Particles Types in a Swarm: Searching for Efficiency

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ABSTRACT

This research is devoted analysis of convergence in particle swarm where each of the particles can represent different dynamic behaviour in search space.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search

Keywords

Particle Swarm Optimization; Convergence Analysis; Discrete-Time Dynamic Systems

1. INTRODUCTION

Particle swarm is one of heuristic optimization approaches being a subject of great interest. This is a population-based method proposed by Kennedy and Eberhart in 1995 [2] where members of population, called particles, represent a sample of solutions of a given problem. A set of all possible solutions, called search space, is a subject of exploration by the algorithm. With use of the population, the algorithm samples the space. During this process the particles exchange information with each other like, for example, a swarm of bees or school of fishes. This way they coordinate their movement through the space searching for more promising regions. The quality of the points in the search space is defined by the evaluation function. The swarm is searching for the best point in the space, that is, for the optimum (maximum or minimum) of the evaluation function. Thus, the swarm can be called a heuristic optimization algorithm.

A simple scheme of the particle swarm optimization algorithm is given in Algorhitm 1. A particle $\mathbf{y}_i$ represents the best solution found by the i-th particle (called particle attractor), and a particle $\mathbf{y}^*$ – the best solution found by the swarm (called swarm attractor).

Numerous implementations are based on the particle parameters values presented by Clerc and Kennedy [1]. This method of balancing global and local searches known as *constriction* defines the particle speed equation as follows:

$$v_j^{t+1} = w[v_j^t + c_1 \cdot r_1^t \cdot (y_j^t - x_j^t) + c_2 \cdot r_2^t \cdot (y_j^{*t} - x_j^t)], \quad (1)$$

where:
v_j^t — particle's velocity,
x_j^t — particle's location,
y_j^t — the best solution the particle $\mathbf{x}^t$ has found so far,
y_j^{*t} — the best value obtained so far by any particle in the neighborhood of $\mathbf{x}^t$.
r_1, r_2 — random values: $U(0,1)$
$c_1 = c_2 = c$ as proposed by Clerc and Kennedy [1]

The value of w is derived from the existing constants in the velocity update equation:

$$w = \frac{2 \cdot \kappa}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \quad \text{where } \varphi = c_1 + c_2 \quad (2)$$

The factor κ controls balance between exploration and exploitation:

1. $\kappa \approx 0$: fast convergence, local exploitation,
2. $\kappa \approx 1$: slow convergence, high degree of exploration.

For $w > 4$ and $\kappa \in [0,1]$ the swarm convergence would be quick and guaranteed. So, using the constant $\varphi = 4.1$ to ensure convergence, Clerc and Kennedy proposed the values $c = 2.05$ and $w = 0.729843788$.

In our paper we study features of particles defined with different values of parameters c and w and propose a new, nonuniform configuration of particle parameters in the swarm. Presented results of experimental tests show that for some classes of problems this configuration is more efficient, that is, less computational cost is needed to find points located in a very close boundary of the global optimum.

