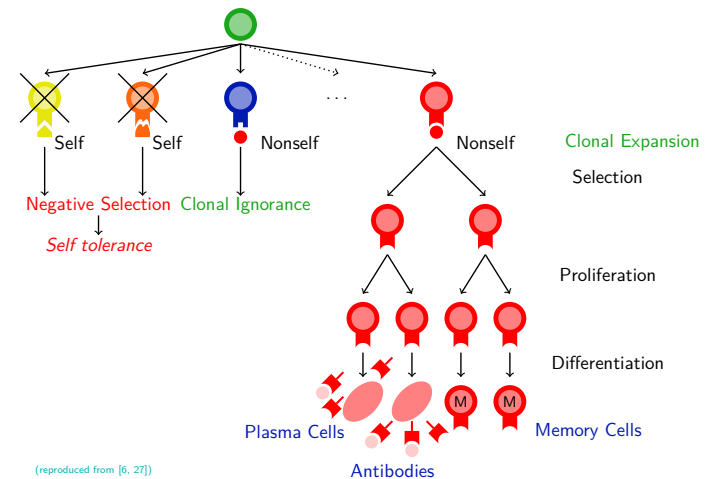
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## Clonal Selection Theory (1) [2]

- Describes basic properties of **adaptive immune response**
- Only cells recognizing an antigen **proliferate** and **differentiate** into effector cells
- B cells:**
  - Subject to **somatic hypermutations**
  - B effector cells secrete **antibodies**
- T cells:**
  - Not subject to mutation
  - T effector cells secrete **lymphokine**
- B cell clonal selection **similar** to **natural selection**
- Learning** through increasing population size and affinity
- Immune repertoire **evolved** from a random base to reflection of actual antigenetic environment

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## Clonal Selection Theory (2) [2]



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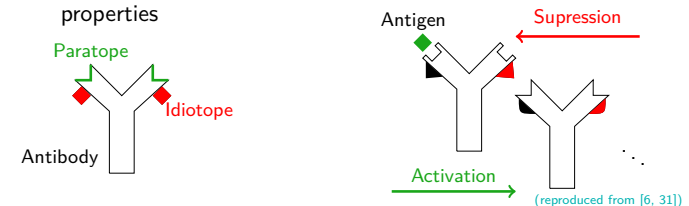
## Clonal Selection Theory (3) [2]

- Diversity via **somatic hypermutations**, **receptor editing** and **newcomer cells**
- Non-functional and harmful anti-self specificities are **eliminated**
- Variants with higher affinity **dominate** immune response and enter **immune memory**
- Some low affinity cells enter repertoire to maintain diversity
- Hypermutations**
  - Point mutations** allow for exploring local regions
  - On average **one** mutation per cell division
  - Short **burst** of somatic hypermutation followed by a **pause** to allow for selection and clonal expansion
  - Regulation** of the hypermutation process by selection depending on receptor affinity
- Receptor editing**
  - Instead of **clonal deletion** development of new receptors
  - Allows for larger steps through the landscape

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## Immune Network Theory [24]

- Remember Clonal Selection Theory:**  
Immune system  $\hat{=}$  set of discrete cells and molecules originally at rest; triggered only by a foreign antigenic stimulation
- Now:** different perspective **Immune Network Theory**  
Immune system  $\hat{=}$  regulated network of cells and molecules that recognize one another even in the absence of antigens
- Network is autonomous, self-regulated and aims at maintaining a specific range of activity
- Immune tolerance, learning and memory as inherent global properties



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## Efficient Negative Selection

**Algorithm** outline, main ingredients  
 use **prefix trees** as main data structure  
 efficiently build **finite automaton** to represent detectors  
 (note: **no** explicit detector set)  
 construction of automaton works in time  $O(|S| \cdot l \cdot r)$   
 classification works in time  $O(l)$   
 (for details see Elberfeld, Textor (2011) [10])  
 (for implementation see  
<http://bioinformatics.bio.uu.nl/textor/negativeselection.html>)

**Lesson Learned** immune metaphor **useful** for ideas  
 algorithmic implementation following the IS  
 may be **very far** from optimal  
 immune-ideas can be implemented **efficiently**  
 using 'classical' algorithmic ideas

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## More Modern AIS-Approaches to Classification

**Remember** self-nonsel self discrimination  
 based on a **simplistic** understanding of the immune system  
 can be **implemented efficiently**  
 using clever algorithms/data structures

**Fact** many more AIS-approaches to classification exist  
 based on different aspects of immunology  
**too many** to cover all here

**Today** two current approaches  
 based on different immunological theories  
**current**  $\hat{=}$  both considered and further developed  
 in current publications

- ① based on **T cell signalling**: **receptor density algorithm**
- ② based on **danger theory**: **dendritic cell algorithm**

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## Receptor Density Algorithm

