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Evolutionary Algorithms Dynamics and its Hidden Complex Network Structures

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***Abstract*—In this participation, we are continuing to show mutual intersection of two completely different areas of research: complex networks and evolutionary computation. Large-scale networks, exhibiting complex patterns of interaction amongst vertices exist in both nature and man-made systems (i.e., communication networks, genetic pathways, ecological or economical networks, social networks, networks of various scientific collaboration etc.) and are a part of our daily life. We demonstrate that dynamics of evolutionary algorithms, that are based on Darwin theory of evolution and Mendel theory of genetic heritage, can be also visualized as complex networks. Such network can be then analyzed by means of classical tools of complex networks science. Results presented here are currently numerical demonstration rather than theoretical mathematical proofs. We open question whether evolutionary algorithms really create complex network structures and whether this knowledge can be successfully used like feedback for control of evolutionary dynamics and its improvement in order to increase the performance of evolutionary algorithms.**

I. Introduction

In this article, we try to merge two at first glance completely different areas of research: complex networks and evolutionary computation.

Large-scale networks, exhibiting complex patterns of interaction amongst vertices exist in both nature and in man-made systems (i.e., communication networks, genetic pathways, ecological or economical networks, social networks, networks of various scientific collaboration, Internet, World Wide Web, power grid etc.). The structure of complex networks thus can be observed in many of those systems. The word complex networks [1], [2] comes from the fact that they exhibit substantial and non-trivial topological features, with patterns of connection between vertices that are neither purely regular nor purely random. Such features include a heavy tail in the degree distribution, a high clustering coefficient, hierarchical structure, amongst other features. Amongst many studies, two well-known and much studied classes of complex networks are the scale-free networks and small-world networks, whose discovery and definition are vitally important in the scope of this research. Specific structural features can be observed in both classes i.e. so called power-law degree distributions for the scale-free networks and short path lengths with high clustering for the small-world networks. Research in the field of complex networks has joined together researchers from many areas, which were outside of this interdisciplinary research in the past like mathematics, physics, biology, chemistry computer science, epidemiology etc..

Evolutionary computation is a sub-discipline of computer science belonging to the bio-inspired computing area. The main ideas of evolutionary computation has been published [3] and widely introduced to the scientific community [4]. The most well known evolutionary techniques are Genetic Algorithms (GA) introduced by J. Holland [4], Evolutionary Strategies (ES), by Schwefel [5] and Rechenberg [6] and Evolutionary Programming (EP) by Fogel [7] for example.

Here we can observe mutual intersection of evolutionary algorithms and complex networks that is a promising interdisciplinary research. Evolutionary algorithms, based on their canonical central dogma (following darwinian ideas) clearly demonstrate intensive interaction amongst individual in the population see [3]- [7], [18], [9], which is, in general, one of the important attributes of complex networks (intensive interaction amongst the vertices).

The main motivation (as well as question) is whether it is possible to visualize and simulate underlying dynamics of evolutionary process like complex network. Reason for this is that today various techniques for analysis and control of complex networks exist and if complex network structure is hidden behind EA dynamics, then we believe, that existing control techniques could be used to improve dynamics of EAs.

The main idea of our research is to show in this article that the dynamics of evolutionary algorithms in general, can be converted to the complex networks and can be analyzed and visualized like complex networks. This article is focused on observation and description of complex networks phenomenon in evolutionary dynamics. Possibilities of its use are discussed at the end.

II. Experiment Design

The main idea of experiments here is that evolutionary experiments will run on selected test functions, interactions amongst the individuals (which individual successfully created an offspring) will be recorded and then this interaction is visualized like network of interactions.

Population size	50
Mutation	0.4
Generations	300
Individual Length	20

TABLE I. GA SETTING.

Algorithms selected for our previous experiments [17], [19], [22] were differential evolution (DERand1Bin), [9] and SOMA (AllToOne), [8]. Application of alternative algorithms like Genetic Algorithms GA and Simulated Annealing (SA), ES and/or Swarm Intelligence are now in process and we would like to present here some preliminary results for GA introduced by J. Holland [4]. Setting of algorithm is in Tab. I.