Algorithm 1 the particle swarm optimization

```
Create and initialize the swarm: x_1, ..., x_N
repeat
  for i = 1 to N do
    if f(x_i) > f(y_i) then
      y_i = x_i
    end if
    if f(y_i) > f(y*) then
      y* = y_i
    end if
  end for
  update location and velocity of all the particles
until stop condition is satisfied
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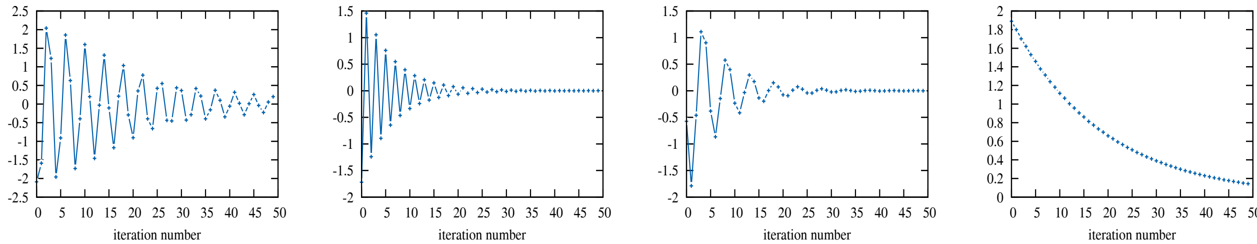

Figure 1: Convergence of particles of type: A, B, C, and D (from left to right).

2. THE SELECTED MODEL OF PARTICLE MOVEMENT

In our research a model of particle movement proposed in [1, 4] is investigated:

$$\mathbf{y}^{t+1} = A\mathbf{y}^t + Bp \tag{3}$$

where:

$$A = \begin{bmatrix} 1-c & w \\ -c & w \end{bmatrix} \quad \mathbf{y}^k = \begin{bmatrix} x^k \\ v^k \end{bmatrix} \quad B = \begin{bmatrix} c \\ c \end{bmatrix} \tag{4}$$

The model is equivalent of the model proposed by Shi and Eberhart [3] with the inertia weight factor. In [1, 4] authors convert the equations into the form applied in the theory of linear, discrete-time dynamic systems. This way particles with different movement characteristics can be identified easily, that is convergent particles, or particles with harmonic oscillatory behaviour or with zigzagging behaviour.

3. PARTICLE BEHAVIOR TYPES

Four types of particles are considered in our research. Their characteristics are depicted in Figure 1.

The safe configuration proposed in [1] represents uniform swarm which consists of particles of the same type. In our research we give careful consideration to the swarms assembled of some numbers of representatives for all the four types.

The most efficient configurations of particle types within a swarm are searched experimentally. The results of experiments for three types of test cases are presented in the next section.

4. BENCHMARKS

Three test-cases are selected for the research:

- a unimodal function defined with a formula same as 1-dimensional Gaussian probability density function:

$$f(x) = 1/(\sqrt{2\pi\sigma^2})\exp(\frac{(x-\mu)^2}{2\sigma^2}) \tag{5}$$

  with a mean μ and a standard deviation σ,

- a multimodal function build of two neighbouring hills (two Gaussian p.d.f.),

- the Ackley function:

$$\begin{aligned} f(\mathbf{x}) \quad &= \quad -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \\ &\quad \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e \end{aligned} \tag{6}$$

5. METHODOLOGY OF EXPERIMENTAL RESEARCH

5.1 Test-cases and evaluation measures

Two groups of experiments were performed:

1. with uni- and duo-modal function in one-dimensional search space,
2. with the Ackley function in two and five-dimensional search space.

For the evaluation of the results: for selected test-case and for a series of 100 executions of particle swarm optimization algorithm the mean values of numbers of execution of evaluation function to converge. The convergence is defined as finding a solution located in euclidean distance not greater than 10^{-3} from the optimum. The maximum number of iterations to converge is set to 1000. In the case of exceeding this limit the result is set to the upper boundary, that is, 1000.

5.2 Tested swarm configurations

A single swarm might consist of four types of particles, identified as A,B,C, and D.

In the first group of experiments the swarms consisted of: 1, 2, 3, and 5 particles of any type. For swarms of size 1 and 2 all the possible sets of particles were tested. In the remaining two just the selected configurations were applied. All the cases, that is, 1, 2, 3, and 5 particles, were also tested with swarms consisting of standard particles, that is, having the parameters of velocity formula defined as advised in [1]. It is worth noting that the behaviour of the standard particles is of type A, however, the proposed configuration of particles of type A has slightly different values of parameters.

