2014 IEEE Congress on Evolutionary Computation (CEC)
July 6-11, 2014, Beijing, China

Linear Sparse Arrays Designed by Dynamic Constrained Multi-Objective Evolutionary Algorithm

Wei Dong, Sanyou Zeng, Yong Wu, Dayue Guo, Lunan Qiao and Zhiqun Liu

***Abstract*— The design of linear sparse array is a constrained multi-objective optimization problem(CMOP). There are three objectives: minimization of peak sidelobe level(PSLL), half-power beam width(HPBW) and spatial aperture. The amplitude coefficients of elements and sensor positions of the array are decision variables. Dynamic constrained multi-objective evolutionary algorithm(DCMOEA) is used to design linear sparse arrays in this paper. It makes a difference that the output is a set of Pareto solutions (antenna arrays), not just only one solution. The users can choose an array from the set to meet their preferences for low PSLL, small HPBW, small spatial aperture or a trade-off among them. Experimental results showed that the DCMOEA performs better than peer state-of-art algorithms referred in this paper, especially on the arrays' spatial aperture optimization.**

Index Terms*— Dynamic constrained multi-objective evolutionary algorithm(DCMOEA), linear sparse arrays, peak sidelobe level(PSLL), half-power beam width(HPBW), spatial aperture

I. Introduction

Antenna array is a group of isotropic radiators such that the currents running through them are of different amplitudes and phases. Antenna arrays constitute one of the most versatile classes of radiators due to their capacity for beam shaping, beam steering and high gain. Antenna arrays have been widely used in different applications such as radar, sonar and communications[1]. In many applications it is necessary to design antenna arrays with very directive characteristics and a small HPBW. Meanwhile, PSLL reduction has a great importance in recent communication systems. It is considered as one of the most important applications of digital beamforming since it reduces the effect of interference arriving outside the main lobe[2].

Sparse array is one where the inter-sensor spacing is larger than half of the signal wave length[3]. In order to obtain lower PSLL, improve spatial resolution and reduce implementation complex and cost, sparse array is better than equally-spaced arrays[4].

The design techniques of sparse arrays which all the element currents are identical have been proposed mainly centered on two problems. Firstly, element position synthesis is a nonlinear problem. Secondly, element spacing constraint has to be placed on the solutions[5] [6].In order to solve these problems efficiently, evolutionary algorithms have been considered and successfully applied to antenna array designing[4] [7]. Meanwhile optimizing the amplitude coefficients of elements and sensor positions of the array simultaneously makes the problem more complex[8] [9]. In addition, many evolutionary algorithms has been successfully applied to the designing of linear arrays. For example, linear arrays designed by genetic algorithm(GA)[2] [5] [10] [11], improved genetic algorithm(IGA)[4], invasive weed optimization(IWO)[12] . However, the simulate results of the algorithms mentioned above were simplex, only one linear array was provided.

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Linear sparse array designing problem is a minimization constrained multi-objective optimization problem(CMOP). To solve this problem, DCMOEA is proposed in this paper. The amplitude coefficients and sensor positions are jointly optimized by using DCMOEA. What make a difference is that the output of DCMOEA is a set of Pareto solutions (antenna arrays), not just only one solution. The users can choose an array from the set to meet their preferences for low PSLL, small HPBW, small spatial aperture or a trade-off among them. The difference between the method proposed in this paper and other methods is that the method in this paper provides not only one solution, which makes the solution meet different preferences of users.

The remaining part of this paper is organized as follows. In the second section, linear sparse array synthesis formulation is briefly introduced. The third section describes the dynamic constrained multi-objective evolutionary algorithm proposed in this paper. The simulation results using DCMOEA is presented in the fourth section. Finally, the last section presents the conclusions of this paper.

II. Linear Sparse Array Synthesis Formulation

Linear sparse array designing problem is a minimization constrained optimization problem(COP), it is a multi-objective problem as well. The PSLL, spatial aperture and HPBW are considered in this paper.