**Motivation** two-class classification performed by **T cells**  
 depending on history

**Basis** model of **T cell receptor** called **receptor**  
 having a state  $c$ , position  $p$ , negative feedback  $n$ ,  
 a negative feedback barrier  $\beta$ , length  $l > \beta$   
**reacts** to input  $u$  by  
 updating the position (adding  $u$ , subtracting  $n$ ),  
 increasing negative feedback for positions above  $\beta$ ,  
 decaying negative feedback otherwise  
**combining** receptors spatially in form of a grid  
 with a stimulation kernel function  
 yields **receptor density algorithm**  
 (for details see Owens (2010) [28], Owens et al. (2013) [29],  
 Hilder et al. (2011) [15])

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## Dendritic Cell Algorithm

**Motivation** **danger theory** specifically dendritic cells  
**model** immune systems responds to **danger/safe signals**  
 (does **not** perform self-nonsel self discrimination)

**Basis** model of **dendritic cells** being either **immature**,  
**semi-mature** or **mature**, having a lifespan  
 processing input classified as either **danger**, **safe** or **PAMP**,  
 computes two values:  
**DCM**, indicating the amount of processed information,  
**K**, indicating the classification as normal or anormal  
 a collection of such cells (with different lifespans) forms  
**dendritic cell algorithm** performing classification  
 (for details see Greensmith (2007) [12])  
 fully formalised, simplified deterministic version  
**deterministic dendritic cell algorithm** available  
 (for details see Gu (2011) [13], Gu et al. (2013) [14])

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## Artificial Immune Systems as Optimisers

- Remember** most natural application  
 $\hat{=}$  classification, pattern recognition, IT security
- Observation** some algorithms also suitable for optimisation tasks
- In particular** clonal selection and immune network theory
- Consider** minimisation/maximisation of  
 pseudo-Boolean functions  $f: \{0, 1\}^n \rightarrow \mathbb{R}$  or  
 real-valued functions  $f: \mathbb{R}^n \rightarrow \mathbb{R}$
- Observation** structure similar to evolutionary algorithms  
 concrete implementation very different
- Today** characteristics and concrete algorithms
- Mutation and Metadynamics in AIS
  - Clonal Selection Algorithms: CLONALG, B-Cell Algorithm, opt-IA
  - Immune Network: opt-aiNet
- Additionally** simple implementation to 'play along'  
<http://www.cs.bham.ac.uk/~zargesc/ais-tutorial/>

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## Mutation in Artificial Immune Systems (1)

Usually Mutations at high rate  $\rightsquigarrow$  Hypermutations

### Inverse Fitness-“Proportional” Hypermutation

- **Idea** Apply mutations with lower mutation rate to good search points
- **Usually** Normalised fitness value  $\hat{f} \in [0, 1]$  used
  - Using optimal function value  $f_{\text{opt}}: \hat{f}(x) = f(x)/f_{\text{opt}}$
  - Using best known function value  $f_{\text{best}}: \hat{f}(x) = f(x)/f_{\text{best}}$
- **Examples** for some parameter  $\rho \in \mathbb{R}^+$ , maximisation
  - **CLONALG**:  $p_m = \exp(-\rho \cdot \hat{f})$
  - **opt-aiNet**:  $p_m = (1/\rho) \cdot \exp(-\hat{f})$
- **Remark**  
 In continuous optimisation  $p_m$  equals the mutation strength

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## Mutation in Artificial Immune Systems (2)

### Contiguous Hypermutations

**Idea** Perform mutations with probability  $r \in (0, 1]$  only in contiguous region.

- Choose hotspot  $p \in \{0, \dots, n-1\}$ , length  $\ell \in \{0, 1, \dots, n\}$ .  
 $p = 8, \ell = 4$

0 1 2 3 4 5 6 7 8 9 not wrapping around

0 1 2 3 4 5 6 7 8 9 wrapping around

- Choose two hotspots  $a, b \in \{0, \dots, n-1\}$ .  
 $a = 8, b = 5$

0 1 2 3 4 5 6 7 8 9

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## Contiguous Hypermutations in the Tiny AIS Box

Name	Current value
Objective function	OneMax
Optimisation goal	Maximisation Minimisation
Problem size	30
Budget per run	0
Runs	5
Population size	10
Offspring population size	3
Hypermutation operator	contiguous mutation potential inverse fitness-proportional EA mutations
Parameter $r$ from (0, 1)	0.95
CHM variant	wrap no wrap two positions
Crossover	
Ageing	

<http://www.cs.bham.ac.uk/~zargesc/ais-tutorial/>

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## Mutation in Artificial Immune Systems (3)

### Hypermutation with Mutation Potential

- **Idea** Determine number of local mutation steps during a single hypermutation.
- **Different** classes: static, inversely proportional, proportional
- **Example** for some constant  $c \in ]0, 1[$ , minimisation  

$$M_c(v) = \lceil (1 - f_{\text{best}}/v) \cdot c \cdot n \rceil$$
- Variants for the hypermutation of  $x \in \{0, 1\}^n$ :  
**tabu**, **stop at first constructive mutation**
  - 1 Set  $y := x$ . Set  $v := f(x)$ .
  - 2 Repeat the following  $M_c(v)$  times:
    - If **tabu** = 0 select  $i \in \{1, \dots, n\}$  uniformly at random  
else select  $i \in \{1, \dots, n\}$  uniformly at random,  $i$  not previously chosen.
    - $y[i] := 1 - y[i]$
    - If **fc** = 1 and  $f(y) < f(x)$  Then break

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## Hypermutation with Mutation Potential in Tiny AIS Box

Name	Current value
Objective function	OneMax
Optimisation goal	Maximisation Minimisation
Problem size	30
Budget per run	0
Runs	5
Population size	10
Offspring population size	3
Hypermutation operator	contiguous mutation potential inverse fitness-proportional EA mutations
Parameter c from (0, 1)	0.5
Variants	stop on success yes no tabu yes no normalisation optimum best
Crossover	<input type="checkbox"/>
Ageing	<input type="checkbox"/>

<http://www.cs.bham.ac.uk/~zargesc/ais-tutorial/>

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## Metadynamics in Artificial Immune Systems

Sometimes worst search points replaced by new random ones.

