2014 IEEE Congress on Evolutionary Computation (CEC)
July 6-11, 2014, Beijing, China

Comparison of Multiobjective Particle Swarm Optimization and Evolutionary Algorithms for Optimal Reactive Power Dispatch Problem

Yujiao Zeng
State Key Laboratory of Hybrid Process Industry Automation System and Equipment Technology
Automation Research and Design Institute of Metallurgical Industry
Beijing, China
Email: zengyujiao010@163.com

Yanguang Sun
State Key Laboratory of Hybrid Process Industry Automation System and Equipment Technology
Automation Research and Design Institute of Metallurgical Industry
Beijing, China
Email: syanguang@263.net

***Abstract*—The optimal reactive power dispatch (ORPD) problem is formulated as a complex multiobjective optimization problem, involving nonlinear functions, continuous and discrete variables and various constraints. Recently, multiobjective evolutionary algorithms (MOEAs) and multiobjective particle swarm optimization (MOPSO) have received a growing interest in solving the multiobjective optimization problems. In this paper, MOPSO, and two highly competitive algorithms of MOEAs, that is, nondominated sorting genetic algorithm II (NSGA-II) and strength Pareto evolutionary algorithm (SPEA2) are presented for solving the ORPD problem. Moreover, a mixed-variable handling method and an effective constraint handling approach are employed to deal with various types of variables and constraints. The proposed algorithms are evaluated on the standard IEEE 30-bus and 118-bus test systems. In addition, several multiobjective performance metrics are employed to compare these algorithms with respect to convergence, diversity, and computational efficiency. The results show the effectiveness of MOEAs and MOPSO for solving the ORPD problem. Furthermore, the comparison results indicate that MOPSO generally outperforms other algorithms for ORPD and has a great potential in dealing with large-scale optimal power flow problems.**

Keywords—optimal reactive power dispatch; evolutionary algorithms; MOPSO; multiobjective optimization

I. INTRODUCTION

The optimal reactive power problem (ORPD) has attracted great attention in the past decades because it can greatly improve economy and security of power system. Generally, the goal of the ORPD is to minimize the network real power loss and improve voltage profile by regulating generator bus voltages, switching on/off static VAR compensator and changing transformer tap-settings, while satisfying various equality and inequality constraints. Therefore, ORPD is formulated as a constrained nonlinear optimization problem.

A large number of methods has been proposed for solving the ORPD problem in the literature. Generally, these reported methods can be divided into three categories.

The first one employs the conventional methods such as nonlinear programming (NLP) [1], Quadratic programming [2], linear programming (LP) [3], and interior point algorithms [4]. However, these techniques are failed in handling non-convexities and non-smoothness and susceptible to be trapped in local minima.

The second one utilizes the intelligent search-based methods, such as simulated annealing (SA) [5], evolutionary algorithms(EAs) [6-7], and tabu search (TS) [8], particle Swarm optimization(PSO) [9].These heuristic methods have been applied to the ORPD problem with impressive success but converge to only a single optimal solution where tradeoffs between different objectives must be fixed in advance of solution. In order to obtain desired Pareto-optimal solutions, these methods require multiple runs and tend to find weakly nondominated solutions.

In the last few years, the use of evolutionary algorithms and particle swarm optimization (PSO) for multiobjective optimization problems has significantly grown. Since these algorithms use a population of solutions in their search, multiple Pareto-optimal solutions can be found in one single run. Currently, there is a number of multiobjective evolutionary algorithms (MOEAs), especially the three competitive algorithms: nondominated sorting genetic algorithm II (NSGA-II) [10], strength Pareto evolutionary algorithm 2 (SPEA 2) [11] and PAES-II [12]. Recently, MOEAs [13-16] and MOPSO [17] have been implemented and applied individually to the ORPD problem with impressive success. However, there is lack of comprehensive comparison between MOEAs and MOPSO, since these algorithms are supposed to find an optimal Pareto-front to a given objective function but employ different strategies and computational effort. Moreover, the quality and diversity of the obtained nondominated solutions by using these algorithms have not been measured and evaluated quantitatively.

In this paper, a comparative study of MOPSO, SPEA2 and NSGA-II has been carried out to assess their potential to solve the real-world multiobjective ORPD problem. The ORPD problem is formulated as a nonlinear constrained

This work was supported by National High Technology Research and Development Program of China ('863' projects) (no.2012AA050215)

multiobjective optimization problem where power losses and voltage profile are treated as competing objectives. In order to deal with various types of variables and constraints, a mixed-variable handling method and an effective constraint handling approach are incorporated into these basic algorithms. The simulations are carried out on the standard IEEE 30-bus and 118-bus power test systems. Furthermore, several multiobjective performance metrics are employed to evaluate and compare these three algorithms with respect to convergence, diversity, and computational efficiency.

II. Problem Formulation

A. Problem Objectives

1) Real Power Loss (P_L)

The objective is to minimize the real power loss in transmission lines that can be expressed as

$$\min P_L = \min \sum_{k=1}^{NE} G_k \left[V_i^2 + V_j^2 - 2V_iV_j \cos\left(\delta_i - \delta_j\right) \right] \quad (1)$$

Where NE is the number of transmission lines; G_k is the conductance of the k^{th} line; $V_i\angle\delta_i$ and $V_j\angle\delta_j$ are the voltages at end bus i and bus j of the k^{th} line.