The test functions applied in this experimentation were selected from

the test bed of 17 test functions. In total 5 test function (in 20 dimensions) were

selected as a representative subset of functions which shows

geometrical simplicity and low complexity as well as functions from

the opposite side of spectra.

Selected functions were: Ackley's function (1), Griewangk's function (2), Rana's function (3), Rastrigin's function (4) and Schwefel's function (5) and each experiment was repeated 100 times, i.e. 500 experiments was done (2 algorithms, 5 test functions, 100 repeated simulations).

All experiments were done in Mathematica 9, on MacBook Pro, 2.8 GHz Intel Core 2 Duo.

$$\sum_{i=1}^{D-1}\left(20+e-\frac{20}{e^{0,2\sqrt{\frac{(x_i^2+x_{i+1}^2)}{2}}}}-e^{0,5(\cos(2\pi x_i)+\cos(2\pi x_{i+1}))}\right) \tag{1}$$

$$1+\sum_{i=1}^{D}\frac{x_i^2}{4000}-\prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}}) \tag{2}$$

$$\sum_{i=1}^{D-1}\left(\begin{array}{l} x_i\sin(\sqrt{|x_{i+1}+1-x_i|})\cos(\sqrt{|x_{i+1}+1+x_i|})+ \\ (x_{i+1}+1)\cos(\sqrt{|x_{i+1}+1-x_i|}) \\ \sin(\sqrt{|x_{i+1}+1+x_i|}) \end{array}\right) \tag{3}$$

$$2D\sum_{i=1}^{D}x_i^2-10\cos(2\pi x_i) \tag{4}$$

$$\sum_{i=1}^{n}-x_i\sin(\sqrt{|x_i|}) \tag{5}$$

III. VISUALIZATION

The most critical point of this research and related simulations was which data and relations should be selected and consequently visualized. Each class of algorithm is based on different principle. The main idea was that each individual is represented by vertex and edges between vertices should reflect dynamics in population, i.e. **interactions between individuals** (which individual has been used for offspring creation,). In the GA the individual is selected according to its fitness. Thus in GA, we have recorded only those individuals-parents, that has been replaced by better offspring (like vertex with added connections). In the GA class of algorithms we have omitted the philosophy that a bad parent is replaced by a better offspring, but accepted philosophical interpretation, that individual (worse parent) is moving to the better position (in classical philosophy it is better offspring). Thus no vertex (individual) has to be either destroyed or replaced in our proposed methodology. If, for example, GA has a parent been replaced by offspring, then it was considered as an activation-improvement (new additional links, edges) of vertex-worse parents from another one.

Experimental data can be visualized in few different ways and as an example, few typical visualizations are depicted here. For example in Fig. 2 - 4 interactions between individuals in the population during entire evolution are described. As mentioned in the previous section, vertices in complex graph are individuals that are improved by other individuals, incrementally from generation to generation. Fig. 2 - 4 shows, that interactions between individuals create (at the first glance) structures, which look like complex networks. However, it has to be said that we have met results whose visualizations looks like net and resemble complex networks but after closer complex network characteristics calculations, those networks did not belong to the class of complex networks with small world phenomenon.

Another kind of visualization is depicted in Fig. 1, in which one can see which individual (out of 50) has been activated for offspring creation (in this case were visualized the best individuals from population). It is visible that after 200 generations almost in all experiments global extreme has been found, because since approx. 200 generations the best individual stay the same in each of all 100 experiments.

IV. RESULTS

A. Structure and dynamics

As already mentioned, the main idea of this participation is to expand our previous experiments from SOMA and DE to GA and another algorithms and make some basic visualization and calculations. For GA 100 experiments on 5 test function in 20 dimensional space have been done and dynamics of GA was recorded from generation to generation and then processed and visualized. Some visualizations are on Fig. 1 that shows all 100 experiments in one figure. Each dot is the best solution from GA generation of given experiment. Visualization of complex network structure are Fig. 2 - 4 that demonstrate our idea of complex network structure in EA based on obtained results. Visualization has been done by means of Mathematica 9.

