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A Replacement Strategy for Balancing Convergence and Diversity in MOEA/D

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***Abstract*—This paper studies the replacement schemes in MOEA/D and proposes a new replacement named global replacement. It can improve the performance of MOEA/D. Moreover, trade-offs between convergence and diversity can be easily controlled in this replacement strategy. It also shows that different problems need different trade-offs between convergence and diversity. We test the MOEA/D with this global replacement on three sets of benchmark problems to demonstrate its effectiveness.**

***Index Terms*—Multiobjective optimization, MOEA/D, selection operator, replacement.**

I. Introduction

A *multiobjective optimization problems (MOP)* has several conflicting objectives. Very often, no single solution can optimize these objectives at the same time. Pareto optimal solutions are of very practical interest to decision makers. Improvement in one objective of a Pareto optimal solution will lead to deterioration in at least one other objective. The set of all the optimal Pareto optimal solutions is called the *Pareto set (PS)* and their corresponding objective vectors in the objective space is the *Pareto front (PF)*. Most multiobjective optimization evolutionary algorithms (MOEAs) aim to find a good approximation of the PF (PS) [1]–[3].

Selection is a main component in an MOEA. Popular selection strategies include Pareto dominance based, indicator based and decomposition based strategies [4]. In the multiobjective evolutionary algorithm based on decomposition (MOEA/D) framework, an MOP in question is decomposed into a number of simple subproblems. These subproblems can be single objective ones (e.g. scalar objective problems, [3], [5]), or multiobjective ones [6], which are optimized in a collaborative manner. As so far, most proposed MOEA/D versions adopt $(\mu + 1)$-selection scheme which selects μ individuals from a population of μ parents and 1 offspring. Therefore, the selection is also called as replacement or update in some papers.

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Selection in MOEA/D plays a vital role in the information exchange among subproblems. We hope the useful information can be transmitted to the appropriate subproblems to improve them, at the same time will not mislead other unsuitable subproblems. Several selection schemes in MOEA/D have been proposed and studied. It has been shown that selection in MOEA/D also plays a key role in balancing the population diversity and convergence in the search process. Moreover, It is clearly that solving different MOPs may need different combinations of efforts for diversity and convergence at different search stages. Therefore, it would be worthwhile developing a selection scheme which can make the communication among subproblems more efficient, meanwhile can achieve different combinations of diversity and convergence efforts easily by adjusting a control parameter. This paper represents our first attempt along this line.

In this paper, we propose a global replacement scheme for the selection in MOEA/D. In our proposed scheme, when the algorithm generates a new solution x^i_{new} from the search on subproblem i, the most appropriate subproblems will be selected from the whole population. Then the new solution will replace the current solutions of some of these subproblems. We have demonstrated that MOEA/D with global replacement scheme works well on some difficult test problems. Our simulation results also show that the replacement neighborhood size (neighborhood should have mentioned before) is a very sensitive parameter for balancing diversity and convergence. Meanwhile, it has been shown that different problems need different trade-offs between diversity and convergence. In other words, the new scheme has the advantage to balance convergence and diversity in solving MOPs by using the MOEA/D algorithm.

The rest of the paper is organized as follows. Section II briefly introduces the MOEA/D framework. Section III gives the details of the proposed global replacement scheme in MOEA/D. Section IV presents the statistical results. And the paper is concluded in Section V.

II. MOEA/D

A. Multiobjective Optimization Problem

An MOP can be defined mathematically as follows.

$$\begin{array}{ll} \text{minimize} & F(x) = (f_1(x), f_2(x), \cdots, f_m(x))^T \\ \text{subject to} & x \in \Omega \end{array} \quad (1)$$

where $x = (x_1, \ldots, x_n)^T$ is a decision variable vector, Ω is the feasible region in the decision space, and $F : \Omega \to R^m$ consists of m objective functions $f_i(x) (i = 1, 2, \cdots, m)$.

The following are definitions of Pareto optimality.

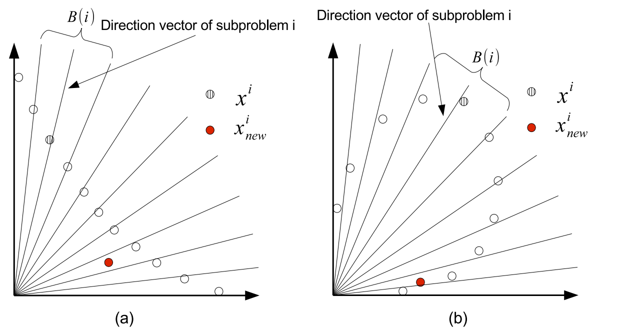

Fig. 1. Two examples of the special cases may occurred in the evolution process

- Let $x, y \in \Omega$, x is said to ***dominate*** y, denoted by $x \prec y$, if and only if $f_i(x) \leq f_i(y)$ for all $i \in \{1, 2, \cdots, m\}$, and $f_j(x) < f_j(y)$ for at least one $j \in \{1, 2, \cdots, m\}$.
- A point $x^* \in \Omega$ is called ***Pareto optimal*** if there is no other $x \in \Omega$ which *dominates* x^*.
- The set of all the Pareto optimal points is called the ***Pareto set (PS)*** and the ***Pareto front (PF)*** is defined as $PF = \{F(x) | x \in PS\}$.

A Pareto optimal solution is a best trade-off candidate solution for a decision maker.