2) Voltage Deviation (VD)

The objective is to minimize the load buses voltage deviation from the nominal value defined as

$$\min VD = \min \sum_{i=1}^{NL} \left| V_i - V_i^{spec} \right| \quad (2)$$

Where V_i^{spec} is the pre-specified reference value at load bus i, which is usually set as 1.0 p.u., and NL is the number of load buses.

B. Problem Constraints

1) Power Flow Equations

$$\begin{cases} P_{G_i} - P_{D_i} - V_i \sum_{j=1}^{NB} V_j \left[G_{ij} \cos\left(\delta_i - \delta_j\right) + B_{ij} \sin\left(\delta_i - \delta_j\right) \right] = 0 \\ Q_{G_i} - Q_{D_i} - V_i \sum_{j=1}^{NB} V_j \left[G_{ij} \sin\left(\delta_i - \delta_j\right) + B_{ij} \cos\left(\delta_i - \delta_j\right) \right] = 0 \end{cases} \quad (3)$$

2) System Operating Constraints

$$V_{G_i}^{min} \le V_{G_i} \le V_{G_i}^{max}, i = 1,2,\ldots,NG \quad (4)$$

$$T_i^{min} \le T_i \le T_i^{max}, i = 1,2,\ldots,NT \quad (5)$$

$$Q_{C_i}^{min} \le Q_{C_i} \le Q_{C_i}^{max}, i = 1,2,\ldots,NC \quad (6)$$

$$V_{PQ_i}^{min} \le V_{PQ_i} \le V_{PQ_i}^{max}, i = 1,2,\ldots,NPQ \quad (7)$$

$$Q_{G_i}^{min} \le Q_{G_i} \le Q_{G_i}^{max}, i = 1,2,\ldots,NG \quad (8)$$

$$S_{L_i} \le S_{L_i}^{max}, i = 1,2,\ldots,NE \quad (9)$$

Where NB NG, NT, NC, NPQ and NG are the numbers of buses, generators, transformers, switchable VAR sources, and PQ buses, respectively. P_{G_i} and Q_{G_i} are the generator real and reactive power; P_{D_i} and Q_{D_i} are the load real and reactive power; G_{ij} and B_{ij} are the transfer conductance and susceptance between i and j; V_{G_i} are generator voltage; T_i is the transformer tap setting; Q_{C_i} is reactive power compensation of the switched VAR source; V_{PQ_i} is the PQ bus voltage; S_{L_i} is the apparent power flow of transmission line. The max and min represent the maximum and minimum values of the corresponding variables.

III. Multiobjective Optimization Approaches

A. Basic Concepts

The general multiobjective optimization is formulated as:

$$\min\left[f_1(X), f_2(X), \ldots, f_{N_{obj}}(X) \right] \quad (10)$$

Subject to

$$\begin{cases} h_i(X) = 0 & i = 1,\ldots,m \\ g_j(X) \le 0 & j = 1,\ldots,p \end{cases} \quad (11)$$

Where $X = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ is a vector of the decision variables; N_{obj} is the number of objective functions; $f_i(X)$ is the i^{th} objective function; $h_i(X)$ and $g_j(X)$ are the functions of the i^{th} equality constraint and the j^{th} inequality constraint, respectively. The constraints define the feasible region and any vector X in the feasible region is called a feasible solution.

For a multi-objective optimization problem, a solution X_1 is said to dominate the other solution X_2 if and only if $f_i(X_1) \le f_i(X_2)$ for all $i = 1,\ldots,N_{obj}$ and $f_j(X_1) < f_j(X_2)$ at least one $j \in \{1,\ldots,N_{obj}\}$. If X_1 dominates X_2, X_1 is called as the nondominated solution. The solutions that are nondominated within the entire search space are denoted as Pareto optimal solutions. The set of all the Pareto optimal solutions is called the Pareto optimal set. The image of the Pareto optimal set under the objective functions is called Pareto front.

B. Non-dominated Sorting Genetic Algorithm (NSGA-II)

NSGA-II, originally developed by Deb [10], is a commonly used multiobjective optimization technique well suited to solve highly constrained optimization problems. The characteristic feature of NSGA-II is its fast non-dominated sorting procedure for ranking solutions in its selection. The population is sorted into several fronts according to the dominance ranks of the individuals. NSGA-II has a fitness assignment based on the estimation density of a solution. The density estimation of NSGA-II is performed by a truncation operator based on crowding distance which for each individual of the population computes values relative to distance between two points on either side of this point along each of the objectives. During selection, NSGA-II uses a crowded-comparison operator that takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred).

C. Strength Pareto Evolutionary Algorithm (SPEA2)

SPEA2, presented by Zitzler and Thiele [11], is another popular MOEA based on Pareto domination. SPEA2 maintains an external archive of the nondominated solutions found during the search and updates it at each generation. Once the archive is full, further solutions are removed based on a clustering technique, to make room for new solutions in successive iterations. The clustering aims to preserve the characteristics of the current Pareto front despite removing solutions. In SPEA2, each solution is associated to a strength value that defines the strength of dominance of an individual in relation to other individuals. The individuals with higher strength values are preserved and maintained in the population. In case two individuals are of equal strength, SPEA-II uses a density measure to distinguish between solutions. This measure is an adaptation of the k-th nearest neighbor method.