The PSLL, spatial aperture and HPBW are not only required to be minimized but also required to satisfy specific constraints. The linear sparse array designing problem is defined as:

$$
\begin{aligned}
min \quad & y_1 = f_{PSLL}(\overrightarrow{x}) \\
min \quad & y_2 = f_{Aperture}(\overrightarrow{x}) \\
min \quad & y_3 = f_{HPBW}(\overrightarrow{x}) \\
st: \quad & g_{PSLL}(\overrightarrow{x}) - C_{PSLL} \leq 0 \\
& g_{aperture}(\overrightarrow{x}) - C_{aperture} \leq 0 \\
& g_{HPBW}(\overrightarrow{x}) - C_{HPBW} \leq 0 \\
where \quad & \overrightarrow{x} = (\overrightarrow{x}_1, \overrightarrow{x}_2) \in (\mathbf{X_1}, \mathbf{X_2}) \\
& \mathbf{X_1} = \{(x_{11}, \cdots, x_{1n}) | l_{1i} \leq x_{1i} \leq u_{1i}, \\
& \qquad i = 1, \cdots, n_1\} \\
& \mathbf{X_2} = \{((x_{21}, \cdots, x_{2n}) | l_{2i} \leq x_{2i} \leq u_{2i}, \\
& \qquad i = 1, \cdots, n_2\} \\
& \overrightarrow{l_1} = (l_{11}, l_{12}, ..., l_{1n_1}) \\
& \overrightarrow{u_1} = (u_{11}, u_{12}, ..., u_{1n_1}) \\
& \overrightarrow{l_2} = (l_{21}, l_{22}, ..., l_{2n_2}) \\
& \overrightarrow{u_2} = (u_{21}, u_{22}, ..., u_{2n_2})
\end{aligned} \tag{1}
$$

where $f_{PSLL}(\overrightarrow{x})$, $f_{Aperture}(\overrightarrow{x})$ and $f_{HPBW}(\overrightarrow{x})$ are objectives, $g_{PSLL}(\overrightarrow{x})$, $g_{aperture}(\overrightarrow{x})$ and $g_{HPBW}(\overrightarrow{x})$ are constraints. ($\overrightarrow{x}_1$, $\overrightarrow{x}_2$) are decision variables, $\overrightarrow{x}$ denotes decision space. $\overrightarrow{x}_1$ are the amplitude coefficients of elements, $\overrightarrow{x}_2$ are the distances between two elements, l_1 and u_1 are the lower bound and upper bound of the amplitude coefficient, respectively. l_2 and u_2 are the lower bound and upper bound of the distance between two elements. Before defining $f_{PSLL}(\overrightarrow{x})$, here gives a definition of array factor:

$$
AF(\theta) = \sum_{n=1}^{N} x_{1n} e^{\vec{j} k d_n cos(\theta)} \tag{2}
$$

Where x_{1n} are amplitude coefficients of the elements, $k = 2\pi/\lambda$, λ represents the wavelength, d_n are the locations of the elements on the X axis and $d_0 = 0$, while $d_n = \sum_{i=0}^{n} x_{2i}, (x_{2i} >= 0.5\lambda)$, θ is the steering angle of the array, $0 < \theta < \pi$. To ensure the array response at the preestablished target direction θ_0, a constraint on array as $AF(\theta_0) = 1$ is set in optimization. Then $f_{PSLL}(\overrightarrow{x})$ is given as

$$
f_{PSLL}(\overrightarrow{x}) = 20 \log \frac{AF_{\theta_{peakside}}}{AF_{\theta_{mainbeam}}} \tag{3}
$$

Where $AF_{\theta_{peakside}}$ and $AF_{\theta_{mainbeam}}$ are the array factor of side beam and main beam. $f_{Aperture}(\overrightarrow{x})$ is a function to calculate the spatial aperture of a array, it is given as:

$$
f_{Aperture}(\overrightarrow{x}) = \sum_{i=1}^{N-1} x_{2i} \tag{4}
$$

where x_{2i} is the distance between the ith element and the $(i+1)th$ element. $\overrightarrow{d} = d_1, d_2, ..., d_N$, in this paper, $\overrightarrow{d}$ is set to be symmetrical which means $d_i = d_{N-i}$. $f_{HPBW}(\overrightarrow{x})$ is the half power beam width of the array pattern which is measured at -3dB.