B. Decomposition Approaches

Several decomposition approaches have been used and studied in the MOEA/D framework [3], [7]–[10]. This paper uses the Tchebycheff approach. In this approach, a scalar optimization subproblem is defined as:

$$\begin{aligned} \text{minimize} \quad & g(x|\lambda, z^*) = \max_{1 \leq i \leq m} \{\lambda_i |f_i(x) - z_i^*|\} \\ \text{subject to} \quad & x \in \Omega \end{aligned} \tag{2}$$

where $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$ is the weight vector of this subproblem, i.e., $\lambda_i \geq 0$ for all $i = 1, 2, \cdots, m$ and $\sum_{i=1}^{m} \lambda_i = 1$. z^* is a reference point. $z_i^* = \min\{f_i(x) | x \in \Omega\}$ for each $i = 1, \cdots, m$. The corresponding direction vector of this subproblem in the objective space should be $(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \cdots, \frac{1}{\lambda_m})^T$.

C. Generating Weighting Vectors

In the case when no preference information of the decision maker is available, it is desirable to find uniformly distributed solutions along the PF. Therefore, we should set weight vectors such that the optimal solutions of their subproblems are uniformly distributed along the PF. Several methods for generating weight vectors for different decomposition approaches in MOEA/D have been proposed [10], [11]. In this paper, we use the method developed in [12] to set the weight vectors.

D. Two Neighborhoods

In MOEA/D, neighborhood relations among its subproblems are defined by the Euclidean distances between their corresponding direction vectors. These relationships can be used for selection of parent solutions and replacement of old solutions. As suggested in [13], [14] , MOEA/D should use two different neighborhoods for these two purposes (i.e. the mating neighborhood and the replacement neighborhood). Moreover, some effort has been made to use different neighborhood sizes during different search stages [15].

E. MOEA/D Framework

For each subproblem i at each generation, MOEA/D adopted in this paper works as follows:

1) Set the mating neighborhood size T_m and the replacement neighborhood size T_r for subproblem i. The mating pool P can be composed by the solutions selected from the mating neighborhood with a large probability δ and whole population with a probability $(1 - \delta)$.
2) Randomly select parent solutions from the mating pool P, and then perform the reproduction operators on them to generate a new solution x^i_{new}. Compute $F(x^i_{new})$.
3) Decide which subproblems should be updated. Replace the current solutions of these subproblems by x^i_{new} if x^i_{new} is better than them.

III. Global Replacement in MOEA/D

In the original version of MOEA/D, the new solution x^i_{new} of subproblem i is used to replace the solutions of its neighboring subproblems $B(i)$. However, it is uncertain whether the new solution x^i_{new} is most suitable for these subproblems. As shown in Fig. 1(a), x^i_{new} is not good for $B(i)$, but very good for some other subproblems. So it is very likely that x^i_{new} will be discarded in the update stage unless a very big replacement neighborhood is used. However, a big replacement neighborhood can easily lead to the loss of population diversity distinctly. In the second example shown in Fig. 1(b), which is MOP1 used in [6], some solutions with high quality are very easily found at very early search stages, and then they will replace most current solutions with the original replacement scheme even a very small replacement neighborhood is adopted. Consequently many subproblems will be trapped at their local optimal solutions. Therefore, the original replacement scheme makes MOEA/D inefficient and ineffective for the kinds of problems like the examples in Fig. 1.

To overcome the above shortcomings, we propose a new replacement scheme called the global replacement for MOEA/D. For a newly generated solution x^i_{new}, it works as follows:

step 1: Find the most appropriate subproblem of x^i_{new}.
step 2: Determine the replacement range.
step 3: Replace the current solutions.

At step 1, we can use the objective functions of subproblems or the distance between x^i_{new} and the direction vectors to find the most appropriate subproblem j for x^i_{new}. The former is adopted in this paper and it can be represented as

$$j = \arg \min_{1 \leq k \leq N} \{g^{te}(x^i_{new} | \lambda^k, z^*)\}.$$

At step 2, T_r closest subproblems to subproblems j are selected to form the replacement neighborhood, i.e., $B_r(j)$ here. And the replacement range is controlled by the replacement neighborhood size T_r.

At step 3, solutions of subproblems $B_r(j)$ will be replaced by x^i_{new}. Actually, we can only discard the worst solution from the group of x^i_{new} and the current solutions of subproblems $B_r(j)$. However, here we replace all the possible solutions of subproblems $B_r(j)$ in order to facilitate the analysis of its performance. For all $k \in B_r(j)$, if $g^{te}(x^i_{new}|\lambda^k, z^*) < g^{te}(x^k|\lambda^k, z^*)$, set $x^k = x^i_{new}$.

In the above scheme, the useful information can be transmitted to the suitable subproblems even a very small neighborhood is used. So it makes the information transfer among subproblems more efficient. In addition, we can easily find that a big replacement neighborhood means that many solutions may be replaced by a new solution and then the diversity will decrease, and vice versa. Therefore, we can control the trade-off between diversity and convergence via the replacement neighborhood size with global replacement.

IV. Experimental Studies

To show the effectiveness of the proposed global replacement, MOEA/D with global replacement (MOEA/D-GR) is compared with original MOEA/D-DE. Both algorithms use the differential evolution (DE) operator as the reproduction operator for a fair comparison. The only difference between two algorithms is the replacement scheme.

A. Test Problems

Three sets of benchmark problems with different characteristics are used in this paper. The first set includes MOP1-MOP7 proposed in [6]. The second set is ZDT and DTLZ problems [16], [17], and the third one includes F1-F9 proposed in [5]. In MOP1-MOP7, some Pareto optimal solutions are much easier to obtain than others at the early stage of evolution. Population diversity is very important for solving these problems. While in ZDT and DTLZ problems, the PS shapes are simple lines or hyperplanes. When a few Pareto optimal solutions are found, other solutions are easily obtained by searching along the PS. For this reason, convergence is very important for solving them. F1-F9 has very complicated PS shapes and it is difficult to search along the PS, so the population diversity is also important.