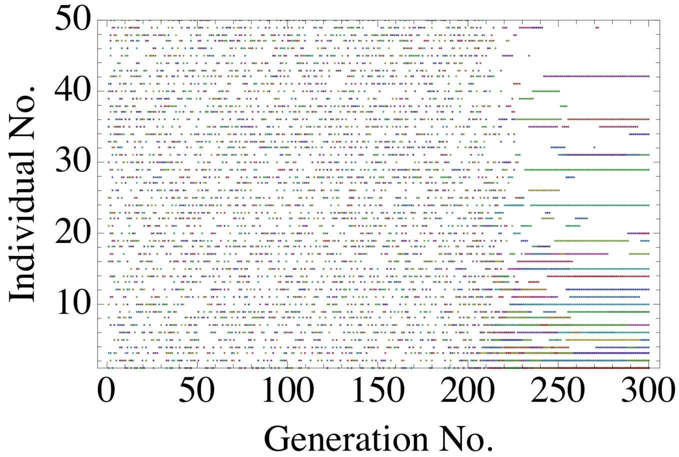

Fig. 1. 100 repeated experiments of GA in one picture

B. Interpretation of the Example Network

As reported above GA algorithms were set according to Table I and have been tested on 5 test functions with constant level of test function dimensionality (i.e. individual length = 20). All data has been processed graphically. Emergence of complex network structure behind evolutionary dynamics depends on many factors. To observe complex network structure it is important to wait certain number of generations, that depend on parameters of evolutionary algorithm and problem dimensionality. The main tools of *Mathematica* software were used on basic analysis and are proposed here. Analyzed graphs have multiple edges (not visualized here) that can be understand like weight of single edge. Attributes of proposed analysis are represented by subgraph colors and vertices sizes in graphs. Our proposed interpretation, based on terms and command from Wolfram *Mathematica* used for all of our experiments is following:

Degree centrality, see Fig. 2, gives a list of vertex degrees for the vertices in the underlying simple graph of g. Degree centrality will give high centralities to vertices that have high vertex degrees. The vertex degree for a vertex v is the number of edges incident to v. For a directed graph, the in-degree is the number of incoming edges and the out-degree is the number of outgoing edges. For an undirected graph, in-degree and out-degree coincide. In the case of evolutionary dynamics, degree centrality shows how many in-coming (support from individuals) or out-coming (support to another individuals) edges, vertex - individual, under study has during the evolution. This quantity can be related to progress of the evolutionary search and used to made conclusion of what set of individuals has maximally contribute to that. On Figure 2 are individuals sized according to that degree.

Graph partition, see Fig. 3, finds a partition of vertices such that the number of edges having endpoints in different parts is minimized. For a weighted graph, graph partition finds a partition such that the sum of edge weights for edges having endpoints in different parts is minimized. In the case of evolutionary dynamics, individuals in population are separated into "groups" according to their interactions with another individuals, based on their success in active individual fitness improvements. "Endpoints" can be understood like successful

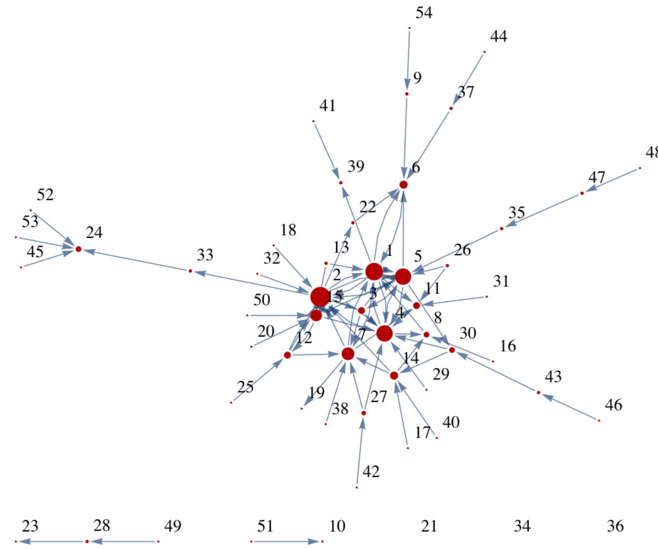

Fig. 2. Degree centrality of GACN in 50th generation.

participation of selected individuals in active individual fitness. On Fig. 3 is partition visualized by colors. This analysis gives view on population structure and shows the set of individuals that got or donate oriented edges (support from / to) the same group of individuals. Based on number of connections or weights (if multiple edges are understood like integer weights) of edge, it can be analyzed what part of population was the most important in the evolutionary dynamics for given case.