As discussed above, linear sparse array designing is a constrained multi-objective problem(CMOP). To solve C-MOP by constrained multi-objective evolutionary algorithm(CMOEA), the key issue is to achieve feasible population.It is impossible to always maintain a feasible population during the evolutionary process. However, we could maintain a population with most solutions feasible which ensures the performance of CMOEA not to decrease too much. It is implemented by adopting dynamic technique as follows:

Assume $\overrightarrow{0}$ is the original boundaries of the linear sparse array designing problem, it is broadened to $\overrightarrow{e}^{(0)}$ at the beginning to achieve a feasible population. Then the broadened boundaries $\overrightarrow{e}^{(0)}$ are shrank gradually back to $\overrightarrow{0}$. Each shrinking should be small enough so that most of the solutions in the population are not destroyed into infeasible ones. This prosess constructs a series of CMOPs ($CMOP^{(k)}$),k=0,1,2,3,...,i.e., a dynamic constrained multi-objective optimization problem(DCMOP) is shown as follows

$$
COMP^0 = \begin{pmatrix} min \overrightarrow{y} = (\overrightarrow{f}(\overrightarrow{x}), \overrightarrow{\phi}(\overrightarrow{x})) \\ st: \overrightarrow{g}(\overrightarrow{x}) \leq \overrightarrow{e}^{(0)} \end{pmatrix}
$$

$$
COMP^1 = \begin{pmatrix} min \overrightarrow{y} = (\overrightarrow{f}(\overrightarrow{x}), \overrightarrow{\phi}(\overrightarrow{x})) \\ st: \overrightarrow{g}(\overrightarrow{x}) \leq \overrightarrow{e}^{(1)} \end{pmatrix}
$$

......

$$
COMP^K = COMP \cong COP = \begin{pmatrix} min \overrightarrow{y} = (\overrightarrow{f}(\overrightarrow{x}), \overrightarrow{\phi}(\overrightarrow{x})) \\ st: \overrightarrow{g}(\overrightarrow{x}) \leq \overrightarrow{e}^{(K)} \end{pmatrix}
$$

Where $\overrightarrow{e}^{(k)} = (\overrightarrow{e_1}^{(k)}, \overrightarrow{e_2}^{(k)}, ..., \overrightarrow{e_m}^{(k)})$, $\overrightarrow{e}^{(0)} > \overrightarrow{e}^{(1)} ... > \overrightarrow{e}^{(K)} = \overrightarrow{0}$., $\overrightarrow{y}$ are objectives and $\overrightarrow{g}$ are the constraints. When there is a solution meet $\overrightarrow{g} < \overrightarrow{e}$, this solution is $\overrightarrow{e}$-feasible, other wise, the solution is $\overrightarrow{e}$-infeasible.

$\overrightarrow{\phi}(\overrightarrow{x})$)=($\phi_1(\overrightarrow{x})$,$\phi_2(\overrightarrow{x})$,$\phi_3(\overrightarrow{x})$) is violations objectives.Given a solution, The violation of a constraint in is usually defined as

$$
G_i(\vec{x}) = \max\{g_i(\vec{x}), 0\}, i = 1, 2, ..., m \tag{5}
$$

$\phi_1(\overrightarrow{x})$ is the first violation objective, it is defined as followed:

$$
\phi_1(\vec{x}) = \sum_{i=1}^{m} \frac{G_i(\vec{x})}{\max_{\vec{x} \in \mathbf{P}(0)} \{G_i(\vec{x})\}}, \tag{6}
$$

$\phi_2(\overrightarrow{x})$ is the second violation objective, it is the max value of the violations of constraints. It is defined as followed:

$$
\phi_2(\vec{x}) = \frac{MAX(G(\vec{x}))}{\max_{\vec{x} \in \mathbf{P}(0)} \{G_i(\vec{x})\}}, \tag{7}
$$

$\phi_3(\overrightarrow{x})$ is the third violation objective, it is the number of the violations of constraints which is grater than 0.