Popular mechanism: Ageing

- **Idea**:  
Increase diversity by removing old and non-improving points
- General implementation:
  - Assign age to each search point
  - Increase age in each round
  - If new search point improves over parent  
Then assign age 0 to new search point  
Else inherit age of parent
  - Optional: Fill up population with new random search points
- Variants for some parameter  $\tau_{\text{max}}$ 
  - **Static Pure Ageing**  
remove search points older than  $\tau_{\text{max}}$
  - **Stochastic Ageing**  
remove each search point with probability  $1 - 2^{-1/\tau_{\text{max}}}$

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## Ageing in the Tiny AIS Box

Name	Current value
Objective function	OneMax
Optimisation goal	Maximisation Minimisation
Problem size	30
Budget per run	0
Runs	5
Population size	10
Offspring population size	3
Hypermutation operator	contiguous mutation potential inverse fitness-proportional EA mutations
Parameter r from (0, 1)	0.95
CHM variant	wrap no wrap two positions
Crossover	<input type="checkbox"/>
Ageing	<input checked="" type="checkbox"/>
Maximal age tau	1000
Ageing variant	static pure ageing evolutionary genotypic
Ageing selection variant	random most frequent smallest age distance

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## CLONALG [8]

**Parameters**  $\mu$ : population size  $d$ : selection pressure  
 $\beta$ : offspring population size factor

1. **Initialisation** Create an initial population  $P = \{x_1, x_2, \dots, x_\mu\}$ .
  2. **Clonal Selection and Expansion**  
 For all  $i \in \{1, 2, \dots, \mu\}$ :  
 a) Create  $\lfloor \beta\mu \rfloor$  clones of  $x_i$  and place them in a clonal pool  
 $C_i = \{y_i^1, \dots, y_i^{\lfloor \beta\mu \rfloor}\}$ .  
 b) For all  $j \in \{1, \dots, \lfloor \beta\mu \rfloor\}$ :  
 Apply inversely fitness-proportional hypermutation to  $y_i^j$ .
  3. **Selection for Replacement**  
 Keep the  $\mu$  best search points from  $P \cup C_1 \cup \dots \cup C_\mu$ ,  
 breaking ties uniformly at random.
  4. **Metadynamics**  
 Replace  $d$  search points with lowest fitness by new random ones.
  5. **Stopping** If stopping criterion not met continue at line 2.
- Variants**
1. Non-elit selection for replacement
  2. Keep best search point from  $x_i \cup C_i$  for all  $i \in \{1, \dots, \mu\}$

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## The B-Cell Algorithm (BCA) [25]

**Parameters**  $\mu$ : population size  $\lambda$ : offspring population size

1. **Initialisation** Create an initial population  $P = \{x_1, x_2, \dots, x_\mu\}$ .
2. **Clonal Selection and Expansion**  
 For all  $i \in \{1, 2, \dots, \mu\}$ :  
 a) Create  $\lambda$  clones of  $x_i$  and place them in a clonal pool  
 $C_i = \{y_i^1, \dots, y_i^\lambda\}$ .  
 b) Select  $j \in \{1, \dots, \lambda\}$  uniformly at random:  
 Flip each bit of  $y_i^j$  with probability  $p_m$ .  
 c) For all  $j \in \{1, \dots, \lambda\}$ :  
 Apply somatic contiguous hypermutation to  $y_i^j$ .
3. **Selection for Replacement**  
 For all  $i \in \{1, 2, \dots, \mu\}$ :  
 If  $\min\{f(y_i^1), \dots, f(y_i^\lambda)\} \leq f(x_i)$ :  
 Replace  $x_i$  by some randomly chosen  $y_i^j$  with minimal  $f$ -value.
4. **Stopping** If stopping criterion not met continue at line 2.

