# A Taxonomy of Heterogeneity and Dynamics in Particle Swarm Optimisation

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**Abstract.** We propose a taxonomy for heterogeneity and dynamics of swarms in PSO, which separates the consideration of homogeneity and heterogeneity from the presence of adaptive and non-adaptive dynamics, both at the particle and swarm level. It supports research into the separate and combined contributions of each of these characteristics. An analysis of the literature shows that most recent work has focussed on only parts of the taxonomy. Our results agree with prior work that both heterogeneity, where particles exhibit different behaviour from each other at the same point in time, and dynamics, where individual particles change their behaviour over time, are useful. However while heterogeneity does typically improve PSO, this is often dominated by the improvement due to dynamics. Adaptive strategies used to generate heterogeneity may end up sacrificing the dynamics which provide the greatest performance increase. We evaluate exemplar strategies for each area of the taxonomy and conclude with recommendations.

## 1 Introduction

There has recently been a sharp rise in interest in heterogeneity of swarms for particle swarm optimisation (PSO). Since early results (e.g. [1]) showed the potential benefit of heterogeneity to PSO, it has been shown to offer a high robustness to unknown problems [2]. In an effort to improve the performance and robustness of heterogeneous PSO variants, more recent work (e.g. [3,4]) has focussed on heterogeneity driven by particle-level adaptation, based on run-time information. However, in this drive to add complexity to PSO by incorporating heterogeneity, behavioural dynamics and run-time adaptation, there is a key question which has not yet been fully addressed: are the observed performance improvements due to better heterogeneity itself, run-time adaptation based on state information, or simply the increase in behavioural dynamics? In this paper we tease out these three components of modern adaptive heterogeneous PSO variants, in order to provide some insight into this question.

Our first contribution is a taxonomy of heterogeneity and dynamics in PSO, into which we place existing PSO variants from prior work. Accordingly we show that most recent research has focussed on only part of the design space arising from our taxonomy, in particular neglecting non-adaptive dynamic PSO in both

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heterogeneous and homogeneous cases. Our second contribution is to show that these neglected regions of the taxonomy contain PSO variants which outperform similar adaptive heterogeneous variants. Furthermore, the introduction of dynamics often has a greater impact on performance than the introduction of heterogeneity. Therefore, this paper provides insight into existing PSO variants and makes recommendations for future PSO research.

Montes de Oca et al. [2] describe a heterogeneous swarm as one in which at least two particles differ from each other. They found that heterogeneous swarms typically outperform the worst, and in some cases the best homogeneous swarm on a particular problem. They propose that heterogeneity mitigates the risk of choosing the "wrong" variant of PSO for an unknown problem. They identify three types of heterogeneity: i) *static heterogeneity*, in which particles in a heterogeneous swarm never change their configuration (i.e. behaviour), ii) *dynamic heterogeneity*, in which particles' configurations change either randomly or according to some predetermined sequence over time, and iii) *adaptive heterogeneity*, where particles' configurations change based on the state of the swarm; we use these three classes as a starting point for our taxonomy. From their analysis, they conclude that future work should focus on adaptive heterogeneity to improve robust performance of PSO across different problems.

Nipomucino and Engelbrecht [5] define dynamic swarms as those in which particles change their behaviours during the search, also drawing the distinction between static, dynamic and adaptive heterogeneous swarms. PSO variants in the above categories have been proposed by Spanevello and Montes de Oca [6], Li and Yang [7], Engelbrecht [3] and Nipomucino and Engelbrecht [8]. While much of the work on dynamic swarms focuses on heterogeneity, dynamics have also proven useful in homogeneous swarms. In one of the most successful early variants of PSO, Shi and Eberhart [9] proposed varying particles' *inertial weight* over a swarm's lifetime. Later variants attempt to improve on this with more complex models. Chatterjee and Siarry [10] propose a non-linear update scheme for inertial weights suggesting, however, that an adaptive mechanism for online parameter choice would make their algorithm more robust. Such adaptive algorithms have shown good empirical performance [11–13]. Other homogeneous PSO variants use feedback to choose between a discrete set of behaviour types. Riget and Vesterstrøm [14] propose a variant which monitors diversity in order to prevent premature convergence, switching behaviour when particles are closely clustered. Similarly, Evers and Ghalia [15] propose a variant which performs a one-time update of particle positions when diversity drops below some threshold.