Here $m = 3$, all the values of objectives($\overrightarrow{y}$) and constraints depend on variables $\overrightarrow{x}$ as well as k in the DCMOP.

Linear sparse array designing is a constrained multi-objective problem. To solve this problem, DCMOEA is proposed.

III. Dynamic Constrained Multi-Objective Evolutionary Algorithm

In order to solve the design problem of linear sparse array, dynamic constrained multi-objective evolutionary algorithm is proposed in this section.

A. The Framework of DCMOEA

DCMOEA for solving the dynamic constrained multi-objective problems(DCMOP) is a dynamic version of constrained multi-objective evolutionary algorithm. The details are as follows.

Algorithm 1 The framework of DCMOEA

step 1 : Initiation
  step 1.1 Create parent population **P**(0) and evaluate **P**(0). Set evolutionary generation counter t=0.
  step 1.2 Set environment parameter k=0. Initialize elastic-boundaries $\overrightarrow{e} = \overrightarrow{e}^{(0)}$ satisfying **P**(0) is $\overrightarrow{e}$-feasible.
step 2 :
  step 2.1 t=t+1.
  step 2.2 if all $\overrightarrow{x} \in$ **P**(t) are $\overrightarrow{e}$-feasible, the set $\overrightarrow{e} = \overrightarrow{e}^{(k+1)}$, k=k+1 and update $\overrightarrow{e}$-feasibility of all $\overrightarrow{x} \in$ **P**(t).
step 3 : Generate offspring population **S**(t) from **P**(t) and evaluate **S**(t), the offspring generation process is shown in **Algorithm** 2.
step 4 : Create next parent population **P**(t+1) by selecting solutions from **P**(t) and **S**(t). It is shown in **Algorithm** 3.
step 5 : If k achieves K or t achieves $AbortingT$ then goto *Step 6*, else goto *Step 2*.
step 6 : Output **P**(t).

Algorithm 1 is a dynamic version of a CMOEA by insertion $Step$1.2 and $Step$2.2.

B. Generate offspring population S(t) from P(t)

Algorithm 2 Generate offspring population

input : Parents $\mathbf{P}(t) = \{P_1, P_2, ..., P_N\}$.
output : Offsprings $\mathbf{S}(t) = \{S'_1, S''_1, ..., S'_N, S''_N\}$.
FOR i=1 TO N
step 1 : Affine crossover on $\mathbf{P}(t) \setminus P_i$:
  Randomly select M different solutions P_{i1},P_{i2},...,P_{in} from **P**(t)$\setminus P_i$. Randomly create M coefficient: a_j, $j = 1, 2, 3, ..., M, \sum_{j=1}^{M} a_j = 1$ and -1 $\leq a_j \leq$ M.
  Create offspring: $O_i = \sum_{j=1}^{M} a_j P_{ij}$.
step 2 : Uniform crossover on O_i and P_i:
  Suppose $O_i = (O_{i1}, O_{i2}, ..., O_{in})$, $P_i = (P_{i1}, P_{i2}, ..., P_{in})$.
  FOR j = 1 TO n, exchange P_{ij} and O_{ij} with exchange probability p_c
  Suppose offspring being S'_i, S''_i.
step 3 :let $S'_i = (S'_{i1}, S'_{i2}, ..., S'_{in})$,$S''_i = (S''_{i1}, S''_{i2}, ..., S''_{in})$. *FOR j=1 TO n*, let $S'_{ij} \Leftarrow rand(l_i, u_i)$, $S''_{ij} \Leftarrow rand(l_i, u_i)$ under the probability of p_m, $[l_i, u_i]$
step 4 : Calculate $(f(S'_i)), \phi(S'_i), f(S''_i)), \phi(S''_i))$
END FOR i
step 5 : Output:$\mathbf{S}(t) = \{S'_1, S''_1, ..., S'_N, S''_N\}$.

Algorithm 2 shows the offspring process in this part, it contains crossover and mutation. The input of this algorithm is parent population which contains N individuals and the output contains 2N individuals.