D. Multiobjective Particle Swarm Optimization (MOPSO)

PSO is a simple and efficient population based optimization method proposed by Kennedy and Eberhart [18]. The basic idea of PSO is the mathematical modeling and simulation of the food searching activities of a swarm of particles. Each particle in a swarm flies around in a multidimensional search space. During the iterative process, each particle adjusts its velocity and position according to its past experiences (the local best $Pbest_i$) and its neighbors' experiences (the global best $Gbest$).

Up to now, there have been several proposals to extend PSO to handle multiobjectives. Here, we only choose one of popular MOPSO algorithms developed by Coello [19]. In order to extend PSO to handle multiobjectives, Coello uses the concept of Pareto dominance to decide $Pbest_i$ and $Gbest$. The $Pbest_i$ of particle i is the position which is nondominated by its own past positions. In order to determine $Gbest$, Coello adopts an external archive to store the nondominated vectors found along the search process. This archive uses a diversity-preserving mechanism to separate the objective function space into a number of hypercubes (or adaptive grids). If the archive exceeds its membership threshold, the hypercubes with the most densely populated hypercubes are truncated. The archive also facilitates the selection of $Gbest$ for any individual. Each hypercube are given a fitness value, which represents the number of the solutions contained in the hypercube. Thus, a more densely populated hypercube is given a lower score. Selection of $Gbest$ for a particle is then based on roulette wheel selection of a hypercube first according to its score, and then uniformly choosing a member of that hypercube.

IV. Approach Implementation

A. Representation of the Decision Variable

In the decision variables of the ORPD problem, V_{G_i} is continuous variable, which can be running at any real number within the limit boundary; T_i and Q_{C_i} are discrete variables, which can only be given a value from a fixed discrete values set. Here, we use the real number to represent continuous variables and the integer to represent discrete variables. Thus, the vector of new control variables can be written as

$$X^{*T} = \left[v_{G_1} \ldots v_{G_{NG}}, t_1 \ldots t_{NT}, q_{C_1} \ldots q_{C_{NC}} \right] \tag{12}$$

Then, the relationship between practical parameters and new control variables can be written as:

$$\begin{cases} V_{G_i} = v_{G_i} \\ T_i = 1.0 + t_i \cdot \Delta t \\ Q_{C_i} = q_{C_i} \cdot \Delta q_{C_i} \end{cases} \tag{13}$$

Where v_{G_i}, t_i and q_{C_i} are the generator bus voltage, the position of transformer tap and the step of reactive compensation capacitor. Δt_i and Δq_{C_i} express the step size of corresponding variables.

B. Mixed-variable Handling

In the basic forms of the proposed multiobjective optimization algorithms, they can only handle continuous variables. Hence, we employ a mixed-variable handling method introduced in [20], which can be briefly described as follows.

In the initialization process, all variables are random generated within their upper and lower bounds at first. Then, the integer part of the variable value is picked to be the value of the integer variable, which is denoted as $\text{int}\left(x_i\left(t\right)\right)$, $i = 1, 2, \ldots, n$.

In the iterative process, there're some differences between MOEAs and MOPSO. For MOEAs, all offspring solutions are produced by parent using crossover and mutation, and then the integer part of the variable value is picked to be value of the current variable. For MOPSO, the new position for the integer variables will be selected in the neighbor integer values of the former position according to the velocity of the particle. If the velocity is more than zero, the new position is forward to plus 1. Otherwise, it is back to minus 1.

C. Constraints Handling

The ORPD problem is subjected to several equality and inequality constraints represented by (3)-(9). The equality constraints (3) can be used to calculate the state variables and objective functions using Newton-Raphson method. The inequality constraints (4)-(6) can be automatically satisfied by setting the boundary of the search space. The inequality constraints (7)-(9) need to be handled during the optimization procedure. This paper adopted an efficient constraint handling method, originally proposed by Deb [21]. The main idea of this approach is to apply a set of simple rules to decide the selection process in MOEAs or MOPSO algorithm, where two solutions are compared at a time, the following criteria are enforced:

1） If both are feasible, nondominance is directly applied to decide who is the winner.
2） If one is feasible and the other is infeasible, the feasible dominates.
3） If both are infeasible, then the one with the lowest value in its total sum of constraint violations dominate.