B. Performance Metric

In our experimental studies, two widely used performance metrics the inverted generational distance metric (IGD) and the Hypervolume indicator (I_H) [18], [19] are adopted in assessing the performance of the compared algorithms.

a) IGD-metric [18], [19]: Let P^* be a set of uniformly distributed Pareto optimal points along the PF in the objective space. Let P be an approximate set to the PF obtained by an algorithm. The inverted generational distance from P^* to P is defined as

$$IGD(P^*, P) = \frac{\sum_{v \in P^*} d(v, P)}{|P^*|} \tag{3}$$

Where $d(v, P)$ is the minimum Euclidean distance between v and the points in P. If $|P^*|$ is large enough $IGD(P^*, P)$ could measure both convergence and diversity of P in a sense. Because the IGD metric has an assumption that the true PF is known, so we select 500 evenly distributed points in PF and let these points be P^* for each test problem with two objectives, and 1,000 points for each test problem with three objectives.

b) I_H-metric [20]: Let $y^* = (y^*_1, y^*_2, \cdots, y^*_m)$ be a point in the objective space which is dominated by any Pareto optimal objective vectors. Let S be the obtained approximation to the PF in the objective space. Then the I_H value of S (with regard to y^*) is the volume of the region dominated by S and bounded by y^*:

$$I_H(S, y^*) = volume(\bigcup_{y \in S} [y_1(x), y^*_1] \times \cdots [y_m(x), y^*_m]) \tag{4}$$

In our experiments, $y^* = (1.0, 1.0)$ for bi-objective test problems, and $y^* = (1.0, 1.0, 1.0)$ for three objective ones.

In general, If the points in P are very close to the PF and no part of the whole PF will be missed, the $IGD(P^*, P)$ will have a small value. Instead, the higher the I_H value, the better approximation to the true PF.

C. Parameter Settings

1) *Control parameters in reproduction operator*: Because the same reproduction operators are used in two algorithms, so the control parameters in DE and polynomial mutation are same with [5].
2) *The population size N*: Both two algorithms have the same population in our experiments, the population size is set to be 100 for the ZDT problems, 300 for DTLZ problems, F1-F5, F7-F9 and MOP1-MOP5, and 595 for F6 and MOP6-MOP7.
3) *Number of runs and stopping condition*: Each algorithm is run 30 times independently for each test problem. The algorithms stop after a given number of function evaluations. The maximal number of function evaluations is set to be $25,000$ for ZDT problems, $100,000$ for DTLZ problems, $150,000$ for F1-F5 and F7-F9, $300,000$ for MOP1-MOP5 and F6, and $900,000$ for MOP6 and MOP7.
4) *The neighborhood sizes*: Because the replacement neighborhood and the mating neighborhood are the same in MOEA/D-DE. To be fair, two neighborhoods are also of the same size in MOEA/D-GR. So $T_r = T_m = 0.1 \times N$ in both algorithms.
5) *Other control parameters*: In MOEA/D-DE, the maximal number of solutions replaced by a offspring solution is set to be $n_r = 2$. Because all the solutions can be replaced in MOEA/D-GR, so it needs no parameters.

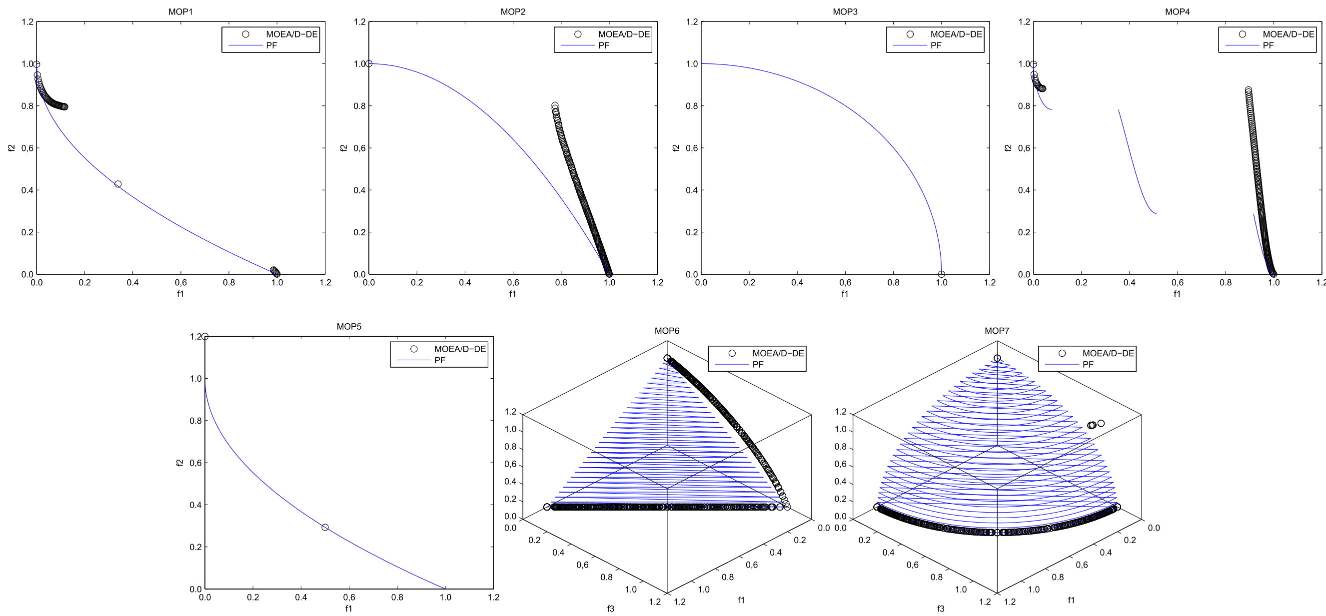

Fig. 2. Plots of the final populations with the lowest IGD-metric values found by MOEA/D-DE in 30 runs in the objective space on MOP1-MOP7.