C. Create next parent population

In this part, next parent population which contains N individuals will be chosen from $2N$ individuals, using constrained $\overrightarrow{e} - Pareto - domination$.

Algorithm 3 Create next parent population

Create next parent population **P**(t+1) by selecting solutions from **R**=**P(t)** $\cup$ **S**(t)
step 1 : Empty P(t+1).
step 2 :Perform a non-dominated sorting aiming at **R**(t) by using constrained $\overrightarrow{e} - Pareto - domination$, and identify different non-dominated sets: $\mathbf{B}_1$, $\mathbf{B}_2$, $\ldots$.
step 3 :Move N solutions from **R**(t) to **P**(t+1) by using the sorting order.
  step 3.1 Set i=1
  step 3.2
  WHILE $|\mathbf{P}(t+1) \cup \mathbf{B}_i| < N$
  $\mathbf{P}(t+1) = \mathbf{P}(t+1) \cup \mathbf{B}_i$; i=i+1.
  END WHILE
  step 3.3
  If$|\mathbf{P}(t+1) \cup \mathbf{B}_i| > N$,then execute the cutoff operator which eliminates $|\mathbf{P}(t+1) \cup \mathbf{B}_i|$ - N solutions from $\mathbf{B}_i$ and assings the reduced $\mathbf{B}_i$ to $\mathbf{P}(t+1)$.
step 4 :Output **P**(t+1).

The comparison of the simulate result depends on the value of objectives $\overrightarrow{y}$ and constraints $\overrightarrow{\phi}$. Array a is better than array b if a and b are all e-feasible and

$$y_{ia} < y_{ib} \ and \ \phi_{ia} < \phi_{ib} \ for \ all \ i$$

or a is e-feasible and b is e-infeasible. Array b is better than array a if a and b are all e-feasible and

$$y_{ib} < y_{ia} \ and \ \phi_{ib} < \phi_{ia} \ \ for \ all \ i$$

or b is e-feasible and a is e-infeasible. If a and b are both e-infeasible, compare the distance between a solution to the current e-boundary, the smaller, the better. Otherwise, a and b are incomparable. If a and b is incomparable, they will be added into the non-dominated set.

IV. Simulate Results and Comparisons

A. The initial parameter settings

In order to start the optimization precess of a linear 19-elements array antenna, the initial value of parameters are set. The populations size is 50, the number of iterations is set to 2500, the mutation probability is 0.01, the exchange probability is 0.9. The dynamic-boundaries $\vec{e}^{(k)} = (e_1^{(k)}, e_2^{(k)}, \ldots, e_m^{(k)})$ $(k = 0, 1, \cdots, K)$ in DCMOP are modelled as:

$$e_i^{(k)} = A_i e^{-(\frac{k}{B_i})^2} - \varepsilon, i = 1, 2, \cdots, m \tag{8}$$

where ε is a given small positive number ($\varepsilon = 0.0001$ in this paper).

$$
\begin{aligned}
A_i &= \max_{\vec{x} \in \mathbf{P}(0)} \{G_i(\vec{x})\} + \varepsilon, i = 1, 2, \cdots, m \\
B_i &= \frac{K}{\sqrt{ln\left(\frac{\max_{\vec{x} \in \mathbf{P}(0)} \{G_i(\vec{x})\} + \varepsilon}{\varepsilon}\right)}}, i = 1, 2, \cdots, m
\end{aligned} \tag{9}
$$

The element number is set to be $N = 19$, the lower bound of amplitude coefficient $l_1 = 0$ while the upper bound $u_1 = 1$, the lower bound of the distance between two element $l_2 = 0.5\lambda$ and the upper bound $u_2 = 2.5\lambda$, the constraints of PSLL is set to be $C_{PSLL} = 14dB$, the constrain of spatial aperture is $C_{aperture} = 22\lambda$ and the constrain of HPBW is $C_{HPBW} = 1.5°$.

To consider the HPBW and PSLL, the steering angle is set to arranging from $\theta_{Start} = 0°(0)$ to $\theta_{End} = 180°(\pi)$ while the sampling number is 1025.