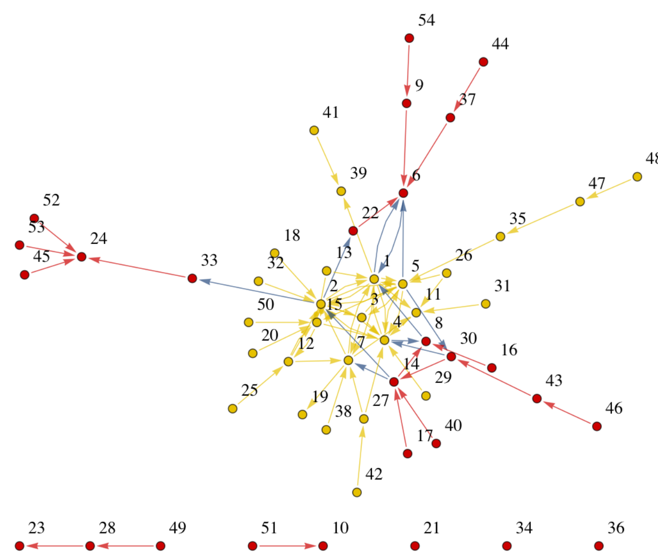

Fig. 3. Graph partition of GACN in 50th generation.

Community, see Fig. 4. Community graph plot attempts to draw the vertices grouped into communities. In the case of evolutionary dynamics, community graph plot showing the individuals grouped into communities. Communities (with border are individuals that communicate amongst themselves (higher density of edges in community, multi edges are not visualized here, rather than between communities) and community are then joined by connections that are ”one-way” and shows flow of information between communities). This kind of visualization can be interesting also in the case of parallel EAs, where islands of subpopulations are formed.

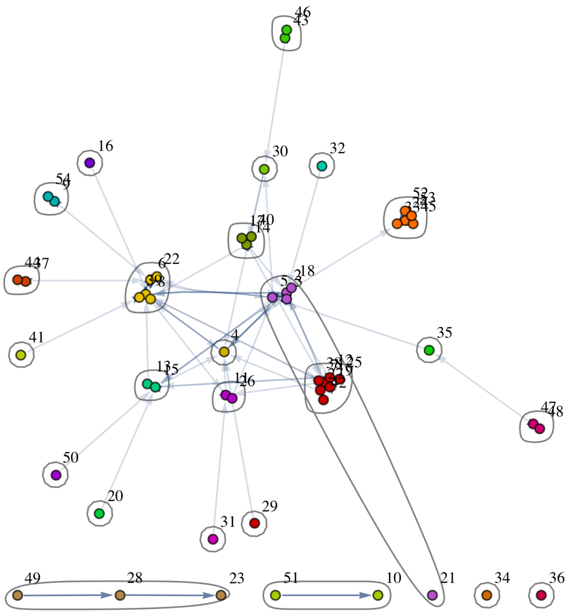

Fig. 4. Community graph of GACN in 50th generation.