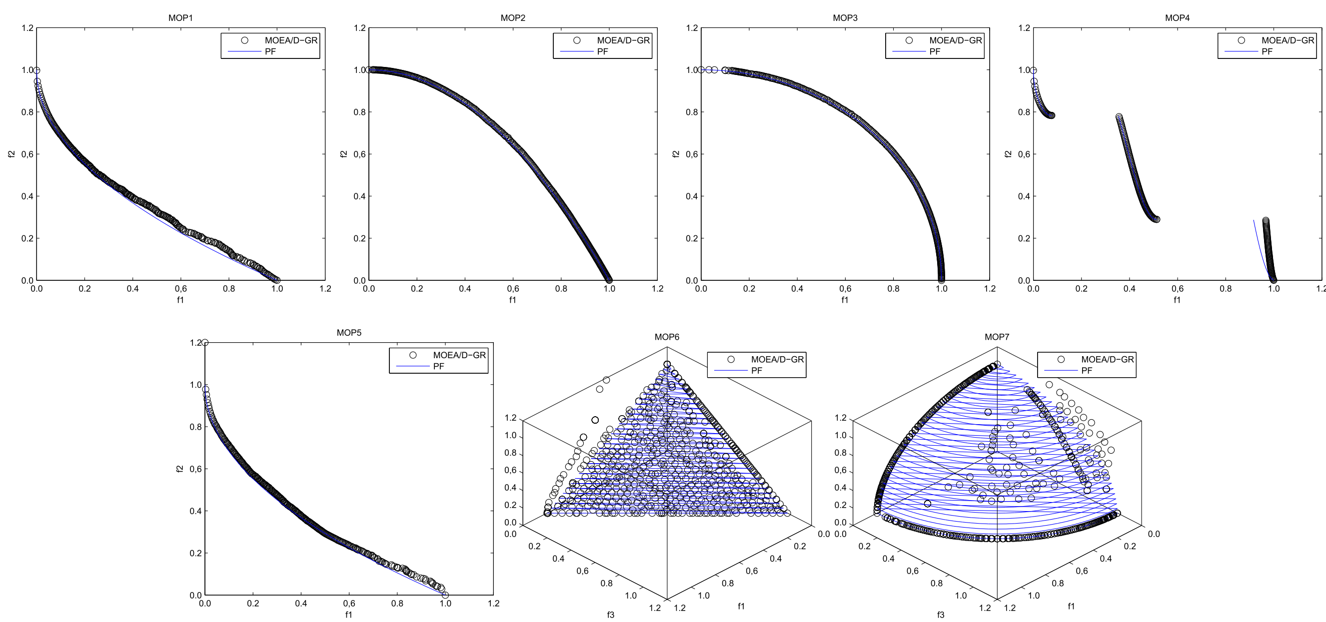

Fig. 3. Plots of the final populations with the lowest IGD-metric values found by MOEA/D-GR in 30 runs in the objective space on MOP1-MOP7.

TABLE I
IGD-metric Values of Solutions Found by MOEA/D-DE and MOEA/D-GR on MOP1-MOP7

IGD-value	MOEA/D-DE			MOEA/D-GR		
Problem	mean	min	std	mean	min	std
MOP1	0.3132	0.1593	0.0765	**0.0210**	**0.0170**	0.0051
MOP2	0.3061	0.1741	0.0655	**0.0637**	**0.0033**	0.0730
MOP3	0.5572	0.4774	0.0496	**0.0497**	**0.0043**	0.0821
MOP4	0.2832	0.2434	0.0234	**0.0824**	**0.0083**	0.0758
MOP5	0.3142	0.2936	0.0076	**0.0177**	**0.0135**	0.0044
MOP6	0.2996	0.2253	0.0174	**0.0446**	**0.0384**	0.0039
MOP7	0.3705	0.3225	0.0146	**0.1075**	**0.0798**	0.0311

TABLE II
I_H Values of Solutions Found by MOEA/D-DE and MOEA/D-GR on MOP1-MOP7

I_H-value	MOEA/D-DE			MOEA/D-GR		
Problem	mean	max	std	mean	max	std
MOP1	0.1699	0.4487	0.1404	**0.6381**	**0.6434**	0.0063
MOP2	0.0236	0.1415	0.0370	**0.2499**	**0.3280**	0.0952
MOP3	0.0012	0.0004	0.0545	**0.1682**	**0.2090**	0.0699
MOP4	0.1465	0.1803	0.0187	**0.3958**	**0.5065**	0.1106
MOP5	0.3536	0.3536	0.0000	**0.6417**	**0.6464**	0.0034
MOP6	0.5026	0.6001	0.0211	**0.7742**	**0.7814**	0.0067
MOP7	0.2139	0.2335	0.0049	**0.3345**	**0.3958**	0.0406