B. Simulate results and comparisons

The optimization process ended when the simulate times reaches 2500. The output of DCMOEA is a non-dominated set which contains 46 different solutions, each solution represents a sparse linear array. The distribution of the the non-dominated set are shown in Fig.1, Fig.2, Fig.3 and Fig.4.

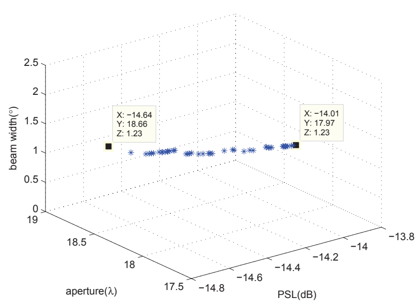

Fig. 1. The distribution of the non-dominated set(1)

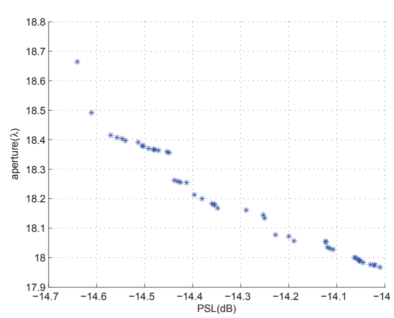

Fig. 2. The distribution of the non-dominated set(2)

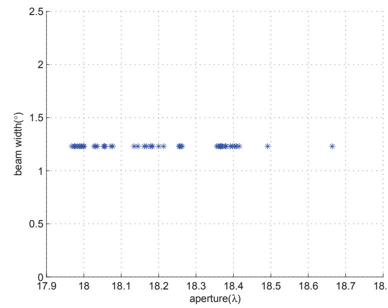

Fig. 3. The distribution of the non-dominated set(3)

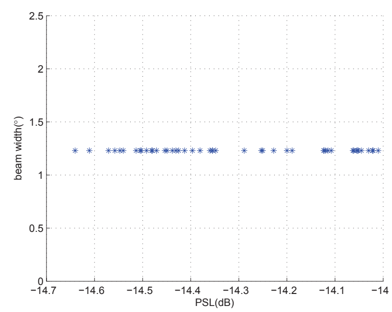

Fig. 4. The distribution of the non-dominated set(4)

The overall conditions of the non-dominated set is shown in Fig.1. The distribution of aperture and PSLL is shown in Fig.2, the distribution of HPBW and aperture is shown in Fig.3 and Fig.4 shows the distribution of HPBW and PSLL.

The apertures of these arrays ranges from 17.97λ to 18.66λ, the HPBWs are all $1.23°$ and the PSLL ranges from -14.64dB to -14.01dB. One sparse linear array in the non-dominated set is shown as follows. Taking PSLL, spatial aperture and HPBW into account, this array has a better performance than the arrays proposed in [3] and [4]. The comparison between the method proposed in [3] and the method proposed in this paper is shown in Table I. The comparison between [4] and the this paper is shown in Table II.

TABLE I

THE COMPARISON BETWEEN THE METHOD PROPOSED IN [3] AND THE METHOD PROPOSED IN THIS PAPER

	Referenced [3]	This Paper
Sensor number	24	19
PSLL	-14.45dB	-14.64dB
Aperture	25λ	18.6641λ
HPBW	$1.23°$	$1.23°$
u_{-3dB}	0.0214	0.0214

The amplitude coefficient of this array is shown in Table III and Fig.5. The location of this array is shown in Table IV and Fig.6.

TABLE II

THE COMPARISON BETWEEN THE METHOD PROPOSED IN [4] AND THE METHOD PROPOSED IN THIS PAPER

	Referenced [4]	This Paper
PSLL	-14.49dB	-14.64dB
Aperture	21.0898λ	18.6641λ
HPBW	1.32°	1.23°
u_{-3dB}	0.0230	0.0214

TABLE III

THE AMPLITUDES COEFFICIENT OF THE ARRAYS DESIGNED BY DCMOEA

element	Amplitude Coefficient	element	Amplitude Coefficient
1	0.3850	11	0.9101
2	0.6312	12	0.7739
3	0.5083	13	0.5875
4	0.5751	14	0.4369
5	0.5076	15	0.5076
6	0.4369	16	0.5751
7	0.5875	17	0.5083
8	0.7739	18	0.6312
9	0.9100	19	0.3850
10	0.6140		