## 2 Forms of Heterogeneity and Dynamics in PSO

Firstly, we focus on whether or not the swarm is homogeneous or heterogeneous. **Homogeneous** swarms are those in which at each point in time, all particles exhibit the same behaviour as each other. **Heterogeneous** swarms are those in which at some point in time, at least two particles exhibit different behaviours from each other. In this description, we focus on particle update behaviour.

However, our taxonomy can also be used to describe other forms of swarm heterogeneity. We further break down homogeneous and heterogeneous PSO variants according to how the distribution of behaviours in the swarm changes over time:

- In **static** swarms, the behaviour of each particle does not change over time.
- In **constrained dynamic** swarms, there is a stationary proportion of each behaviour, under expectation, between time windows of a predefined size. Therefore, in constrained dynamic homogeneous swarms, the entire swarm might progress through a static sequence of behaviours in synchrony.
- In **dynamic** swarms, the proportion of each behaviour changes over time. In dynamic homogeneous swarms the entire swarm might progress through a sequence of behaviours in synchrony, and this sequence varies over time.
- In **adaptive** swarms, the proportion of each behaviour changes over time in response to the state of the algorithm as perceived by the particles.

The majority of PSO variants use **static homogeneous** swarms. Most recent work on heterogeneity in PSO has focussed on the use of adaptive strategies to generate particle behaviour, and therefore use **adaptive heterogeneous** swarms. However, these results have often been used to argue that heterogeneity of a swarm *per se* is beneficial, despite the characteristics of heterogeneity, particle-level and swarm-level dynamic behaviour and adaptivity being conflated. By dividing both homogeneous and heterogeneous swarms into the above groups, we can study heterogeneity separately from dynamics and adaptation. Table 1 shows a classification of existing literature in terms of the taxonomy. This classification includes the earlier categorisation of PSO variants with update rule heterogeneity due to Montes de Oca et al. [2]. It builds on it firstly by considering work in the context of our expanded taxonomy which accounts for dynamics and adaptivity apart from any heterogeneity present, and secondly by including the significant amount of work on heterogeneous PSO since 2009. It is clear that, despite the recent work on heterogeneity in PSO, a great deal of the space defined by the taxonomy remains to be explored.

## 3 Experimental Analysis of Heterogeneity in PSO

Table 1 shows that despite the recent work on heterogeneity in PSO, there is a large part of our taxonomy yet to be explored. Next, we present an experimental study which establishes that such exploration would be fruitful. We differentiate between update particles on the basis of parameters and by using two qualitatively different behaviours: Standard PSO [16] and Barebones PSO [17]. These use different information (Standard PSO requires velocity while Barebones PSO does not); however, so that a particle can switch freely between behaviours, we require that particles maintain all information required by either behaviour. We say that this is the particle's **cognitive information**: the set $\mathcal{C}_p(t) = \{\boldsymbol{x}_p(t), \boldsymbol{v}_p(t), \boldsymbol{h}_p(t)\}$ where: $\boldsymbol{x}_p(t)$ is the particle's **position**, $\boldsymbol{v}_p(t)$ is the particle's **velocity** and $\boldsymbol{h}_p(t)$ is the particle's **historic best position**. We assume that the aim of a PSO algorithm is to find an input that minimizes the