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## opt-IA [3, 4, 5]

**Parameters**  $\mu$ : population size  $\lambda$ : offspring population size  
 $H, M$ : flags for mutation operators

1. **Initialisation** Create an initial population  $P = \{x_1, x_2, \dots, x_\mu\}$ .
2. **Clonal Selection and Expansion**  
 For all  $i \in \{1, 2, \dots, \mu\}$ :  
 a) Create  $\lambda$  clones of  $x_i$  and place them in a clonal pool  $C_i = \{y_i^1, \dots, y_i^\lambda\}$ .  
 b) For all  $j \in \{1, \dots, \lambda\}$ :  
 If (H) Then Apply hypermutation with mutation potential to  $y_i^j \rightsquigarrow C_i^H$ .  
 Else  $C_i^H = \emptyset$ .  
 If (M) Then Apply contiguous hypermutations to  $y_i^j \rightsquigarrow C_i^M$ .  
 Else  $C_i^M = \emptyset$ .
3. **Metadynamics** Apply aging to  $P$ ,  $C_i^H$ , and  $C_i^M$ .
4. **Selection for Replacement**  
 Set  $P = P \cup C_1^H \cup \dots \cup C_\mu^H \cup C_1^M \cup \dots \cup C_\mu^M$ .  
 If  $|P| \geq \mu$  Then Keep the  $\mu$  best search points from  $P$ ,  
 breaking ties u.a.r. and removing duplicates.  
 Else Keep all search points in  $P$ ; fill up  $P$  with random points.
5. **Stopping** If stopping criterion not met continue at line 2.

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## opt-aiNet [7]

**Parameters**  $\mu$ : initial population size  $\lambda$ : offspring population size  
 $\sigma$ : affinity threshold  $\delta$ : average fitness threshold  
 $d$ : selection pressure

1. **Initialisation** Create an initial population  $P = \{x_1, x_2, \dots, x_\mu\}$ .
2. **Clonal Selection and Expansion**  
 For all  $i \in \{1, 2, \dots, \mu\}$ :  
 a) Create  $\lambda$  clones of  $x_i$  and place them in a clonal pool  $C_i = \{y_i^1, \dots, y_i^{\beta\mu}\}$ .  
 b) For all  $j \in \{1, \dots, \lambda\}$ : Apply inversely fitness-proportional mutation to  $y_i^j$ .  
 c) For all  $i \in \{1, 2, \dots, \mu\}$ : Keep the best search point from  $x_i \cup C_i$ .
3. **Network Dynamics**  
 If change of average normalised fitness less than  $\delta$   
 Then Calculate pairwise affinity  $d(x_i, x_j)$  of all search points.  
 If  $d(x_i, x_j) < \sigma$  Then remove worse search point. Update  $\mu$ .  
 Else continue at line 2
4. **Metadynamics** Introduce  $\lfloor d\mu \rfloor$  new search points in the network.
5. **Stopping** If stopping criterion not met continue at line 2.

**Variants** Non-elit selection for replacement

**Remark** Population size not fixed during optimisation

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## Analysing Artificial Immune Systems as Optimisers

- Observation** artificial immune systems as optimisers  
are randomised search heuristics used for optimisation just as evolutionary algorithms, ant colony optimisation, particle swarm optimisation, simulated annealing, iterated random local search, ...
- Consequence** AIS as optimisers should be considered the same way as other RSH as optimisers  
≡ applied as other RSH  
analysed as other RSH
- Fact** analysis of RSH as optimisers is important topic  
runtime analysis of EAs (intro) Pietro Oliveto, Per Kristian Lehre  
bio-inspired computation Carsten Witt  
parameterized complexity analysis Frank Neumann, Andrew Sutton  
theory of swam intelligence Dirk Sudholt  
black-box complexity Benjamin Doerr, Carola Doerr

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## Analysing Artificial Immune Systems

Why?

Because 'gaining a better understanding'

- of general limitations (black-box complexity)
- of behaviour in typical situations (example functions)
- of impact of specific operators (operators in (1+1)-frame)
- of parameter settings (simple algorithms with 1 parameter)
- for particular problem classes (classes of functions; combinatorial optimisation problems)

Because 'design of better randomised search heuristics'

- know when not to apply
- have an idea of when to apply
- have an idea of 'good' operators
- have an idea of 'good' parameter values
- have an idea of what kind of RSH
- ...

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## Analysing Randomised Search Heuristics – What?

**Observation** most important efficiency

≡ time  
Measuring time in randomised search heuristics

counting what	advantage	disadvantage	remark
computation steps	very precise	very tedious	rarely done (see [21])
function eval.	often good enough easier to handle	not exactly time still tedious	very common
rounds	convenient	imprecise can be misleading	very common

**Usually** count  $X$  until optimum found  
≡ optimisation time (RV  $\rightsquigarrow$  expectation ...)

**Sometimes** count  $X$  until good enough solution found  
≡ approximation time (RV  $\rightsquigarrow$  expectation ...)

**Alternative** analyse solution quality after  $X$   
≡ fixed budgeted computation (RV  $\rightsquigarrow$  expectation ...)  
(see [9], [22], [23])

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## Analysing Randomised Search Heuristics

Artificial Immune Systems?

Yet another class of Randomised Search Heuristics?

Why should I care?