Based on the above criteria, objective and constraint violation information are considered separately. Thus, there is no need of any penalty factors. Whenever two solutions are compared, we firstly check their constraints. If they are all infeasible, we need to calculate the total sum of constraint violations. In this paper, the sum of all the normalized constraint violations of an infeasible solution is calculated as:

$$F_V = \sum_{i \in NPQ} \frac{\left|V_{PQ_i} - V_{PQ_i}^{\lim}\right|}{\Delta V_{PQ_i}^{\max}} + \sum_{i \in NG} \frac{\left|Q_{G_i} - Q_{G_i}^{\lim}\right|}{\Delta Q_{G_i}^{\max}} + \sum_{i \in NE} \frac{\left|S_{L_i} - S_{L_i}^{\max}\right|}{\Delta S_{L_i}^{\max}} \tag{14}$$

$$V_{PQ_i}^{\lim} = \begin{cases} V_{PQ_i}^{\max}; V_{PQ_i} > V_{PQ_i}^{\max} \\ V_{PQ_i}^{\min}; V_{PQ_i} < V_{PQ_i}^{\min} \end{cases} \tag{15}$$

$$Q_{G_i}^{\lim} = \begin{cases} Q_{G_i}^{\max}; Q_{G_i} > Q_{G_i}^{\max} \\ Q_{G_i}^{\min}; Q_{G_i} < Q_{G_i}^{\min} \end{cases} \tag{16}$$

where $\Delta V_{PQ_i}^{\max}$, $\Delta Q_{G_i}^{\max}$ and $\Delta S_{L_i}^{\max}$ are the largest violation of the i^{th} PQ bus voltage constraint, the i^{th} generator reactive power output constraint, and the i^{th} line apparent power flow constraint achieved by any individual in the current population.

V. RESULTS AND DISCUSSION

The effectiveness and efficiency of the different multiobjective optimization algorithms for solving the ORPD problem are tested on the standard IEEE 30-bus and 118-bus power systems. Owing to the randomness of the proposed MOEAs and MOPSO, 10 independent trials are conducted when applied to the test systems. All the techniques and simulations developed in this study are implemented on 1.83 GHz PC using MATLAB language. The load flow is run using MATPOWER 4.1 software [22] with necessary alterations in the coding.

A. Parameter Settings

For successful implementation of the proposed algorithms, the optimum settings of different input parameters are required to be determined. Different trials have been made for solving the ORPD problem of both test systems. Based on the trials, the optimal parameters are selected as follows. The maximum size of the Pareto-optimal set is selected as 50 solutions for both two test systems. The values of the common parameters used in each algorithm such as population size N_{pop} and the maximum number of

TABLE I BEST EXTREME SOLUTIONS OUT OF TEN RUNS BY THE THREE ALGORITHMS FOR IEEE 30-BUS SYSTEM

Variable	Initial	MOPSO		NSGA-II		SPEA2	
		Best P_L	Best VD	Best P_L	Best VD	Best P_L	Best VD
V_{G_1}	1.05	1.0708	1.0183	1.0658	1.0294	1.0687	1.0369
V_{G_2}	1.04	1.0630	1.0084	1.0561	1.0086	1.0586	1.0125
V_{G_5}	1.01	1.0415	1.0031	1.0346	1.0001	1.0333	0.9986
V_{G_8}	1.01	1.0405	1.0	1.0351	1.0048	1.0368	0.9994
$V_{G_{13}}$	1.05	1.0744	1.0300	1.0417	1.0988	1.0168	1.0270
V_{G_1}	1.05	1.0620	1.0418	1.0694	0.9974	1.0543	1.0177
T_{6-9}	1.078	1.078	1.0	1.025	1.0	1.0125	0.975
T_{6-10}	1.069	0.9	0.9125	0.9	0.9625	0.9125	0.975
T_{4-12}	1.032	1.0	1.0	0.9875	0.9875	0.9875	1.0
T_{27-28}	1.068	0.9625	0.925	0.95	0.9375	0.95	0.9625
Q_{10}	0.0	0.05	0.01	0.01	0.04	0.01	0.02
Q_{24}	0.0	0	0.05	0.02	0.0	0.03	0.01
$P_L(MW)$	5.8327	**4.9849**	5.5968	**5.0465**	5.7431	**5.1067**	5.6821
$VD(pu)$	0.9520	0.7096	**0.1222**	0.7248	**0.1274**	0.6581	**0.1315**

iterations $Iter_{max}$ for each test system are chosen to be the same. For the IEEE 30 bus system, N_{pop} and $Iter_{max}$ are set at 50 and 100. For the IEEE 118 bus system, $N_{pop} = 100$, $Iter_{max} = 500$. The other parameters are given below:

1) MOPSO settings: inertia weight factor $w = 0.729$; acceleration constants $c_1 = c_2 = 2.05$, the number of grids per each dimension $N_{grid} = 50$.

2) NSGA-II settings: crossover probability $p_c = 0.9$; mutation probability $p_m = 1/n$ (n is the number of decision variables);the distribution indices for crossover and mutation operators are set as $\eta_c = 20$ and, $\eta_m = 20$ respectively.

3) SPEA2 settings: the external archive size $N_A = 50$; crossover probability $p_c = 0.9$; mutation probability $p_m = 1/n$ (n is the number of decision variables); the distribution indices for crossover and mutation operators are set as $\eta_c = 20$ and $\eta_m = 20$, respectively.