**Table 1.** A classification of existing PSO variants, in terms of the proposed taxonomy

| ***Characteristic*** → <br> ***Behaviour*** ↓ | **Homogeneous** | **Heterogeneous** |
|---|---|---|
| **Static** | Static homogeneity<br>– *Many PSO variants, including standard PSO [16], barebones PSO [17] etc.* | Static heterogeneity<br>– *Static heterogeneous PSO (details not available) [6].*<br>– *sHPSO: (random assignment) [3].*<br>– *Static heterogeneous PSO (various fixed proportions) [2].*<br>– *Predator & prey particles [1].*<br>– *Neutral & charged particles [18].*<br>– *Fitness-distance-ratio and standard particles [19].*<br>– *Quantum particles [20].*<br>– *Extra central particle [21].* |
| **Constrained Dynamic** | Constrained dynamic homogeneity<br>– *None.* | Constrained dynamic heterogeneity<br>– *Different maximum velocities after restarts (constrained after initialisation phase) [22].* |
| **Dynamic** | Dynamic homogeneity<br>– *Inertia weight decay: time-based linear [9] and non-linear [10] update.* | Dynamic heterogeneity<br>– *None.* |
| **Adaptive** | Adaptive homogeneity<br>– *Fuzzy adaptive PSO [11] and fuzzy adaptive informed PSO [13]: inertia weight of entire swarm updated based on fuzzy system.*<br>– *Adaptive PSO: swarm parameters updated based on evolutionary state estimation [12].*<br>– *ARPSO: particles simultaneously switch between two behaviours based on diversity [14].*<br>– *RegPSO: all particles perform a one-time position update at low diversity [15].* | Adaptive heterogeneity<br>– *Stagnation threshold [6].*<br>– *Difference proportional probability [6].*<br>– *dHPSO: win-stay-lose-shift [3].*<br>– *pHPSO and pHPSO-lin: inspired by ants [5].*<br>– *$f_k$-PSO: probability of behaviour based on prior performance [8].*<br>– *ALPSO: particle-level probability matching [7].*<br>– *SLPSO: biased probability matching & super-particle [4].*<br>– *Cooperator and defector particles [23].*<br>– *Various adaptive heterogeneous parameters (see [2]).* |

result of a **cost function** $f$, and so a particle's historic best position is simply the lowest cost position it has visited so far. When updating cognitive information, particles may make use of information from their **neighbourhood**: a set of particles whose states they can observe. In this paper, we assume that all particles neighbour each other, allowing particles to make use of the **global historic best position** $\hat{\boldsymbol{h}}_p(t)$: the lowest cost position discovered by any particle.

In **Standard PSO** a particle $p$ updates its velocity in dimension $d$ as follows:

$$v_{p,d}(t+1) = \eta \ v_{p,d}(t) + \phi_1 \cdot r_{1,d} \ (h_{p,d}(t) - x_{p,d}(t)) + \phi_2 \cdot r_{2,d} \ (\hat{h}_d(t) - x_{p,d}(t))$$

where $\eta$ is the inertial weight coefficient and $r_{1,d}, r_{2,d}$ are independent random numbers drawn from $U[0,1]$. Particles are attracted to cognitively and socially determined positions ($\boldsymbol{h}_p(t)$ and $\hat{\boldsymbol{h}}(t)$ respectively) and the constants $\phi_1, \phi_2$ determine the relative importance of these positions. If $\boldsymbol{h}_p(t) = \hat{\boldsymbol{h}}(t)$, then the social component of equation 3 is ommited (effectively $\phi_2$ is set to 0).

In **Barebones PSO** a particle updates its position in dimension $d$ as follows:

$$x_{p,d}(t+1) \sim N\left(\frac{h_{p,d}(t) + \hat{h}_d(t)}{2}, |h_{p,d}(t) - \hat{h}_d(t)|\right)$$

where $N(\mu, \sigma)$ is the Normal distribution with mean $\mu$ and standard deviation $\sigma$. Barebones PSO does not make use of a velocity component but, for it to be compatible with standard PSO, we set $\boldsymbol{v}_p(t+1) = \boldsymbol{x}_p(t+1) - \boldsymbol{x}_p(t)$.

We say that a particle's **behaviour** at time $t$, $b_p(t)$ is the update function it is using at that time. In order for a swarm to be dynamic or heterogeneous its particles must be capable of expressing more than one behaviour. A PSO variant is comprised of a set of update functions and a strategy for selecting between these functions. In our study, we make use of two such **behaviour sets** composed of variants of standard PSO and barebones PSO.