V. Conclusion

The main motivation of this research is whether it is possible to visualize and simulate underlying dynamics of an evolutionary process as a complex network. Based on previous results and results here, it can be stated, in fact the same as in [18] or [19], that:

1) No. of generations: occurrence of the complex network structure (CNS) sensitively depends on the number of generations. If the number of generations was small, then no CNS was established. This effect can be easily understood so that low number of generations means that EAs has no time, long enough, to establish CNS. This is quite a logical observation in complex network dynamic when CNS is not observable at the beginning of linking process. During our experiments it has been observed that the moment of CNS establishing depends on cost function and its dimension, population size and used algorithm. Very generally, EAs searching for global extreme is quite random-like in the beginning and when domain of global extreme is discovered, then CNS is quite quickly established and is global extreme is found then individual that stay at this position start to ”pick up” more and more connections and become to be ”rich and richer”, see [1].
2) Dimensionality: In this paper we have used only dimension 20 for selected test functions but based on [17] in can be concluded that dimensionality impact on CNS forming has been observed when the dimension of the cost function was big and number of generations was too low, the selected EA was not able to finish successfully the global extreme search not all connections had been properly established. Thus if high dimensional cost functions are used, then number of generations has to be selected so that at least domain of the global extreme is found. On the other side, if number of generations is very big, then it is possible observe effect of complex network forming.
3) Test functions: dependence of CNS forming on the test function was not strictly observed, the general consensus being that for more complex test functions, like Schwefel (5), etc, the algorithm needs more generations to establish CNS, i.e. more complex function requires more generations and/or bigger population size. In the case of simpler functions and low dimensions global extreme is quickly found and phase of CNS creation is very short and then effect of ”rich become to be richer” is visible soon, that has impact on CNS structure formation. It is important to say that this last phase depend on algorithm structure (i.e. how individuals are handled for parents selection etc.) and we have observed it in the case of SOMA [8] algorithm and DE [9] too. Term algorithm structure means that for example in the case of the SOMA (or generally swarm algorithms) is like Leader (winning vertex, the best solution for generation-migration-iteration) selected the first individual of population with the best fitness, no matter how many other individuals in the population has the same fitness. It is demonstrated in [17].
4) Population size: CNS forming was observed usually from population size of 100 and more individuals for dimensions 50. Again, it is parameter, which does not influence CNS forming alone, but in the combination with another parameters, as mentioned in the previous items. For that fact and based on our experiments we have selected dimension 20 that show itself (in combination of other GAs parameters) enough. Unfortunately there is no ”cook book” for such setting, it depends on heuristic experiences.
5) Used algorithm: CNS forming has also been clearly observed with algorithms, that are more or less based on swarm philosophy or partly associated with it. For example DERand1Bin did not show CNS formatting so often like another versions (in principle each individual is selected to be parent), see [17], while in the case of the DELocalToBest in which the so called best solution in the population plays an important role, CNS has been observed, as well as in the SOMA strategies. In contrary we have recorded effect of CNS formatting also for GA, that promote idea, that algorithm setting play an important role for that. The easier CNS formatting observable with swarm like algorithms is close to the idea of preferred linking in the complex networks modeling social behavior (citation networks, etc).
6) Evaluation and visualization: Evaluation and visualization has been done here by a few basic figures, exhibiting CNS and its selected attributes. More visualizations and evaluations were done for GA results, like for example Fig. 5 that was not reported here due to the limited space.

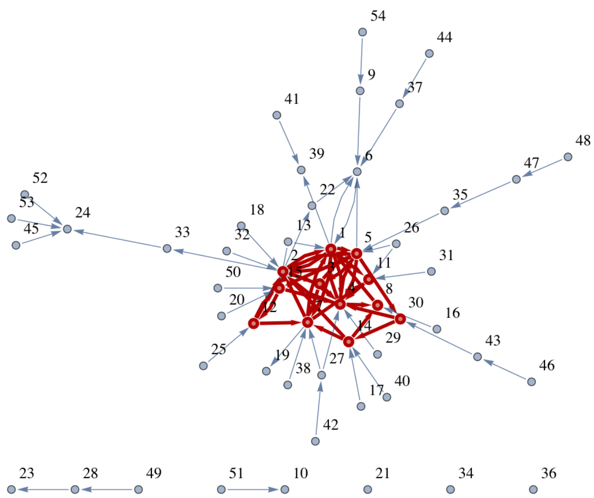

Fig. 5. K Core Components of GACN in 50th generation.