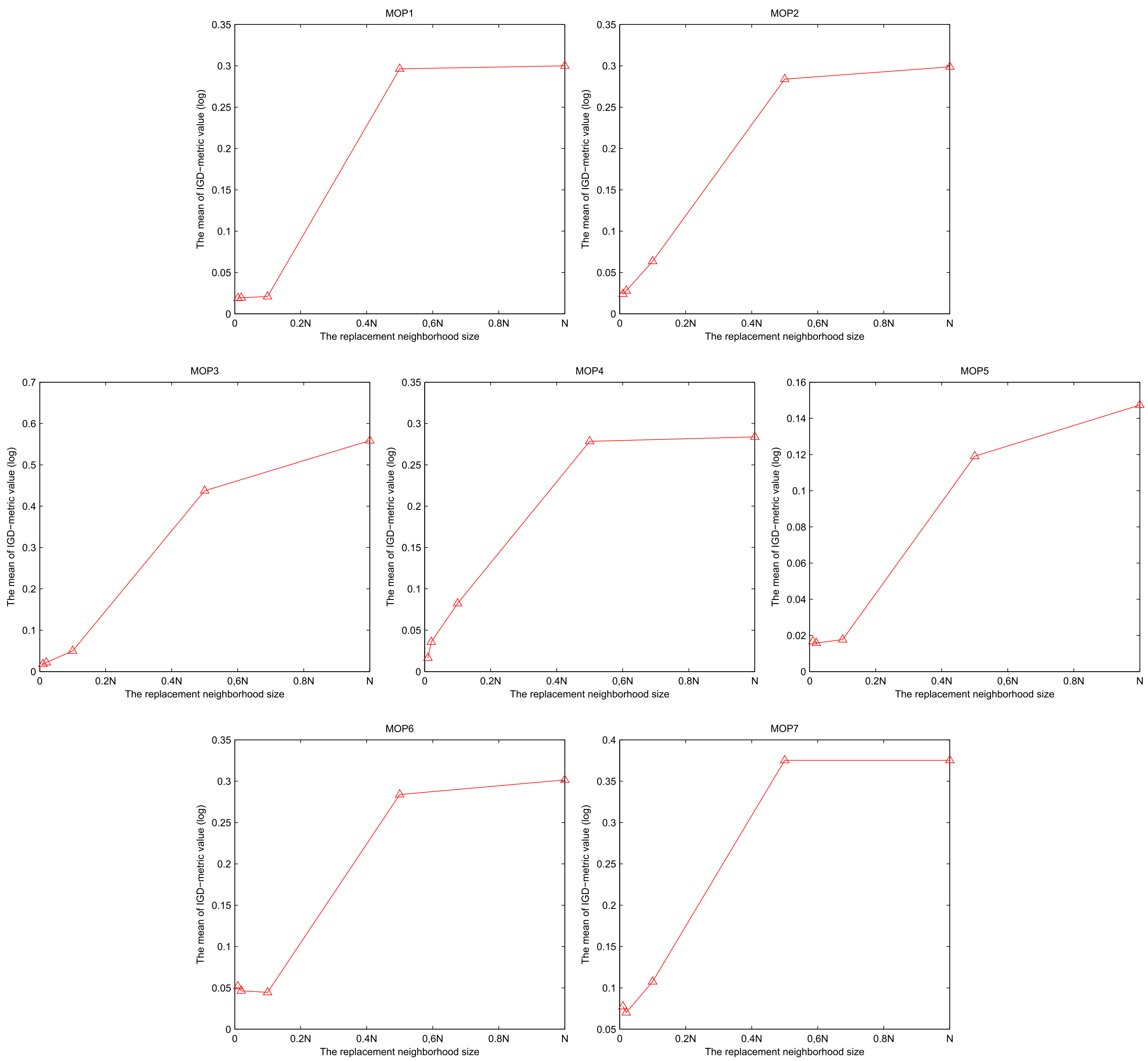

Fig. 4. Mean IGD of MOEA/D-GR with different replacement neighborhood sizes on MOP1-MOP7.

TABLE III
IGD-METRIC VALUES OF SOLUTIONS FOUND BY MOEA/D-DE AND MOEA/D-GR ON ZDT AND DTLZ PROBLEMS

IGD-value	MOEA/D-DE			MOEA/D-GR		
Problem	mean	min	std	mean	min	std
ZDT1	0.0127	0.0053	0.0071	**0.0054**	**0.0043**	0.0011
ZDT2	0.0149	0.0060	0.0038	**0.0050**	**0.0042**	0.0007
ZDT3	0.0271	0.0135	0.0132	**0.0123**	**0.0110**	0.0006
ZDT4	0.3149	0.0660	0.2252	**0.0176**	**0.0053**	0.0275
ZDT6	0.0132	0.0054	0.0075	**0.0047**	**0.0032**	0.0013
DTLZ1	0.4852	0.0690	0.5424	**0.2587**	**0.0447**	0.0638
DTLZ2	**0.0287**	**0.0282**	0.0003	**0.0287**	**0.0282**	0.0003

TABLE IV
I_H VALUES OF SOLUTIONS FOUND BY MOEA/D-DE AND MOEA/D-GR ON ZDT AND DTLZ PROBLEMS

I_H-value	MOEA/D-DE			MOEA/D-GR		
Problem	mean	max	std	mean	max	std
ZDT1	0.6463	0.6578	0.0105	**0.6575**	**0.6601**	0.0021
ZDT2	0.3064	0.3219	0.0061	**0.3243**	**0.3266**	0.0017
ZDT3	0.7258	0.7639	0.0284	**0.7675**	**0.7743**	0.0043
ZDT4	0.3191	0.5764	0.1821	**0.6389**	**0.6577**	0.0392
ZDT6	0.2512	0.2611	0.0090	**0.2624**	**0.2652**	0.0020
DTLZ1	0.7536	0.9756	0.3843	**0.9267**	**0.9744**	0.1194
DTLZ2	**0.4391**	**0.4397**	0.0003	0.4367	0.4392	0.0015

D. Experimental Result on MOP1-MOP7

The statistical results on MOP1-MOP7 are presented and compared in Table I and Table II. It can be clearly observed that MOEA/D-GR has much better performance than MOEA/D-DE in terms of both metrics on all the seven test problems. Fig. 2 and Fig. 3 show the final populations with the lowest IGD-metric values obtained by MOEA/D-DE and MOEA/D-GR on MOP1-MOP7 in the 30 runs. From the comparison of Fig. 2 with Fig. 3, we can easily find that MOEA/D-DE has resulted in the loss of diversity and can only find several points of the PFs. In contrast, MOEA/D-