The first behaviour set, **cognitive-biased and social-biased (CBSB)** contains two parametrically different versions of standard PSO. The cognitive-biased function (let $\phi = 0.5 + \log 2$ then $\phi_1 = \frac{5\phi}{3}$, $\phi_2 = \frac{\phi}{3}$, $\eta = \frac{1}{2\log 2}$) makes more use of cognitive information and is suited to exploration, while the social-biased function ($\phi_1 = \frac{\phi}{3}$, $\phi_2 = \frac{5\phi}{3}$, $\eta = \frac{1}{2\log 2}$) makes more use of social information and is more suited to exploitation. This set allows us to investigate if an algorithm expressing two behaviours which, intuitively, are suited to performing different search tasks, is capable of improving on the best static homogeneous variant. The second behaviour set, **cognitive-biased and barebones (CBBB)** contains two quantitatively different functions: the cognitive-biased function above and barebones PSO. Unlike the CBSB set, the roles of the two functions during search is not so clearly complementary. This allows us to investigate if any results obtained with CBSB only apply when we have clearly complementary behaviours or whether they apply more generally.

### 3.1 Exemplar Strategies

We now consider the concrete strategies which particles use to select behaviours. The strategies used are simple exemplars, allowing us to realise the full range of swarm-level characteristics described in section 2. With the exception of the adaptive strategies they are equally applicable to homogeneous and heterogeneous swarms to allow a direct comparison. Particles in **static** swarms, by definition, never update their behaviour and so set $b_p(t+1) = b_p(t)$.

Particles in **constrained dynamic** swarms update their behaviour such that the proportion of behaviours is static over some defined time window. For homogeneous swarms, a constrained dynamic selector must be deterministic. We use a time-based selector which cycles through all possible behaviours, with the time spent using each given by $\boldsymbol{\tau} = (\tau_1, ..., \tau_{N_u})$ where $\tau_b$ is the number of time-steps a particle can spend in behaviour $b$ before switching. After this time has expired, particles deterministically change to the next behaviour $b$ with $\tau_b \neq 0$.

For comparability with the constrained dynamic selector, we use a deterministic time-based **dynamic** selector. Similarly to the inertia weight decay used in [9], it is based on the intuition that certain behaviours are advantageous at the start of search while others are advantageous at the end. $\boldsymbol{\tau}$ becomes non-static, given by a linear progression from $\boldsymbol{\tau}^{\text{start}}$ to $\boldsymbol{\tau}^{\text{end}}$ based on the fraction of the evaluation budget used. Behaviour is then updated as in the constrained dynamic case. We choose $\boldsymbol{\tau}$ such that we have 10 behavioural cycles per run ($10 \cdot \sum_{\tau \in \boldsymbol{\tau}} \tau = budget$ where *budget* is the swarm's budget of function evaluations) which we have empirically found to be reasonable. Unless specified, $\tau_1 = \tau_2$ in the constrained dynamic case and $\tau_1^{\text{start}} = |\boldsymbol{\tau}|$, $\tau_2^{\text{end}} = |\boldsymbol{\tau}|$ in the dynamic case.

For the above selectors, homogeneity and heterogeneity differ only in **initialization**. Homogeneous swarms are initialized uniformly with the desired initial behaviour while heterogeneous swarms are initialized according to a swarm fraction $\boldsymbol{\rho} = (\rho_1, ..., \rho_{N_u})$ where $\rho_b$ is the fraction of particles initialized with $b_p(1) = b$. This fraction is analagous to the time vector $\boldsymbol{\tau}$ with $\rho_b = \frac{\tau_b}{|\boldsymbol{\tau}|}$.

We use an **adaptive** selector based on *win-stay-lose-shift* [3]. Homogeneous (respectively heterogeneous) particles keep track of the time $\upsilon_p(t)$ since the global best (respectively, their historic best) position improved. If this time exceeds a threshold $\theta$ then the particle will choose another behaviour uniformly at random. Note that an adaptive selector can only be guaranteed to produce a homogeneous swarm if it acts based on global information. In both the homogeneous and the heterogeneous cases, particles are initialized with $b_p(1) = b$.