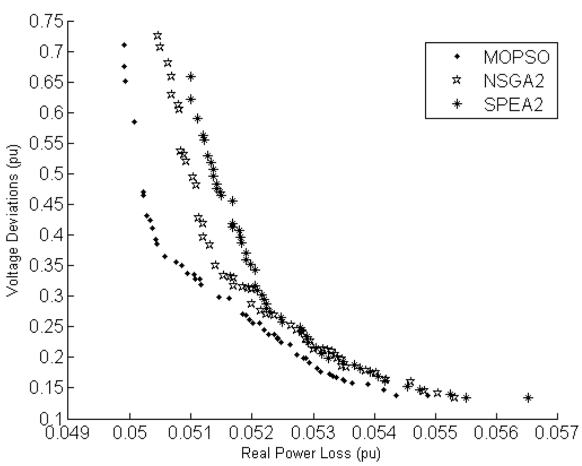

Fig. 1. The best obtained Pareto-fronts of MOPSO, NSGA-Ⅱ and SPEA2 for the base load of the IEEE30-bus system

B. IEEE 30-bus Power System

This system consists of 6 generators (located at bus 1, 2, 5, 8, 11 and 13), 4 transformers, and 2 VAR compensators. Thus the number of decision variables is 12 in this problem. Four branches (6, 9), (6, 10), (4, 12) and (27, 28), are under load tap setting transformer branches. The VAR compensators are installed at buses 10 and 24. The detailed data is given in [23-24].The lower voltage magnitude limits at all buses are 0.95 p.u. and the upper limits are 1.1 p.u. for generator buses 2, 5, 8, 11, and 13, and 1.05 p.u. for the remaining buses including the slack bus 1. The transformer tapping is in the range of [0.9, 1.1] with the step size of 0.0125 and the shunt capacitors have the rating between 0 and 5 MVAR with the step size of 1 MVAR. The initial settings of the control variables and initial objective values are given in Table I.

Tables I represents the best P_L and best VD solutions which are the extreme solutions of Pareto-front, obtained out of ten runs by three different techniques for IEEE 30-bus system. It is clear that the results are almost identical. In addition, MOPSO can find slightly better extreme solutions. The best obtained Pareto-fronts of all techniques are shown in Fig. 1. It can be seen that all the three algorithms are able to locate the Pareto-optimal solutions with excellent diversity. Moreover, MOPSO can find better non-dominated solutions.

It is well known that in particle, there're usually different scenarios (such as peak, off-peak and low load) for a practical reactive power dispatch in a real power system. Thus, four different scenarios are considered, that is, 90%, 95%, 105% and 110% of the original load level. The best obtained Pareto-fronts obtained out of ten runs using different techniques are shown in Fig. 2. It is clear that these algorithms can maintain good diversity among the solutions for all the

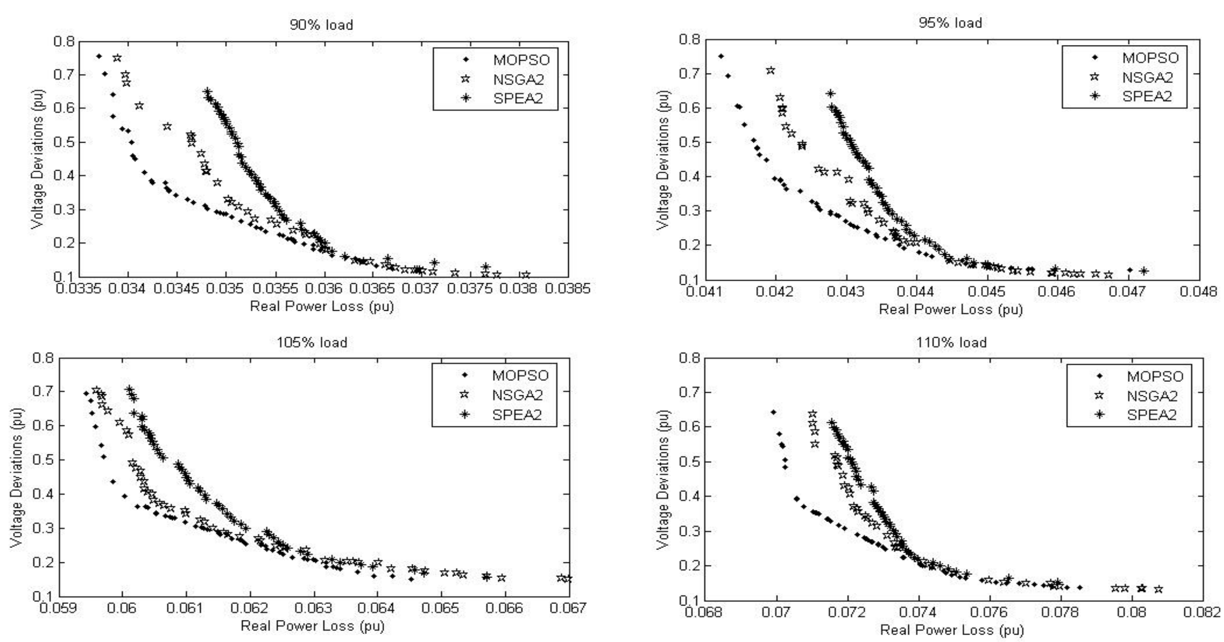

Fig. 2. The best obtained Pareto-fronts of MOPSO, NSGA-Ⅱ, and SPEA2 for different loading levels of the IEEE30-bus system

loading levels. Further, it can be seen that if we gradually add the system load, the range of the Pareto front for the ORPD problem increases in the real power-loss objective and decreases in the voltage-deviation objective. This observation implies that at a heavier load level, more system loss has to be consumed for the same promotion of the voltage profile. This means that the same amount of voltage changes in the heavier load system may bring much more system loss.