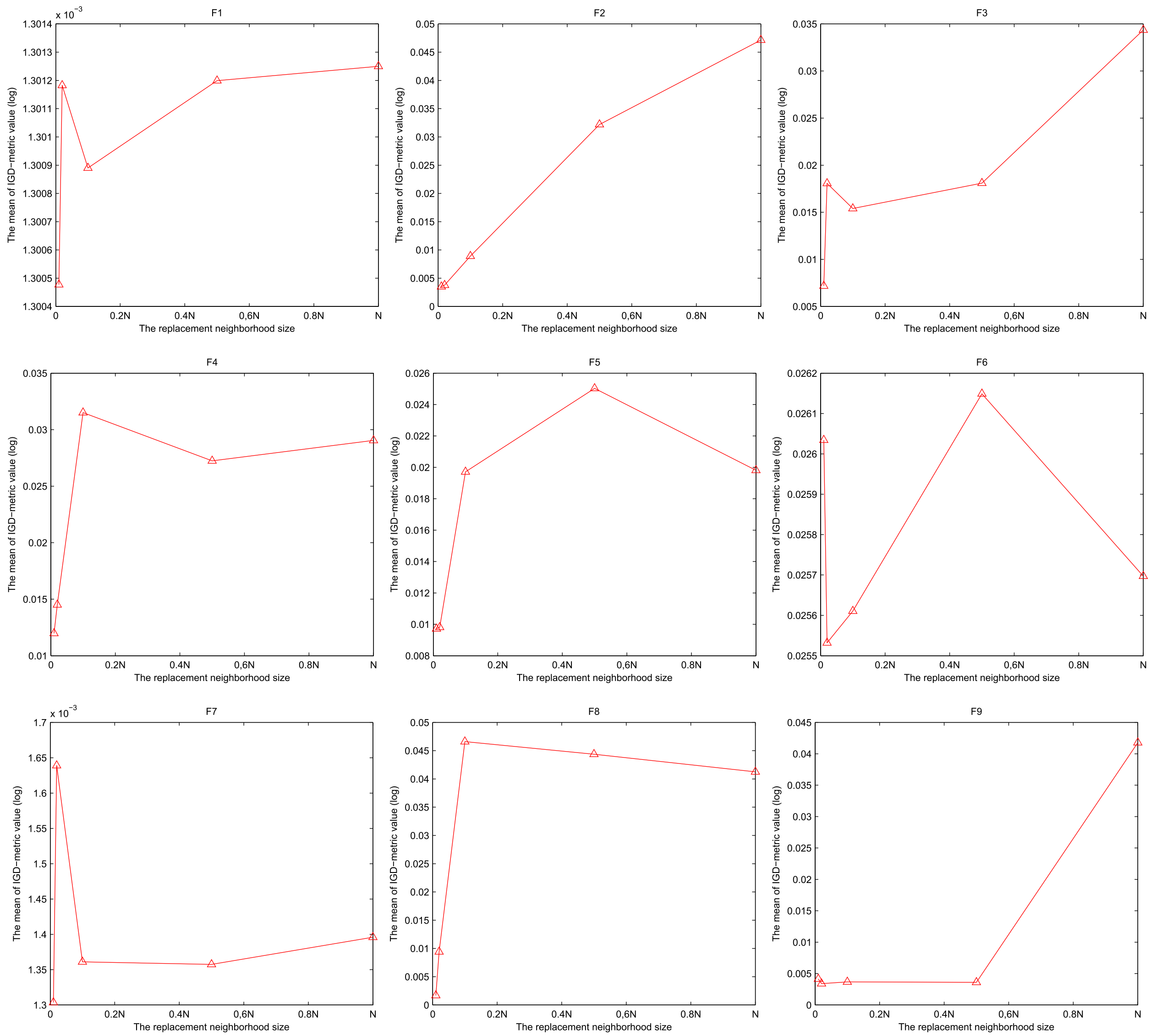

Fig. 5. Mean IGD of MOEA/D-GR with different replacement neighborhood sizes on F1-F9.

TABLE V
IGD-METRIC VALUES OF SOLUTIONS FOUND BY MOEA/D-DE AND MOEA/D-GR ON F1-F9

IGD-value	MOEA/D-DE			MOEA/D-GR		
Problem	mean	min	std	mean	min	std
F1	**0.0013**	**0.0013**	0.0000	**0.0013**	**0.0013**	0.0000
F2	**0.0037**	0.0032	0.0004	0.0089	**0.0028**	0.0206
F3	**0.0043**	0.0030	0.0022	0.0154	**0.0027**	0.0262
F4	**0.0067**	0.0036	0.0061	0.0315	**0.0029**	0.0434
F5	**0.0101**	0.0068	0.0020	0.0197	**0.0063**	0.0293
F6	**0.0254**	**0.0240**	0.0011	0.0256	**0.0240**	0.0011
F7	**0.0013**	**0.0013**	0.0000	0.0014	**0.0013**	0.0001
F8	**0.0017**	**0.0013**	0.0008	0.0466	0.0019	0.0258
F9	0.0042	0.0032	0.0007	**0.0037**	**0.0029**	0.0004

TABLE VI
I_H VALUES OF SOLUTIONS FOUND BY MOEA/D-DE AND MOEA/D-GR ON F1-F9

I_H-value	MOEA/D-DE			MOEA/D-GR		
Problem	mean	max	std	mean	max	std
F1	**0.6647**	0.6647	0.0000	**0.6647**	**0.6648**	0.0000
F2	**0.6603**	0.6611	0.0006	0.6576	**0.6619**	0.0113
F3	**0.6606**	0.6621	0.0015	0.6524	**0.6625**	0.0170
F4	**0.6577**	0.6614	0.0063	0.6425	**0.6622**	0.0264
F5	**0.6527**	0.6574	0.0033	0.6457	**0.6584**	0.0161
F6	**0.4344**	**0.4375**	0.0020	0.4236	0.4293	0.0022
F7	**0.6646**	**0.6647**	0.0001	0.6645	0.6646	0.0001
F8	**0.6633**	**0.6645**	0.0020	0.6035	0.6627	0.0388
F9	0.3266	0.3279	0.0010	**0.3275**	**0.3284**	0.0006

GR can almost approximate these PFs. By comparing these results, it demonstrate that MOEA/D-DE can not maintain diversity of population for these special problems while MOEA/D-GR can avoid this situation.