**Facts** artificial immune systems offer

- useful alternative design paradigm for RSHs
- have different operators with different properties  
↪ useful in different situations
- can be a simpler and at least equally efficient alternative to crossover-based EAs

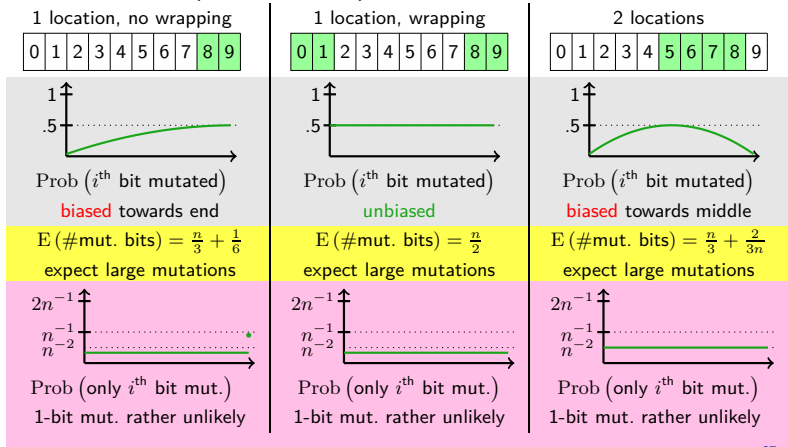
**Now**

- overview of three different types of AIS-specific mutation
- considering ageing as example for a 'meta-dynamic'
- example of a complete AIS in combinatorial optimisation

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## Contiguous Hypermutations

**Remember** (here with  $r = 1$ )



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## Analysing Contiguous Hypermutations

### Method

- Insert mutation operator in (1+1)-framework.  
→ study of effects in isolation
- Prove general observations.  
→ knowledge of general properties
- Compare with (1+1) EA (with mutation probability  $1/n$ ) on well-known example functions.  
→ assessment of effects under well-known circumstances
- Find examples with extremely differing performance.  
→ clear understanding of benefits and drawbacks

**Observation** method not unique to contiguous hypermutations but **generally applicable** for study of operators

**Remember** use Tiny AIS Box, try for yourself  
<http://www.cs.bham.ac.uk/~zargesc/ais-tutorial/>

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## Results About Contiguous Hypermutations (Part 1)

**General Observation**  $\forall f$  with unique global optimum:

$$E(T_{\text{CHM}_{1, \text{no w.}, f}}) = \Omega(n)$$

$$E(T_{\text{CHM}_{1, \text{w.}, f}}) = \Omega(n^2)$$

$$E(T_{\text{CHM}_{2, f}}) = \Omega(n^2)$$

due to probability of final mutation  
(all bounds tight)

**Comparison** for  $\text{ONEMAX}(x) = \sum_{i=1}^n x[i]$

$$E(T_{(1+1) \text{ EA}, \text{ONEMAX}}) = \Theta(n \log n)$$

$$E(T_{\text{CHM}_{1, \text{no w.}, \text{ONEMAX}}}) = O(n^2 \log n)$$

$$E(T_{\text{CHM}_{1, \text{w.}, \text{ONEMAX}}}) = \Theta(n^2 \log n)$$

$$E(T_{\text{CHM}_{2, \text{ONEMAX}}}) = \Theta(n^2 \log n)$$

due to difficulty of making 1-bit improvements

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## (1+1) EA and CHM on ONEMAX in the Tiny AIS Box

Name	Current value	Name	Current value
Objective function	OneMax 1	Objective function	OneMax 1
Optimisation goal	Maximisation	Optimisation goal	Maximisation
Problem size	30	Problem size	30
Budget per run	0	Budget per run	0
Runs	1	Runs	1
Population size	1	Population size	1
Offspring population size	1	Offspring population size	1
Hypermutation operator	contiguous mutation potential inverse fitness-proportional EA mutations	Hypermutation operator	contiguous mutation potential inverse fitness-proportional EA mutations
Parameter $r$ from (0, 1)	1	Parameter $r$ from (0, 1)	1
CHM variant	wrap no wrap two positions	CHM variant	wrap no wrap two positions
Crossover		Crossover	
Ageing		Ageing	
<b>Empirical Results</b>		<b>Empirical Results</b>	
Run started on	OneMax	Run started on	OneMax
Current run	8 (out of 8)	Current run	8 (out of 8)
Current best function value	31 (best is 31)	Current best function value	31 (best is 31)
Number of function evaluations	200	Number of function evaluations	1800
Average of finished runs	177.40	Average of finished runs	2516.80
Standard deviation of finished runs	55.34	Standard deviation of finished runs	1063.73

<http://www.cs.bham.ac.uk/~zargesc/ais-tutorial/>

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## (1+1) EA, CHM on ONEMAX: Fixed Budget Perspective

**Remember**  $E(T_{(1+1) \text{ EA}, \text{ONEMAX}}) = \Theta(n \log n)$  for any initialisation  
 $E(T_{\text{CHM}_{1,w}, \text{ONEMAX}}) = \Omega(n^2)$  for any initialisation

**Now** fixed budget perspective delivering richer picture  
 by giving expected function value after  $b$  steps for any  $b$