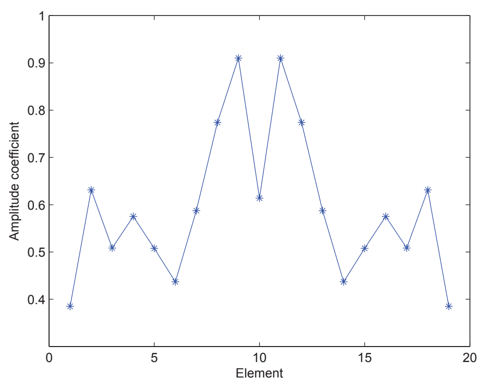

Fig. 5. The amplitudes coefficient of the arrays designed by DCMOEA

TABLE IV

THE LOCATION OF THE ARRAYS DESIGNED BY DCMOEA

element	Distance to the 1st one	element	Distance to the 1st one
1	0	11	10.2267
2	0.8992	12	12.1545
3	1.8564	13	12.9645
4	2.5785	14	13.7622
5	3.5683	15	15.0957
6	4.9018	16	16.0855
7	5.6995	17	16.8076
8	6.5095	18	17.7648
9	8.4374	19	18.6641
10	9.3320		

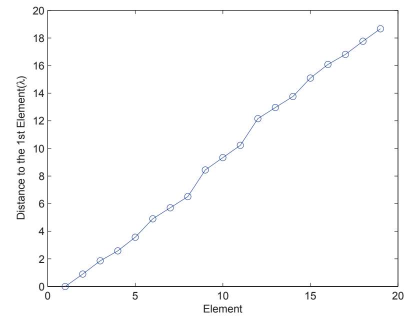

Fig. 6. The location of the element of the array designed by DCMOEA

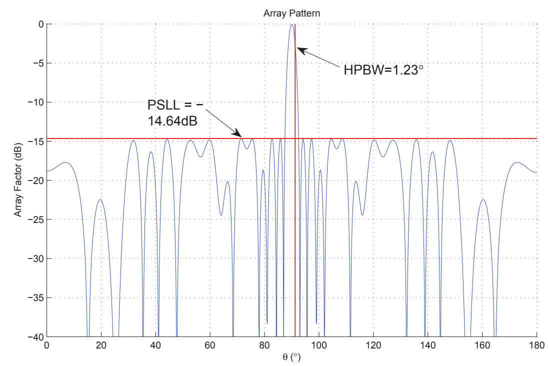

Fig. 7. The radiation pattern of the array designed by DCMOEA

This sparse linear array has a lower PSLL, a smaller spatial aperture and a smaller HPBW compared with the result in [4]. In [4], the steering angle is set to arranging from $\theta_{Start} = 0$ to $\theta_{End} = \pi$ while the sampling number is 1024. the HPBW in [4] is measured at $AF = -3dB$ where $cos(\theta_{-3dB}) = 0.0203$. If the sampling number is 1024 and the array response at $\theta = 0.5\pi$, $\theta = 0.5\pi$ is not a sampling point. Because of this, when calculating PSLL, it will be not too accurate. Besides, in these 1024 points, there does not exist any point θ_k around $\theta = 0.5\pi$ such that $cos(\theta_k) = 0.0203$. However, there is a sampling point $\theta_k = 1.5938$ that $abs(cos(\theta_k)) = 0.0230$ (The HPBW here is 1.32°). We suppose that this is an error in [4], mistakenly written 0.0230 to 0.0203.

V. CONCLUSION

In order to solve constrained multi-objective problem, DCMOEA is proposed in this paper. Dynamic technique is used to ensure the performance of CMOEA do not decrease too much.

Linear sparse array designing problem is a minimization constrained optimization problem(COP), and it is a multi-objective problem as well. The PSLL, spatial aperture and HPBW are not only required to be minimized but also required to satisfy specific constraints.