C. IEEE 118-Bus Power System

In this section, the comparison between the three algorithms is moved to a larger scale power system such as IEEE 118 bus system. The power system consists of 54 generators, 9 transformers and 12 capacitor banks. Thus, the dimension of control variables in this case is 75. For more information about the system, one can refer to [25]. The operating limits of all the control variables are the same as in IEEE 30 bus system.

The best P_L and best VD solutions obtained out of ten runs using three different algorithms for IEEE 118-bus system are given in Table II. Due to the space limitation, the final optimal settings of 75 decision variables are not given. The best Pareto-fronts out of ten runs are show in Fig. 3. From the results, it is clear that all the three algorithms are able to generate Pareto-front in a single simulation run. Moreover, MOPSO can find better extreme points and nondominated solutions.

D. Multiobjective Performance Metrics Analysis

In multiobjective optimization processes, two goals are normally taken into account: 1) convergence to the true Pareto-optimal set; 2) maintenance of diversity in solutions of the Pareto-optimal set. In order to obtain an accurate quantitative comparison of the performances of the proposed approaches, this paper implements three different performance metrics: Generational Distance (GD) [26], Minimal Spacing (MSP) [27], and Hypervolume (HV) [28]. A brief introduction of these metrics is given here:

Generational Distance (GD): GD is used to evaluate the closeness of the nondominated set obtained by an algorithm to the reference Pareto-optimal front.

$$GD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \quad (17)$$

Where n is the number of vectors in the reference Pareto-optimal front and d_i is the Euclidean distance (measured in objective space) between each of these and the nearest member of the Pareto optimal set.

Minimal Spacing (MS): MS is used to evaluate how evenly the nondominated solutions are distributed in the objective.

$$MS = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2} \quad (18)$$

Where $d_i = min_k \sum_{m=1}^{N_{obj}} \left| f_m^i - f_m^k \right|$, N_{obj} the number of objectives, i is the index of the solution which is marked as the current seed, and k is the index of the solution which is stilled unmarked at the current iteration.

Hypervolume (HV): This metric provides the combined qualitative information about closeness and diversity in obtained Pareto-optimal fronts. It calculates the volume covered by the members of the approximate Pareto-optimal Q for problems where all objectives are to be minimized. Mathematically, for each solution $i \in Q$, a hypercube v_i is constructed with a reference point W and the solution i is the diagonal corners of the hypercube. The reference point can simply be found by constructing a vector of worst objective function values. Thereafter, a union of all hypervolume (HV) is calculated as follows:

$$HV = volume\left(\bigcup_{i=1}^{|Q|} v_i\right) \quad (19)$$

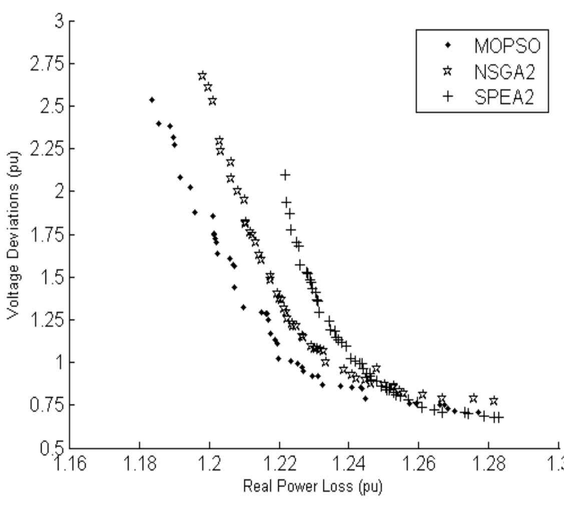

Fig. 3. The best obtained Pareto-fronts of MOPSO, NSGA-II, and SPEA2 for the IEEE118-bus system

TABLE II BEST EXTREME SOLUTIONS OUT OF TEN RUNS BY DIFFERENT APPROACHES FOR IEEE118-BUS SYSTEM

Object	Initial	MOPSO		NSGA-II		SPEA2	
		Best P_L	Best VD	Best P_L	Best VD	Best P_L	Best VD
$P_L(MW)$	132.8629	**117.8686**	127.5240	**119.8287**	128.3485	**122.1903**	129.2778
$VD(pu)$	2.3316	2.9212	**0.7183**	2.7116	**0.7765**	2.1671	**0.7482**

1) *Generation of reference Pareto front*

Before implementing these performance metrics, the reference Pareto-front should be generated. Here, the covariance matrix adapted evolution strategy (CMA-ES) is employed to obtain the Pareto-front by using multiple run. This can be obtained by treating this problem as a single objective optimization with weighted sum of objectives as follows.

$$F = w * P_L + (1 - w) * VD \qquad (20)$$

Where w is the weight factor which varies randomly between 0 and 1. To get 50 nondominated solutions, the algorithm CMA-ES is applied 50 times with varying with a linear increment of 0.02, each run having 50,000 function evaluations.