### 3.2 Experimental Set Up

We use the homogeneous and heterogeneous versions of each of the strategies defined previously to represent the areas of our taxonomy. Concrete variants of PSO are created by combining these strategies with the two update function sets. We investigate these variants using the well-known set of test functions described by Hansen et al. [24]. As we are conducting a qualitative investigation rather than attempting to establish the best possible variant, we omit the full set of functions in favour of analysing six functions in more detail: *Sphere*, *Ellipsoidal*, *Rosenbrock*, *Rastrigin*, *Weierstrass* and *Schaffer F7*. The problems have been chosen so that we have three unimodal, three multi-modal, three separable and three non-separable functions. The bounds of the search space for all functions are set to $[-5, 5]^D$, where $D$ is the dimensionality. A trial terminates after an evaluation budget of $D * 1000$. All results are based on an average over 50 trials.

### 3.3 Experimental Results

Here we summarise key results from our experiments, giving more detailed results in our accompanying technical report [25]. Our initial experiment investigates whether heterogeneity or dynamics are sufficient in themselves to improve upon static homogeneous solutions. We evaluate static homogeneous, static heterogeneous, constrained dynamic homogeneous and constrained dynamic heterogeneous variants using the CBSB and CBBB behaviour sets. Comparing these

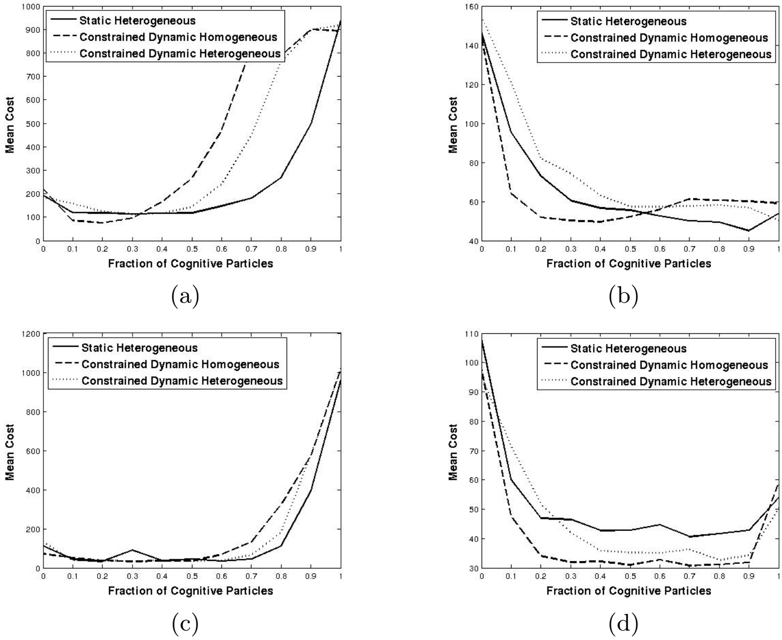


**Fig. 1.** Mean of best cost across 50 trials as a function of the fraction of cognitive particles in a swarm for the *Rosenbrock* (1a and 1c) and *Rastrigin* (1b and 1d) functions. 1a and 1b use the CBSB update function set, 1c and 1d use the CBBB update function set. Note that with fraction 0 or 1, the swarm exhibits static homogeneity.

strategies allows us to ask whether it is more important that particles express different behaviours over their lifetimes, or are different from each other. We look at the performance of our variants on the 30 dimensional *Rosenbrock* and *Rastrigin* functions, controlling for swarm composition by running the experiment for varying values of $\boldsymbol{\rho}$ and $\boldsymbol{\tau}$ (as described in section 3.1) from $(0.0, 1.0)$ (no cognitive-biased particles) to $(1.0, 0.0)$ (all cognitive-biased particles).

The results, shown in figure 1 indicate that heterogeneity and dynamics improve on pure homogeneity, particularly compared to the worst of their two component behaviours. While the type of strategy has a small effect on maximum performance, some strategies are feasible over wider ranges of swarm composition than others. However, a relatively wide range of swarm compositions perform well, confirming results from the literature on heterogeneous swarms and allowing us to draw these same conclusions about constrained dynamic swarms.