We would like to propose more significant expansion of ideas presented here at the end. This idea was already presented in [19] and we would like to propose it here again as possible use and joining of EAs, CNS and CML (Coupled Map Lattices) systems, see [10], [11] and Fig. 7, that allow, in principle, to analyze and control EAs (or CNS itself) dynamics.

CML structure is given by Eq. (6). Typical example is CML based on so called logistic equation, [23], [24], [25] which is used to simulate behavior of system which consists of n mutually joined cells via nonlinear coupling, usually noted like ε. Mathematical description of CML system is given by eq. (6). Function, which is represented by $f(x_n(i))$ is "arbitrary" discrete system - in this case study logistic equations have been selected to substitute $f(x_n(i))$. CML description based on eq. (6) is on Fig. 6 .

$$x_{n+1}(i) = (1-\varepsilon)f(x_n(i)) + \frac{\varepsilon}{2}(f(x_n(i-1)) + f(x_n(i+1))) \tag{6}$$

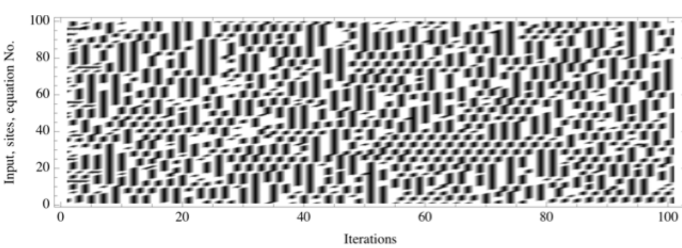

Fig. 6. CML system based on Eq. 6.

Some methods on CML systems control, especially by means of evolutionary algorithms, exist today. The spirit of this idea is to create a closed loop in the following schematic: evolutionary dynamics $\rightarrow$ complex network $\rightarrow$ CML system $\rightarrow$ control CML $\rightarrow$ control evolutionary dynamics. Reason for this is that this proposed techniques can be used for analysis and control of complex networks exists and if complex network structure would be hidden behind EA dynamics, then we believe, that for example above mentioned control techniques could be used to improve dynamics of EAs.

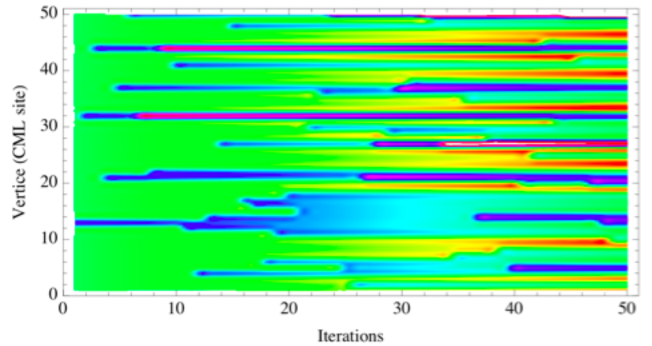

Fig. 7. CML system reflecting complex network structure.

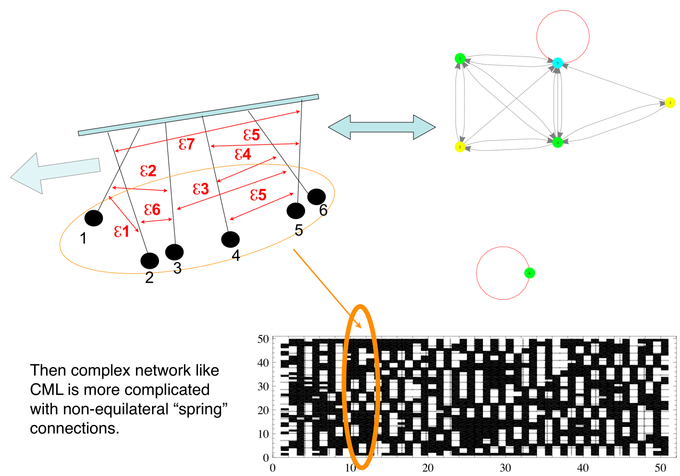

Fig. 8. The mechanical principle of CML from CNS.