In the second group of experiments the swarm configurations originated from the output of a genetic algorithm (GA). First, GA searched for the most promising sets of particles where the chromosome represented the entire single swarm of a given size. The chromosomes sizes: from 1 to 20 with step 1 for 2-dimensional search space and from 30 to 50 with step 5 and also for a size of 100 for 5-dimensional search space. The value of a chromosome is defined as the mean value of numbers of iterations to converge for a series of 100 executions. The population consisted of 100 chromosomes. Every time, a single GA execution consisted of 15 iterations. For each of the chromosome sizes GA was executed 20 times for 5D search space and 50 times for 2D.

Then the obtained best chromosomes, that is, the best configurations of swarms were subject of more detailed analysis. For each of the test-cases from the series of 20 (for 5D)

Table 1: Convergence for single particles of the four types and two types of fitness landscape

landscape\type	A	B	C	D
Unimodal	**23.84**	39.88	24.91	67.47
Duomodal	**24.37**	37.12	31.72	80.33

Table 2: Convergence for swarms of size two and unimodal fitness landscape

type\type	A	B	C	D
A	**257.0**	566.46	501.34	598.7
B		1808.32	1201.46	1728.88
C			514.04	1116.72
D				1291.42

or 50 (for 2D) outputs (that is, the best chromosomes in the final population) the best among them was selected. For the obtained chromosome the number of particles for each of the four types is counted. The histogram with the counted numbers is the final result of experiment for a given swarm size.

6. RESULTS OF EXPERIMENTS

6.1 First group of experiments

Experiments from the first group are presented in Tables 1, 2, 3, 4 and 5.

Tables 1, 2 and 3 show the convergence for swarms consisting of one and two particles. Particles of type A give the best convergence, however, for the duo-modal landscape and two particle-swarm the winner configuration are ex aequo AA and AC. Dominance of A type which is the same type as proposed in [1] is evident.

For the swarms of size 3 and 5 and the unimodal landscape the highest convergence can be obtained with mix of A, C and D type, however, the second best results is without C, which means that the role of C particle is very similar to A. This can be observed also in Table 3 where the winning configuration also contains particle of type C. For the remaining cases (see Table 4) the common observation is that A particles should be the majority of a swarm, but for better results a small fraction of C particles should appear. For the swarm of size 5 in both cases there is also a particle of type B.

Table 5 shows the convergence of the swarms of size 1, 2, 3 and 5 which are build of standard particles. The standard swarm convergence is worse for all the cases except from the swarm of size 5 where the convergence of standard swarm significantly outperforms nonuniform swarms.

Table 3: Convergence for swarms of size two and duomodal fitness landscape

type\type	A	B	C	D
A	636.52	908.12	**635.0**	740.16
B		1927.12	1411.52	1871.36
C			1012.56	1277.78
D				1658.94

Table 5: Convergence for four swarm sizes with standard particles

landscape\size	1	2	3	5
Unimodal	43.49	639.3	342.69	**166.25**
Duomodal	**42.49**	927.72	821.61	439.1

Table 6: Configurations of swarms: numbers of particles of a given type for the types A, B, C, and D for different swarm sizes in 2-dimensional search space

size\type	A	B	C	D	convergence
2	2	0	0	0	1325
3	3	0	0	0	916.5
4	3	0	1	0	510.8
5	3	0	1	1	526,5
6	0	0	6	0	243
7	2	0	5	0	233.8
8	2	0	6	0	264.0
9	6	1	2	0	248.4
10	4	0	6	0	268.0
11	3	2	5	1	288.2
12	3	1	7	1	254.4
13	6	1	4	2	257.4
14	4	1	7	2	312.2
15	3	2	9	1	285.0
16	5	2	8	1	286.4
17	6	1	8	2	294.1
18	4	5	9	0	306.0
19	4	3	11	1	321.1
20	5	5	10	0	300.0

6.2 Second group of experiments

Experiments from the second group with the Ackley function are presented in Tables 6, 7 and 8.

Table 6 shows the best configurations and convergence levels for swarms of size from two to 20 found by GA for the Ackley function defined in 2-dimensional search space. One can see, that there is a significant rise of the number of A particles as the swarm size grows. The proportion between the types of particles for the very small swarm sizes shows the dominance of particle A. For larger swarm sizes the number of particles A remains constant whereas the number of particles C grows. In both cases the numbers of particles B and D stay low.

Table 7 shows the best swarm configurations and convergence levels obtained as previously but for the 5-dimensional search space and swarm sizes from 20 to 50 with step 5. As in the case of two-dimensional search space the numbers of particles A and C are higher than the numbers of particles B and D.