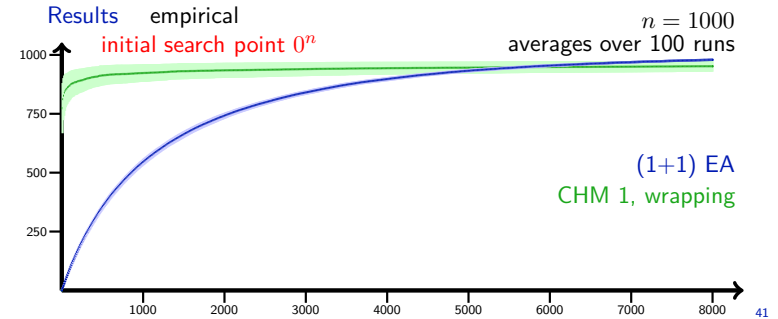
**Results** analytical  
 $\forall b \in \{1, 2, \dots, \lfloor (13/24)n \rfloor\}$ :  
 $E(\text{ONEMAX}(x_{b, \text{CHM}_{1,w}})) > E(\text{ONEMAX}(x_{b, (1+1) \text{ EA}}))$   
 $\forall b > (4/5)n \ln n$ :  
 $E(\text{ONEMAX}(x_{b, \text{CHM}_{1,w}})) < E(\text{ONEMAX}(x_{b, (1+1) \text{ EA}}))$

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## (1+1) EA, CHM on ONEMAX: Fixed Budget Perspective

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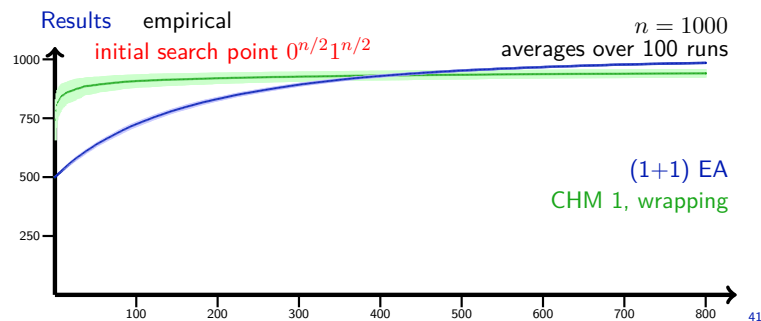


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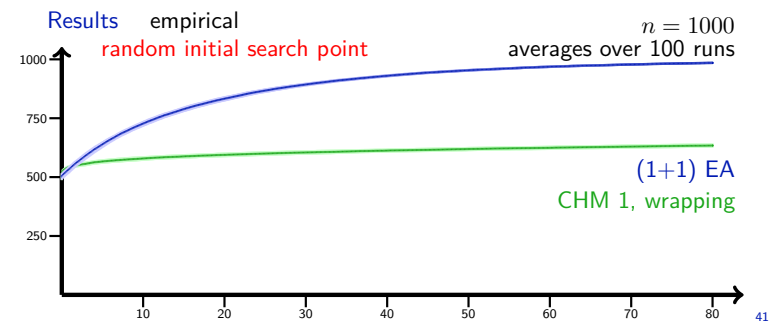


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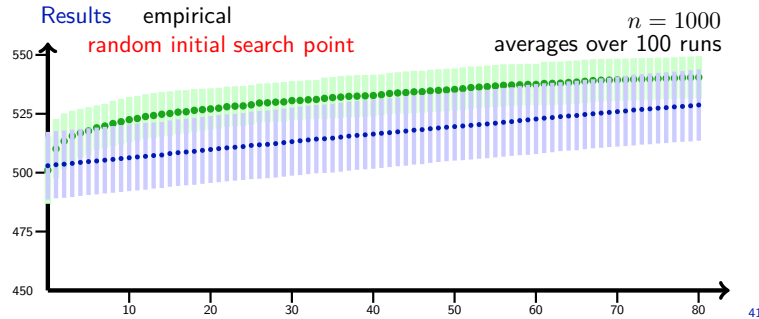


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## (1+1) EA, CHM on ONEMAX: Fixed Budget Perspective

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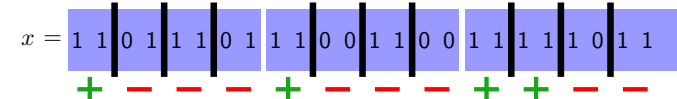
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## Results About Contiguous Hypermutations (Part 2)

**Demonstrate** very large performance difference  
 to demonstrate understanding of benefits and drawbacks

$$\text{CLOB}_{b,k}(x) = n \cdot \left( \sum_{h=1}^k \sum_{i=1}^{n/(bk)} \prod_{j=1}^{i \cdot b} x[(h-1) \cdot (n/k) + j] \right) - \text{OneMax}(x)$$

**Example** with  $n = 24, k = 3, b = 2$



$$\text{CLOB}_{2,3}(x) = 24 \cdot (1 + 1 + 2) - 17 = 79$$

**Comparison** for  $\text{CLOB}_{b,k}$  (with  $n/(k \cdot b) \in \mathbb{N}, l := n/k$ )  
 $E(T_{(1+1) \text{ EA, CLOB}_{b,k}}) = \Theta(k \cdot l^b \cdot (l/b + \log k))$   
 $E(T_{\text{CHM, CLOB}_{b,k}}) = O(n^2 \log n)$  (all 3 variants)  
 since length of block does not matter

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## Summary Contiguous Hypermutations