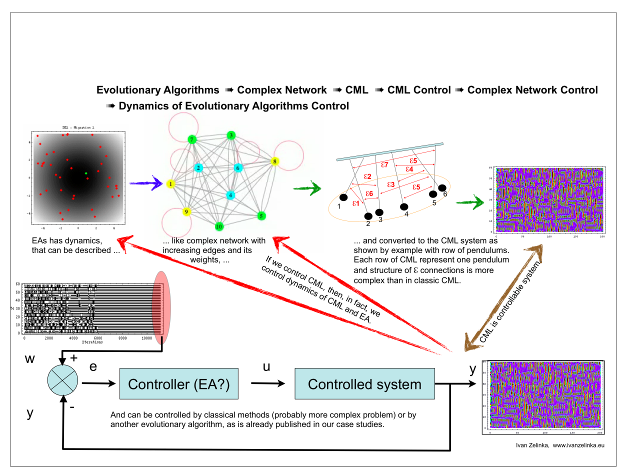

Fig. 9. The schematic principle of proposed feedback control EAs and CNS: evolutionary dynamics complex network CML system control CML control evolutionary dynamics.

Complex network is depicted as a set of vertices, mutually joined by single and multiple edges. Each edge can be added or cancelled during the evolution of the network, or importance of an edge can be modified by weights associated to the each edge. Adding or canceling of the edges (or its weights) represents, in fact, dynamics of the algorithm, i.e. network.

Our method of visualization is based on fact that simplest version of CML (i.e. 1D version, [11], [15], [16], [10]) is usually depicted like a row of mutually joined sites, where each site is nonlinearly joined with its nearest sites, see Fig. 8 for mechanical analogy. The idea of equivalence between CML and complex network is quite simple. Each vertex is equivalent to the site in the CML (via mechanical principle at Fig. 8). Comparing to the standard CML its sites (inputs, rows) are in complex network CML (CNCML) not joined to the nearest site, but to the sites equal to the complex network vertices connections. Thus sites in CNCML **are not joined symmetrically** (i.e. from site x_1 to x_2 to x_3 and vice versa to x_n) and between different sites is random pattern of connections, which can change in the time. As it is visible, based on our interpretation (CN $\rightarrow$ CNCML), CNCML is more complex version of the classical simplified version of the CML, however still CML in general, so all techniques of control and analysis [11] shall be working on such a version of CML. As depicted on Fig. 9 it is possible to convert EAs dynamics to the CNS, then via mechanical analogy to CML. CML is controllable today by means of classical methods [11], [15], [16] as well as evolutionary techniques [10], so if both are joined together, then we have possibility to control dynamics of EA used for this transformation and if EA is omitted from this scheme, then also CNS itself (does not matter of what nature, i.e. citation net, social net, ...) can be also in principle controlled by this approach.

Also, for CML systems there are techniques for analysis of chaos and routes to chaos behavior, so it is obvious that this is also open field of another research that join EAs, CNS and CML systems. We have already published both main part of feedback at Fig. 9, i.e. evolutionary dynamics complex network (in [19]) CML system (in [22], and also at [20], [20]) control CML by means of EAs (in [10], [12], [13], [14]). The main and remaining part to control evolutionary dynamics or CNS is now under investigation.

In this paper we have suggested possible interpretation of selected well known tools and terminology from complex networks analysis to the evolutionary algorithms dynamics converted to the complex network structures. The volume of this article is too small to mention and explain all the possible interpretations and tools. This is only a mid-step in our research presented in above mentioned papers, where we proposed all necessary steps joining evolutionary dynamics, complex networks and CML systems.

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References

[1] S. N. Dorogovtsev and J.F.F. Mendes, Evolution of Networks, Adv. Phys. 51, 1079 (2002)

[2] S. Boccaletti et al., Complex Networks: Structure and Dynamics, Phys. Rep., 424 (2006), 175-308.