DCMOEA is proposed to solve this problem, and the output of DCMOEA are Pareto optimal solutions which is a non-dominated set. The set contains 46 different solutions, each solution represents a linear sparse array. The apertures of these arrays ranges from 17.97λ to 18.66λ, the HPBWs are all $1.23°$ and the PSLL ranges from -14.64dB to -14.01dB.Compared to the arrays designed in [3] and [4], an array designed by DCMOEA has lower PSLL, spatial aperture and HPBW. Notably, the spatial aperture is much smaller than that in [3] and [4].

ACKNOWLEDGMENT

Acknowledgement This work was supported by the National Natural Science Foundation of China and other foundations(No.s: 61271140, 61203306, 2012001202, 60871021, D20122701, 2011ba0268, 2011CDC028, 2011CD229).

REFERENCES

[1] C.A.Balanis,"Antenna Theory: Analysis and Design, Second Edition" ,John Wilely and Sons(Asia), Singepore, 2003.

[2] Amr H. Hussein, Haythem H. Abdullah, Salah Khamis, Ahmed M.Attiya, and Mohammed Nasr, "Side Lobe Level reduction of Linear Antenna Arrays Using a Hybrid Approach Based on MoM/GA algorithms", *PIERS Proceedings, Kuala Lumpur, MALAYSIA*, pp: 1328-1332, March 2012.

[3] A. Trucco and V. Murino, "Stochastic Optimization of Lineaer Sparse Arrays." *IEEE J.Oceanic Eng.*, vol. 24, no. 3, pp. 291-299, Jul.1999.

[4] Ling Cen, Wee Ser, Zhu Linag Yu, Susanto Rahardja and Wei Cen, "Linear Sparse Array Synthesis With Minimum Number of Sensors." *IEEE Transactions on Antennas And Propagation.*, vol. 58, no. 3, pp: 720-726, March.2010.

[5] B. P. Kumar and G. R. Branner, "Design of unequally spaced arrays for performance improvement.",*IEEE trans. Antennas Propag., Special Issue on Phased Arrays.*, vol. 47, no. 3, pp. 511-523, Mar. 1999.

[6] Kumar, B. P. and G. R. Branner, "Generalized analytical technique for the synthesis of uneaually spaced arrays with linear, planar, cylindrical of spheical geometry." *IEEE Trans. Antennas Propag.*, vol. 53, no. 2, pp. 621-634, Feb. 2005.

[7] Kesong Chen, Zishu He and Chunlin Han, "A modified Real GA for the Sparse Linear Array Synthesis With Multiple Constraints." *IEEE Transactions on Antennas And Propagation.*, vol. 54, no. 7. pp: 2169-2173, July 2006.

[8] Ling Cen, Wee Ser, Zhu Liang Yu, Susanto Rahardja, "An Improved Genetic Algorithm for Aperiodic Array Synthesis." *IEEE Trans. Antennas. Propagations*, vol. 60, no. 2, pp: 895-902, Feb. 2012.

[9] D. G. Kurup, M. Himdi and A. Rydberg, "Synthesis of uniform amplitude unequally spaced antenna arryas suing the defferential evolution algorithm.", *IEEE Transactions on Antennas And Propagation.*, vol. 51, no. 9, pp: 2210-2217, Sep. 2003.

[10] B. Eldosouky, A. H. Hussein, H. H. Abdullah, and S. Khamis, "Synthesis of Pencil Beam Linear Antenna Arrays using Simple FFT/CF/GA Based Technique.", *International Journal of Engineering & Technology*, vol. 13, no. 5, Oct. 2013.

[11] Sudipta Das, Somen Bhattachcrjee, Durbadal Mandal , Anup Kumar Bhattacherjee, "Optimal Sidelobe Reduction of Symmetric Linear Antenna Array Using Genetic Algorithm.", *IEEE India Conference, INDICON*, pp: 1-4, Dec.2010.

[12] Shaya Karamkashi, Ahmed A. Kichk, "Invasive Weed Optimization and its Featiures in Electromagnetic", *IEEE Transactions on Antennas And Propagation.*, vol. 58, no. 4, PP: 1269-1278, April 2010.