- **difficulties** with flipping single bits  
 $\rightsquigarrow$  bad at locating optima precisely  
 $\Rightarrow$  combine with other operators if locating optima precisely matters
- in expectation mutate  $\Theta(n)$  bits  
 $\rightsquigarrow$  **advantages** when huge mutations are needed  
 $\Rightarrow$  worth a try when hill-climbing not effective
- some variants with strong positional bias  
 $\rightsquigarrow$  **advantages/disadvantages** depending on function  
 $\Rightarrow$  only use variants with positional bias if known facts about objective function make that appear useful
- all noticeable effects rely on  $r \approx 1$   
 $\rightsquigarrow$  even  $r = 1 - \varepsilon$  ( $\varepsilon > 0$  constant) **not useful**  
 $\Rightarrow$  use  $r = 1 - o(1)$ , e.g.,  $r = 1 - 1/n$

(for details see Jansen/Zarges (2011) [18])

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## Inverse Fitness-“Proportional” Hypermutations

**Remember**

**Normalisation** **opt**  $\hat{f}(x) = f(x)/f_{\text{opt}} \in [0, 1]$   
**best**  $\hat{f}(x) = f(x)/f_{\text{current best}} \in [0, 1]$

**Mutation probabilities** **CLONALG**  $e^{-\rho \hat{f}(x)}$   
**opt-aiNet**  $e^{-\hat{f}(x)}/\rho$

**resulting** in four variants

- $\text{CLONALG}_{\text{opt}}$
- $\text{CLONALG}_{\text{best}}$
- $\text{opt-aiNet}_{\text{opt}}$
- $\text{opt-aiNet}_{\text{best}}$

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## Results About Inverse Fitness-“Prop.” Hypermut. (Part 1)

**Results** for ONEMAX

- $E(T_{(1+1) \text{ EA, ONEMAX}}) = \Theta(n \log n)$
- $E(T_{\text{CLONALG}_{\text{opt}}, \text{ONEMAX}}) = 2^{\Omega(n)}$  for  $\rho = O(1)$   
(even with high prob.) since mutation probability too large
- $E(T_{\text{CLONALG}_{\text{opt}}, \text{ONEMAX}}) = 2^{\Omega(n)}$  for  $\rho = \Omega(n)$   
(even with high prob.) since mutation probability too small
- $E(T_{\text{CLONALG}_{\text{opt}}, \text{ONEMAX}}) = 2^{\Omega(n^{5-\epsilon})}$  for  $\rho = \ln n$   
(even with high prob.)
- but  $O(n \log n)$  once  $\text{ONEMAX}(x) = n - O(n/\log n)$
- $E(T_{\text{CLONALG}_{\text{best}}, \text{ONEMAX}}) = \Theta(\mu n + n \log n)$  for  $\rho = \ln n$   
using population of size  $\mu$
- $E(T_{\text{opt-aiNet}_{\text{opt}}, \text{ONEMAX}}) = 2^{\Omega(n)}$  for  $\rho = 1$
- (even with high prob.) since mutation probability too large
- $E(T_{\text{opt-aiNet}_{\text{opt}}, \text{ONEMAX}}) = \Theta(n \log n)$  for  $\rho = \Theta(n)$

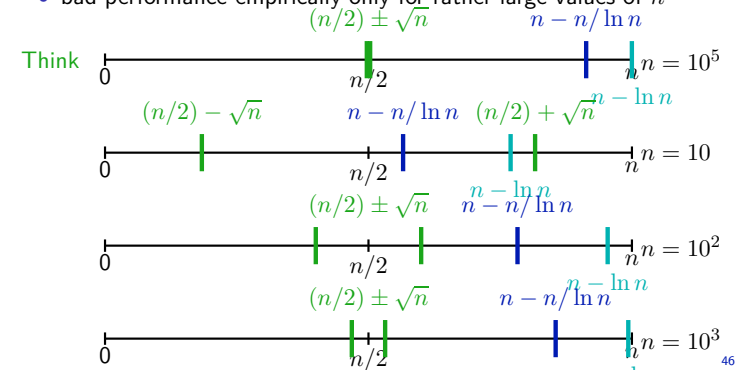
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## Results About Inverse Fitness-“Prop.” Hypermut. (Part 2)

**Remember**  $\text{CLONALG}_{\text{opt}}$  **inefficient** even with  $\rho = \ln n$

How is this possible in practice?

- under-estimating opt improves (see  $\text{CLONALG}_{\text{best}}$ )
- OneMax not necessarily realistic
- bad performance empirically only for rather large values of  $n$



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## Summary Inverse Fitness-“Proportional” Hypermutations

- can be very inefficient in simple situations  
→ e.g., bad at hill climbing  
⇒ use only when needed
- using ‘current best’ appears superior to ‘optimal value’ for normalisation  
→ populations useful  
⇒ prefer population-based approaches and ‘current best’ for normalisation
- $\text{CLONALG}$  **very sensitive** with respect to  $\rho$   
→ very bad performance easy to achieve  
⇒ prefer opt-aiNet
- only analytical results for ONEMAX  
→ most points **open**  
⇒ investigate more

(for details see Zarges (2008), (2009), (2011) [32, 33, 34])