2) *Implement of performance metrics*

In order to evaluate the performance metrics of these algorithms, 100 trials runs are conducted for each system. These metrics will be helpful for evaluating closeness to the true or reference Pareto-optimal front and also for evaluating diversity among non-dominated solutions.

The statistic comparisons of performance metric are showed in Table III. It can be seen that the average and stand deviation performance of both metrics for MOPSO are the best. It proves that MOPSO gives better convergence and diversity solutions than SPEA2 and NSGA-II.

E. Computational efficiency

Computational complexity of NSGA-II is $O(MN^2)$ as in each iteration, it calculates set of solutions dominated by solution k as well as the set of solutions dominated the solution k. Where N is the number of solutions and M is number of objectives. Computational complexity of SPEA2 is $O(MN^2 \log N)$ as in each iteration it calculates near neighbors for each solution. Computational complexity of MOPSO is also $O(MN^2)$ as nondominated sorting is required. Table IV shows the statistic results of computational time of the three algorithms in a single simulation for the IEEE 30 and 118 bus systems. Though computational complexities of NSGA-II and MOPSO proposed method are same, NSGA-II takes more time because PSO based approach (MOPSO) converges quickly compared to GA based approach (NSGA-II). SPEA 2 takes largest computational time because in each iteration, it calculates inter-distance between all individuals.

VI. CONCLUSIONS

In this paper, we have compared three elitist multiobjective optimization methods, namely MOPSO, NSGA-II and SPEA2 for solving the ORPD problem. The problem has been formulated as a bi-objective problem with minimization of the real power loss and the bus voltage deviations. Several

Table III STATISTICAL RESULTS OF PERFORMANCE METRIC FOR IEEE 30-BUS IEEE 118-BUS SYSTEMS

Performance Measures		**IEEE 30-bus system**			**IEEE 118-bus system**		
		MOPSO	NSGA-II	SPEA2	MOPSO	NSGA-II	SPEA2
GD	Best	0.012871	0.057375	0.093126	0.011491	0.102942	0.142735
	Mean	**0.016473**	0.153627	0.165919	**0.017236**	0.228537	0.344697
	Worst	0.023865	0.243776	0.255901	0.025514	0.478526	0.513841
	Std. Dev.	0.003036	0.057408	0.061057	0.004283	0.074689	0.088215
MSP	Best	0.208284	0.316984	0.268075	0.223417	0.265961	0.276804
	Mean	**0.238587**	0.329747	0.305155	**0.298814**	0.403119	0.367682
	Worst	0.265783	0.355727	0.348737	0.325622	0.489737	0.398957
	Std. Dev.	0.021495	0.012701	0.02606	0.036145	0.047227	0.029396
HV	Best	0.148573	0.141977	0.142138	0.284508	0.264875	0.275931
	Mean	**0.142676**	0.137651	0.135746	**0.265727**	0.233248	0.217574
	Worst	0.138175	0.1334836	0.1318645	0.224731	0.209645	0.197866
	Std. Dev.	0.017541	0.041701	0.06513	0.032356	0.053217	0.070472

TABLE IV COMPUTATIONAL TIME OF DIFFERENT ALGORITHM FOR IEEE 30-BUS AND 118-BUS SYSTEMS

Time	**IEEE 30-bus system**			**IEEE 118-bus system**		
	MOPSO	NSGA-II	SPEA2	MOPSO	NSGA-II	SPEA2
Best (s)	40.16	176.86	205.342	137.469	418.891	467.203
Mean (s)	**43.38**	181.52	210.733	**158.376**	448.417	528.813
Worst (s)	51.37	188.281	215.687	181.687	507.281	563.578
Std. Dev.	3.176	6.747	5.316	13.8352	21.8039	18.7585

optimization runs of these three algorithms are carried out on the standard IEEE 30-bus and 118-bus power test systems. In addition, several quality performance measures are employed to evaluate and compare the different techniques with respect to convergence, diversity, and computational time. The results show the effectiveness of MOEAs and MOPSO for handling the ORPD problem. In addition, the comparison results demonstrate MOPSO generally outperforms NSGA-II and SPEA2 on these test instances and implies its potential to deal with the complicated power system multiobjective optimization problems.

REFERENCES

[1] R. Billington and S. S. Sachdev, "Optimum network VAR planning by nonlinear programming," IEEE Trans. Power Appar. Syst., Vol.PAS-92, No.4, pp. 1217-1225, July. 1973.

[2] N. Grudinin, "Reactive power optimization using successive quadratic programming method," IEEE Trans. Power Syst., Vol.13, No.4, pp. 1219-1225, Nov. 1998.

[3] G. T. Hyedt and W. M. Grady, "Optimal Var sitting using linear load flow formulation," IEEE Trans. Power Appar. Syst., Vol.PAS-92, No.5, pp. 1214-1222, May. 1983.

[4] S. Granville, "Optimal reactive dispatch through interior point methods," IEEE Trans. Power Syst., Vol.9, No.1, pp. 136-146, Feb. 1994.

[5] W. S. Jwo, C. W. Liu and C. C. Liu, "Hybrid expert system and simulated annealing approach to optimal reactive power planning," IEEE Proc. Generation, Transmission and Distribution, Vol.142. 1995, pp. 381-385.