E. Experimental Result on ZDT and DTLZ problems

The results on the ZDT and DTLZ problems are presented in Table III and Table IV. The results show that MOEA/D-GR performs significantly better on all the problems except

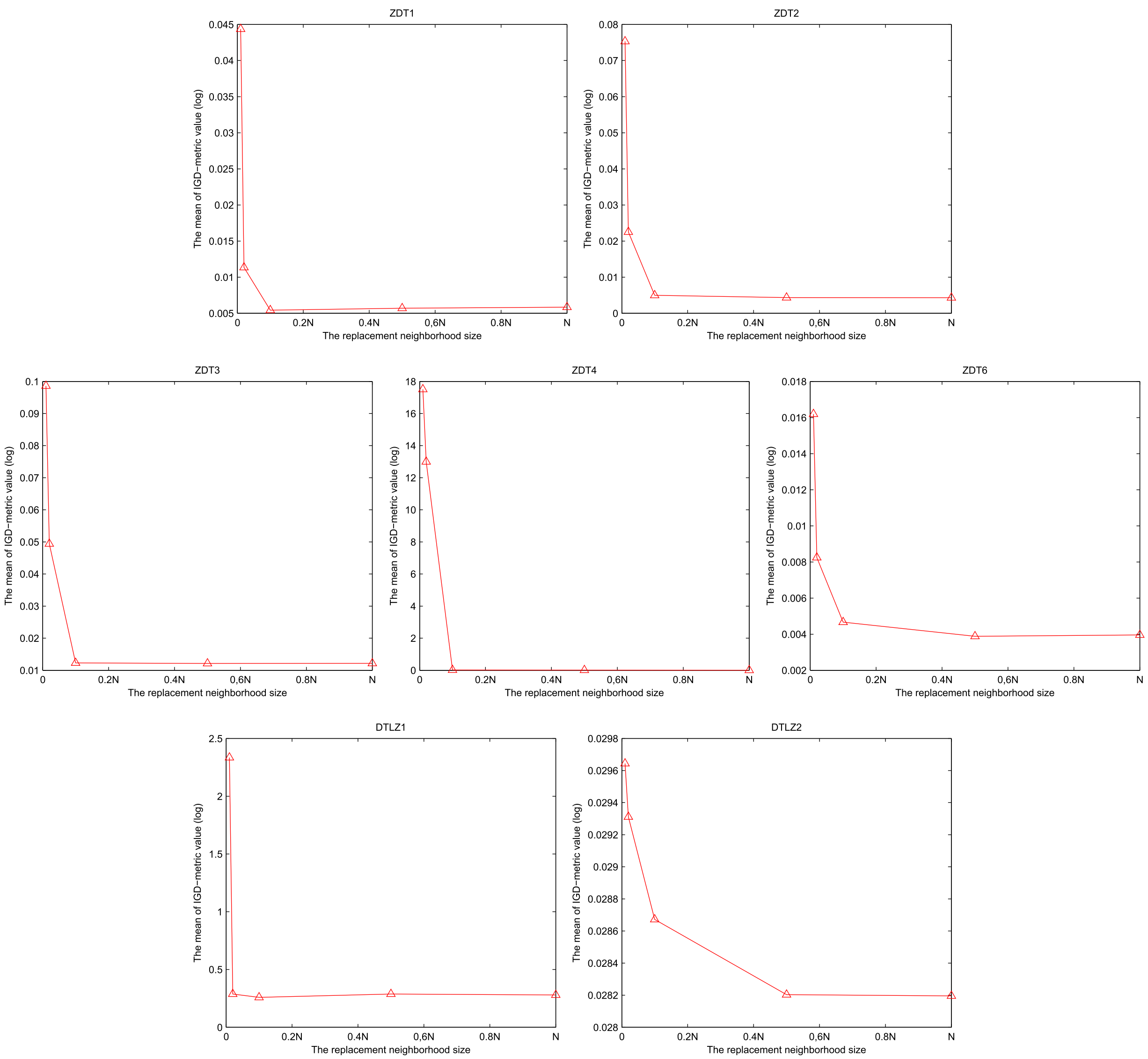

Fig. 6. Mean IGD of MOEA/D-GR with different replacement neighborhood sizes on ZDT and DTLZ problems.

DTLZ2. For DTLZ2, two algorithms have similar performances in terms of IGD-metric while MOEA/D-GR slightly performs slightly worse in terms of I_H-metric.

F. Experimental Result on F1-F9

Table V Table VI give the results on the F1-F9. It is clear that the best results ($i.e.$ minimum IGD-metric values and maximal I_H-metric values) obtained by MOEA/D-GR in all the 30 runs are better than MOEA/D-DE on almost all the problems. However, the mean IGD-metric values of MOEA/D-DE are better than MOEA/D-GR for almost all the problems. It indicates that MOEA/D-GR has better search ability on the best run but is less stable than MOEA/D-DE on this set of test problems. The stability of MOEA/D-GR may be improved if it just allows a limited number of solutions are replaced by a offspring solution as in MOEA/D-DE.