The results for the final part of experiments is presented in Table 8. Based on the observation obtained so far, the structure of swarms applied in these experiments was: 3 parts of particles A and 2 parts of particles C and with no particles B and D regardless of the search space dimensionality (briefly 3A2C-swarm). In particular, for the swarm of size 20 there are 12 A particles and 8 C particles, and for the swarm of size 50 there are 30 A particles and 20 C particles and so on. The obtained number is a mean of numbers of evaluation function executions in 100 runs of particle swarm

Table 4: Convergence for 10 best swarms of size 3 and 5 for uni- and duomodal fitness landscape

rank\landscape	three particles		five particles	
	Unimodal	Duomodal	Unimodal	Duomodal
The best – 1	354.42- ACD	365.88 - AAC	504.75 - AAABC	560.65 - AAABC
2	357.12 - AAD	527.43 - AAB	506.05 - ABCCD	575.55 - ABBCC
3	362.07 - CCC	547.23 - ACD	507.3 - AABCD	590.08 - AACCD
4	416.79 - AAA	548.85 - ACC	508.15 - AAACD	597.7 - AAAAC
5	432.27 - ABB	597.6 - ABC	509.95 - AAADD	617.7 - ACCCD
6	432.93 - ABD	668.94 - AAD	515.3 - AAAAB	642.7 - AABBC
7	456.15 - CCD	734.64 - AAA	515.6 - ACCCD	643.7 - AAACD
8	480.45 - ADD	824.91 - ABD	517.2 - AADDD	646.15 - AABCD
9	511.83 - BCC	837.18 - BCC	526.8 - ACCCC	696.15 - ACCCC
10	789.99 - BCD	873.78 - CCD	528.65 - AAAAD	704.35 - AAACC

Table 8: No. of evaluation function executions of swarms (mean value, the std. deviation and Student's t-test value) with the best configurations compared with the convergence of standard swarms

size\conf.	two dimensional search space			five dimensional search space		
	3A2C	Std	t-test	3A2C	Std	t-test
20	701.36 ±38.86	818.66 ±273.26	6,68246E-07	13280.24±821.02	12545.98 ±847.16	0.20790
25	825.625 ±47.2	937.725 ±70.469	5,80947E-32	12829.225±1213.75	11982.325 ±1121.55	0.06603
30	967.53 ±285.3	1087.56 ±46.77	4,92E-05	13103.73±1223.04	11758.74 ±1120.89	0.00759
35	1049.265 ±55.615	1235.885 ±58.205	0,000369	13486.48±1503.88	12071.29 ±1484.91	0.37055
40	1172.8 ±52.84	1371.12 ±72.36	2,24579E-36	13498.92±1639.56	12519.36 ±1819.28	0.21277
45	1270.8 ±57.105	1511.955 ±62.46	9,66967E-49	13962.69±1877.04	13026.96 ±2008.755	0.08600
50	1375.35 ±57.05	1651.1 ±69.4	6,22448E-48	14622.35±2063.2	13228.50 ±1874.05	0.01534

Table 7: Configurations of swarms: numbers of particles of a given type for the types A, B, C, and D for different swarm sizes in 5-dimensional search space

size\type	A	B	C	D	convergence
20	5	5	9	1	11326.0
25	8	3	9	5	21159.0
30	7	5	9	9	11787.0
35	11	6	10	8	9943.5
40	13	3	17	7	5460.0
45	14	10	15	6	8347.5
50	15	5	17	13	6855.0

optimization algorithm until solution is located in euclidean distance not greater than 10^{-3} from the optimum. This is compared with the same mean for the uniform swarm consisting only of standard particles. In addition there is also standard deviation for the given mean.

For the two-dimensional search space in every case the 3A2C-swarm outperforms the uniform one.

For the five-dimensional search space mean values for the uniform swarm has a slight advantage over the values for the 3A2C-swarm. However, due to large values of standard deviations we decided to do additional statistical tests. Student's t-test for the compared convergence means revealed lack of statistical difference for 5 out of 7 swarm sizes selected for experiments. Therefore, assuming normal distribution of the results we may say that for 5-dimensional search space the two swarm types offer similar efficiency of the algorithm.

7. CONCLUSIONS

This paper demonstrates the performance of nonuniform swarms, consisting of 4 types of particles, compared to uniform consisting only of standard particles.

We tested nonuniform swarms configurations in order to find the fastest converging one. Aforementioned experiments show clearly that nonuniform swarms can be optimized to perform similar or even better than uniform swarms consisting solely of particles proposed in [1].

Additional research can be conducted to test other values for parameters w and c as well as other swarms' compositions in order to improve convergence.

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