[6] I. Kenji, "Reactive power optimization by genetic algorithm," IEEE Trans. Power Syst., Vol.9, No.2, pp. 685-692, May. 1994.

[7] Q. H. Wu, "Power system optimal reactive power dispatch using evolutionary Programming," IEEE Trans. Power Syst., Vol.10, No.3, pp. 1243-1248, Aug. 1995.

[8] M. Abido, "Optimal power flow using tabu search algorithm," Elect. Power Compon. Syst., Vol.30, No.5, pp. 469-483. 2002.

[9] H. Yoshida, K. Kawata and Y. Fukuyama, "A particle swarm optimization for reactive power and voltage control considering voltage security assessment," IEEE Trans. Power Syst., Vol.15, No.4, pp. 1232-1239, Nov. 2000.

[10] K. Deb, A. Pratap and S. Agrwal, "A fast and elitist multiobjective genetic algorithm: NSGA-Ⅱ," IEEE Trans. Evol. Comp., Vol.6, No.2, pp. 182-197, Apr. 2002.

[11] E. Zitzler, M. Laumanns and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm," Computer Engineering and Networks Laboratory, ETH, Zurich, Switzerland, Tech. Rep. 103, 2001.

[12] D. W. Corne, N. R. Jerram and J. D. Knowles, "PEAS-Ⅱ: region-based selection in evolutionary multiobjective Optimization," Proceedings of the genetic and evolutionary computation conference (GECCO-2001), Morgan Kaufmann. 2001, pp. 283-290.

[13] M. A. Abido, "Optimal VAR dispatch using a multiobjective evolutionary algorithm," Int. J. Elect. Power Ener. Syst., Vol.27, No.1, pp. 3-20, Jan. 2005.

[14] F. Shi Gang and Q. Ai, "Multi-objective reactive power optimization using SPEA2," High Voltage Engineering, Vol.9, No.27, Sep. 2007.

[15] S. Jeyadevi, S. Baskar and C. K. Babulal, "Solving multiobjective optimal reactive power dispatch using modified NSGA-II," Int. J. Elect. Power Ener. Syst., Vol.33, No.2, pp. 219-228. 2011.

[16] M. A. Abido and J. M. Bakhashwain, "A novel multiobjective evolutionary algorithm for optimal reactive power dispatch problem," Proceedings of the 10th IEEE International Conference on Electronics, Circuits and Systems. 2003, pp. 1054-1057.

[17] M. A. Abido, "Multiobjective particle swarm optimization for optimal power flow problem," Proceedings of the 12th International Middle-East Optimization Power System Conference (MEPCON 2008), Aswan, Vol.8. 2010, pp. 241-268.

[18] J. Kennedy and R. Eberhart, "Particle swarm optimization," Proceedings of IEEE international conference on neural networks (ICNN, 95), Perth, Western Australia, Vol.4. 1995, pp. 1942-1948.

[19] C. A. C. Coello, G. T. Pulido and M. S. Lechuga, "Handling multiple objectives with particle swarm optimization," IEEE Trans. Evol. Comp., Vol.8, No.3, pp. 256-279, June. 2004.

[20] C. L. Sun and J. C. Zeng, "A Particle swarm optimization with feasibility-based rules for mixed-variable optimization problems," Proceedings of 9th Int. Conf. on Hybrid Intelligence Systems, Shenyang, China. 2009, pp. 543-547.

[21] K. Deb, "An efficient constraint handling method for genetic algorithms," Comput. Methods Appl. Mech. Eng., Vol.186, No.4, pp. 311-338, June. 2000.

[22] R. Zimmerman, C. E. Murillo Sanchez and D. Gan, "MATPOWER 3.0: a matlab power system simulation package,", Ithaca, NY: Cornell University Press, 1997. Available: http://www.pserc.cornell.edu/matpower.

[23] K. Y. Lee, Y. M. Park and J. L. Oritz, "A united approach to optimal real and reactive power dispatch," IEEE Trans. Power Appar. Syst., Vol.PAS-104, pp. 1147-1153, May. 1985.

[24] O. Alsac and B. Stott, "Optimal load flow with steady state security," IEEE Trans. Power Appar. Syst., Vol.PAS-93, No.3, pp. 745-751, May. 1974.

[25] R. Christie, "Power system test case archive," University of Washington, 1993. Available: http://www.ee.washington.edu/research/pstca.

[26] D. A. Van Veldhuizen and G. B. Lamont, "On measuring multiobjective evolutionary algorithm performance," Proceedings of the 2000 Congress on Evolutionary Computation, La Jolla, CA. 2000, pp. 204-211.

[27] S. Bandyopadhyay, S. K. Pal and B. Aruna, "Multiobjective GAs, quantitative indices, and pattern classification," IEEE Trans. Syst. Man Cybern., Vol.34, No.5, pp. 2088-2099, Oct. 2004.

[28] E. Zitzler, L. Thiele, M. Laumanns and V. G. Da Fonseca, "Performance assessment of multiobjective optimizers: an analysis and review," IEEE Trans. Evol. Comp., Vol.7, No.2, pp. 117-132. 2003.