G. Effects of Replacement Neighborhood Size on Different Problems

This subsection investigates the effect of the replacement neighborhood size on the algorithm performance on different problems. T_r is set to be $\{0.01, 0.02, 0.1, 0.5, 1\} \times N$ respectively. Figs. 4-6 plot the average IGD values in 30 independent runs of the MOEA/D-GR with different replacement neighborhood sizes on different problems.

It is clear from Fig. 4 that only a very tiny replacement neighborhood can produce a acceptable result for MOP1-MOP7. It is because most subproblems are very easily trapped at a local optimal solutions. A small replacement neighborhood can avoid it.

As shown in Fig. 5, the effects of the replacement neighborhood size depends on problems. On F1 and F7, the replacement neighborhood size has little impact on algorithm performances. On F2-F5 and F9, a very small replacement neighborhood (smaller than 10% of the population size) is needed to obtain good performances. On F8, which have many local PSs, a very small replacement neighborhood works very well. On F6 with three objectives, very big or very small sizes of replacement neighborhood cannot produce good results. It should be around 10% of the population size.

From Fig. 6, it can be observed that MOEA/D-GR works very well when the replacement neighborhood is larger than $10\%N$ on ZDT and DTLZ problems. It means that more effort should be spent on converge in MOEA/D for ZDT and DTLZ problems.

In conclusion, different problems need different trade-offs between convergence and diversity, meanwhile the global replacement can easily control the trade-off by the replacement neighborhood size.

V. Conclusion

In this paper, we proposed a new replacement strategy named global replacement under the framework of MOEA/D. We studied this strategy experimentally on three different sets of benchmark problems. Experimental results showed that the global replacement can improve the performance of MOEA/D. Moreover, the trade-off between convergence and diversity can be easily controlled by the replacement neighborhood size. We also demonstrated that different problems need different trade-offs between convergence and diversity. Therefore, self-adaption of the replacement neighborhood size is worthwhile exploring in the future.

References

[1] K. Deb, *Multi-Objective Optimization using Evolutionary Algorithms*. John Wiley & Sons LTD, 2001.

[2] C. A. Coello Coello, D. A. van Veldhuizen, and G. B. Lamont, *Evolutionary algorithms for solving multi-objective problems*. New York: Kluwer Academic Publishers, 2002.

[3] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, 2007.

[4] A. Zhou, B.-Y. Qu, H. Li, S.-Z. Zhao, P. N. Suganthan, and Q. Zhang, "Multiobjectiveevolutionary algorithms: A survey of the state of the art," *Swarm and Evolutionary Computation*, vol. 1, no. 1, pp. 32–49, 2011.

[5] H. Li and Q. Zhang, "Comparison between NSGA-II and MOEA/D on a set of multiobjective optimization problems with complicated Pareto sets," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 2, pp. 284–302, 2009.

[6] H.-L. Liu, F. Gu, and Q. Zhang, "Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems," *IEEE Transactions on Evolutionary Computation*, 2013, in press.

[7] J. Jahn, *Mathematical vector optimization in partially ordered linear spaces*. Lang Frankfurt/Main Bern, 1986.

[8] K. Miettinen, *Nonlinear Multiobjective Optimization*. Kluwer Academic Publishers, 1999.

[9] R. T. Marler and J. S. Arora, "Survey of multi-objective optimization methods for engineering," *Structural and multidisciplinary optimization*, vol. 26, no. 6, pp. 369–395, 2004.

[10] I. Giagkiozis, R. C. Purshouse, and P. J. Fleming, "Generalized decomposition," in *Evolutionary Multi-Criterion Optimization (EMO)*. Springer, 2013, pp. 428–442.

[11] Y.-Y. Tan, Y.-C. Jiao, H. Li, and X.-K. Wang, "A modification to moea/d-de for multiobjective optimization problems with complicated pareto sets," *Information Sciences*, vol. 213, pp. 14–38, 2012.

[12] Y. Qi, X. Ma, F. Liu, L. Jiao, J. Sun, and J. Wu, "MOEA/D with adaptive weight adjustment," *Evolutionary computation*, 2013, in press.

[13] H. Ishibuchi, Y. Sakane, N. Tsukamoto, and Y. Nojima, "Effects of using two neighborhood structures on the performance of cellular evolutionary algorithms for many-objective optimization," in *IEEE Congress on Evolutionary Computation, 2009. (CEC)*. IEEE, 2009, pp. 2508–2515.

[14] H. Ishibuchi, N. Akedo, and Y. Nojima, "Relation between neighborhood size and moea/d performance on many-objective problems," in *Evolutionary Multi-Criterion Optimization (EMO)*. Springer, 2013, pp. 459–474.

[15] S.-Z. Zhao, P. N. Suganthan, and Q. Zhang, "Decomposition-based multiobjective evolutionary algorithm with an ensemble of neighborhood sizes," *IEEE Transactions on Evolutionary Computation*, vol. 16, no. 3, pp. 442–446, 2012.

[16] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: Empirical results," *Evolutionary computation*, vol. 8, no. 2, pp. 173–195, 2000.

[17] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, *Scalable test problems for evolutionary multiobjective optimization*. Springer, 2005.

[18] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. Da Fonseca, "Performance assessment of multiobjective optimizers: An analysis and review," *IEEE Transactions on Evolutionary Computation*, vol. 7, no. 2, pp. 117–132, 2003.

[19] M. R. Sierra and C. A. Coello Coello, "Improving PSO-based multiobjective optimization using crowding, mutation and e-dominance," in *Evolutionary Multi-Criterion Optimization (EMO)*, vol. LNCS 3410, 2005, pp. 505–519.

[20] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, 1999.