[3] Turing, A.: Intelligent machinery, unpublished report for National Physical Laboratory. In: Michie, D. (ed.) Machine Intelligence, vol. 7 (1969); Turing, A.M. (ed.): The Collected Works, vol. 3, Ince D. North-Holland, Amsterdam (1992)

[4] Holland, J.: Adaptation in natural and artificial systems. Univ. of Michigan Press, Ann Arbor (1975)

[5] Schwefel, H.: Numerische Optimierung von Computer-Modellen, PhD thesis (1974); Reprinted by Birkhauser (1977)

[6] Rechenberg, I.: (1971) Evolutionsstrategie - Optimierung technischer Systeme nach Prinzipien der biologischen Evolution (PhD thesis), Printed in Fromman-Holzboog (1973)

[7] Fogel,D.B.:Unearthinga Fossil from the History of Evolutionary Computation. Fundamenta Informaticae 35(1-4), 116 (1998)

[8] Zelinka I. SOMA – Self Organizing Migrating Algorithm, in New Optimization Techniques in Engineering, Eds.: Babu B. V., Onwubolu G. (Springer-Verlag, New York), pp 167-218, 2004

[9] Price K. An Introduction to Differential Evolution, New Ideas in Optimization, Ed.: Corne D., Dorigo M. Glover F. (McGraw-Hill, London, UK), pp 79-108, 1999

[10] Zelinka I, Celikovsky S, Richter H and Chen G., (2010) Evolutionary Algorithms and Chaotic Systems, (Eds), Springer, Germany, 550s, 2010.

[11] Schuster H. G. [1999] Handbook of Chaos Control (Wiley-VCH, New York)

[12] Zelinka I., Investigation on Evolutionary Deterministic Chaos Control, IFAC, Prague 2005

[13] Zelinka, I., Real-time deterministic chaos control by means of selected evolutionary algorithms Engineering Applications of Artificial Intelligence (2008), doi:10.1016/j.engappai.2008.07.008

[14] Zelinka I., Investigation on realtime deterministic chaos control by means of evolutionary algorithms, Proc. First IFAC Conference on Analysis and Control of Chaotic Systems, Reims, France, 211-217, 2006

[15] Deilami M.Z., Rahmani Ch.Z.,Motlagh M.R.J., Control of spatio-temporal on–off intermittency in random driving diffusively coupled map lattices, Chaos, Solitons, Fractals, Available online 21 December 2007

[16] Zahra R.Ch., Z. R., Motlagh M.R.J., Control of spatiotemporal chaos in coupled map lattice by discrete-time variable structure control, Physics Letters A, 370, 3-4, 302-305

[17] Zelinka I., Davendra D., Chadli M., Senkerik R., Dao T.T. and Skanderova L., Evolutionary Dynamics and Complex Networks, In: Zelinka I, Snasel V., Ajith A.,(Eds), Handbook of Optimization, Springer, Germany, 1100s, 2012

[18] Zelinka I, Snasel V., Ajith A.,(Eds), Handbook of Optimization, Springer, Germany, 1100s, 2012

[19] Zelinka I., Davendra D., Senkerik R., Jasek R., Do Evolutionary Algorithm Dynamics Create Complex Network Structures? Complex Systems, 2, 0891-2513, 20, 127-140, 2011

[20] Zelinka I., Mutual Relations of Evolutionary Dynamics, Deterministic Chaos and Complexity, tutorial at IEEE Congress on Evolutionary Computation 2013, Mexico, 2013

[21] Zelinka I., On Close Relations of Evolutionary Dynamics, Chaos and Complexity, keynote at International Workshop on Chaos-Fractals Theories and Applications, Dalian, China, 2012

[22] Zelinka I., Controlling Complexity, In Sanayei A., Zelinka I., Rossler O. E. (Eds.), ISCS 2013: Interdisciplinary Symposium on Complex Systems, Emergence, Complexity and Computation, Vol. 8, Springer 2014

[23] May R. [1976] Simple mathematical model with very complicated dynamics, Nature, 261, 45-67

[24] Hilborn R.C. [1994]. Chaos and Nonlinear Dynamics, Oxford University Press, ISBN 0-19-508816-8, 1994

[25] Chen G. [2000] Controlling Chaos and Bifurcations in Engineering Systems (CRC Press, Boca Raton)