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## Metadynamics in Artificial Immune Systems

**Remember** metadynamics influence behaviour of algorithm in a more global way

→ more difficult to analyse than an operator

**Example** ageing

**Remember** ageing

has **parameter maximal age**  $\tau_{\text{max}}$

comes in different variants (static pure, stochastic, ...)

depends non-trivially on implementation details

**Remember** **method** for analysis/work programme

- insert in simple algorithmic framework
- prove general observations
- compare with known algorithms on known problems
- find extreme examples to understand benefits and drawbacks

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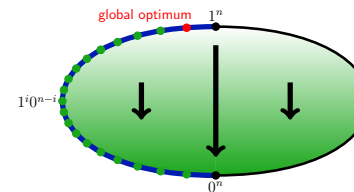
## A Simple Framework for Ageing

- use population of  $\mu$  search points  
Reason ageing only effective in populations
- increase age of all search points deterministically in each round  
Reason most commonly used ageing variant
- create only one new search point per round, using a well understood variation operator  
Reason introduce as little other complexity as possible
- implement ageing variant as simple as possible  
Static Pure Ageing
  1. new search point gets age 0 in case of an improvement, otherwise inherits age.
  2. remove all search points exceeding  $\tau_{\max}$ ; fill population with new random search points as needed

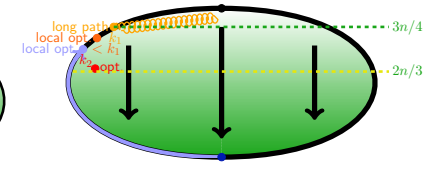
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## Parameter Study: The Maximal Age

Note maximal age  $\tau_{\max}$  must not be too small



Note maximal age  $\tau_{\max}$  must not be too large



In fact Appropriate range for  $\tau_{\max}$  can be extremely narrow.

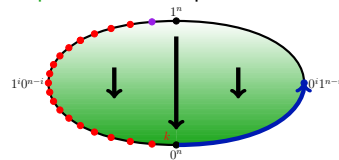
(for details see Horoba, Jansen, Zarges (2009) [16])

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## Comparing Ageing Variants

### Static Pure Ageing

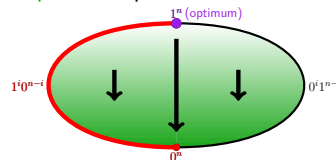
new search points get age 0  
if they improve  
superior in local optima



(Pothole in Tiny AIS Box)

### Evolutionary Ageing

new search points get age 0  
always  
superior on plateaus



(Plateau in Tiny AIS Box)

Combine both into genotypic ageing  
'new search points get age 0 unless they are copy or worse'  
combines advantages, good on plateaus and at local optima

(for details see Jansen, Zarges (2011) [19])

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## Summary Ageing

- ageing adds new dynamics and new capabilities  
⇒ increased potential at the price of additional parameter  
⇒ use with care
- ageing very sensitive with respect to maximal age  
⇒ difficult to set additional parameter  
⇒ perform careful parameter study
- different ageing variants have different capabilities  
⇒ no 'one size fits all' solution  
⇒ try different variants
- ageing very sensitive with respect to implementation details

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Introduction ○○	Natural Immune System ○○○○○○○○○○	AIS as Classifiers ○○○○ ○○○	AIS as Optimisers ○○○○○○○○ ○○○○	Analysing AIS ○○○○○○○○○○○○○○○●○○	Conclusion & References ○○○
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## Summary

We have seen overviews and introductions of

- the natural immune system
- application of AIS as classifiers
- application of AIS as optimisers
- analysis of AIS as optimisers

all as invitation to

- learn more about AIS
- apply AIS
- explore and understand AIS

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Introduction ○○	Natural Immune System ○○○○○○○○○○	AIS as Classifiers ○○○○ ○○○	AIS as Optimisers ○○○○○○○○ ○○○○	Analysing AIS ○○○○○○○○○○○○○○○●○○	Conclusion & References ○○○
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## Conclusions

Artificial Immune Systems are

- models of the natural IS  $\hat{=}$  tool for research in immunology
- heuristic approach to classification based on an example of complex classification from nature
- randomised search heuristics capable of optimisation
  - based on a quite different natural metaphor (compared to EAs)
  - an alternative approach to optimisation, with different characteristics and capabilities
  - an alternative solution if your favourite approach fails
- randomised search heuristics like many others
  - another field of study, worthy of analysis just like EAs/ACO/PSO/...
  - another example of a complex class of RSHs  $\hat{=}$  another opportunity to study differences and similarities hopefully some day leading to useful taxonomy
- a fascinating area of research

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Introduction ○○	Natural Immune System ○○○○○○○○○○	AIS as Classifiers ○○○○ ○○○	AIS as Optimisers ○○○○○○○○ ○○○○	Analysing AIS ○○○○○○○○○○○○○○○●○○	Conclusion & References ●
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Introduction ○○	Natural Immune System ○○○○○○○○○○	AIS as Classifiers ○○○○ ○○○	AIS as Optimisers ○○○○○○○○ ○○○○	Analysing AIS ○○○○○○○○○○○○○○○●○○	Conclusion & References ●
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