To validate the above results more generally and to evaluate the benefits of the dynamic and adaptive models, we test all strategies on 10− 30− and 100−dimensional versions of all evaluation problems. The results in this paper are for the CBBB set, but qualitatively similar results were obtained for the CBSB set, albeit with lower absolute performance. The absolute results do not show a clear pattern by inspection, except for that the best static homogeneous algorithm is typically worse than all heterogeneous or dynamic variants. For simpler analysis of the variants, we present the median improvement over the best homogeneous case (here defined as $\frac{h-v}{h}$, where $h$ is the mean cost of the best homogeneous variant and $v$ is the mean cost of a variant on a given problem) in table 2.

**Table 2.** Median improvement of each variant over the *best* static homogeneous case

| *Characteristic →* *Behaviour ↓* | **Homogeneous** | **Heterogeneous** |
|---|---|---|
| **Static** | N/A | 21% |
| **Constrained Dynamic** | 56% | 46% |
| **Dynamic** | 33% | 45% |
| **Adaptive** | 37% | 23% |

Similarly to our first experiment, all dynamic and heterogeneous strategies improved upon the best static homogeneous behaviour, indicating benefits to dynamics. However, we do not see clear improvements as we move to the more complex areas of our taxonomy: neither our dynamic model of the problem nor our adaptive mechanism (both inspired by successful algorithms from the literature) have improved on constrained dynamic heterogeneous behaviour. In contrast, all non-static strategies are improvements on static ones. Note that we do not claim that a constrained dynamic strategy is optimal (e.g. in comparison to the best possible adaptive strategy), however it strongly indicates that dynamics per-se are making an important contribution to performance and that the benefits of introducing more complex strategies may be outweighed by the loss of dynamics. Even if an adaptive algorithm drives the swarm to some optimal static heterogeneous composition, we have seen from figure 1 that the benefits compared to a sub-optimal but reasonable composition are minimal.

Finally, we invesigate the percentage of problems on which adding one level of dynamics/adaptivity improves performance (table 3, note that a figure of 50% indicates equivalent performance). This supports our previous analysis, showing that our dynamic strategy is typically worse than our constrained dynamic, while our adaptive strategy is similar to our dynamic. However, the initial addition of dynamics (static to constrained dynamic) results in a significant improvement. Similarly, heterogeneous variants improve upon their homogeneous counterparts in 60% of tests (fairly uniformly across strategies); it is clear that most of the observed improvements over the static homogeneous case are due to dynamics.

**Table 3.** The percentage of problems in which moving from one level to the next in our taxonomy of swarm behaviours led to improved performance.

| *Characteristic →* *Behaviour comparison ↓* | **Homogeneous** | **Heterogeneous** |
|---|---|---|
| **Static to Constrained Dynamic** | 78% | 94% |
| **Constrained Dynamic to Dynamic** | 17% | 28% |
| **Dynamic to Adaptive** | 61% | 50% |

## 4 Conclusions

In this paper we have proposed a taxonomy for heterogeneity and dynamics of swarms in PSO, which acts as a design space. The taxonomy builds upon prior classifications of heterogeneous PSO variants [2, 3], by separating the consideration of homogeneity and heterogeneity from that of adaptive and non-adaptive dynamics, both at the particle and swarm level. It supports research into the separate and combined contributions of these characteristics. In prior work, such questions were difficult to pose, leading to the conflation of the effects of heterogeneity, dynamics and adaptation in some research. An analysis of the literature showed that most recent work focuses on only some regions of the design space; however, other regions may be worthy of more attention. Specifically, while our results agreed with prior work that heterogeneity and dynamics are both useful, with the behaviours we tested, the introduction of dynamics typically had a larger impact on performance than the introduction of heterogeneity. Furthermore, our results show that the recent drive to find optimal forms of heterogeneity at run-time using adaptation may sacrifice the very dynamics which provide the greatest performance increase. It will be important to assess the generality of these conclusions on a wider range of PSO variants and problems.

Our results suggest that future work should focus on dynamics, which have the ability to encode a model of the problem. Furthermore, we believe that there is significant scope for the development of adaptation mechanisms which, rather than adapt particles' behaviours directly, search online for better forms of dynamics which in turn determine behaviour. It also seems appropriate that adaptive PSO variants should not only be compared against static ones, but also against uninformed dynamic variants. Only by doing this can any observed improvement be attributed to the adaptation mechanism and not only dynamics.

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