2014 IEEE Congress on Evolutionary Computation (CEC)
July 6-11, 2014, Beijing, China

Memetic Algorithm with Adaptive Local Search Depth for Large Scale Global Optimization

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***Abstract*—Memetic algorithms (MAs) have been recognized as an effective algorithm framework for solving optimization problems. However, the exiting work mainly focused on the improvement for search operators. Local Search Depth (LSD) is a crucial parameter in MAs, which controls the computing resources assigned for local search. In this paper, an Adaptive Local Search Depth (ALSD) strategy is proposed to arrange the computing resources for local search according to its performance dynamically. A Memetic Algorithm with ALSD (MA-ALSD) is presented, its performance and the effectiveness of ALSD are testified via experiments on the LSGO test suite issued in CEC'2012.**

Keywords—Memetic algorithms; Local Search Depth; Differential Search Algorithm; Solis and Wets' Algorithm

I. INTRODUCTION

Many real-life problems from different fields can be formulated as continuous optimization problems. These problems have been tackled using Evolutionary Algorithms (EAs) [1]-[4]. Unfortunately, most of the available evolutionary algorithms suffer from "the curse of dimensionality" [5], their performance deteriorates quickly with the grown of dimensionality [6]. Therefore, it's important to study the way to improve the ability of solving Large Scale Global Optimization (LSGO) problems for modern Evolutionary Algorithms.

Memetic Algorithms (MAs) are a kind of population-based stochastic heuristics composed of an evolutionary framework accompanied with a set of problem-specific Local Search (LS) operators [7]. The earliest formal definition has been presented in [8]. In recent decades, MAs have been used to solve various optimization problems and shown good performances, therefore have been becoming a popular idea for various engineering optimization tasks. [9]-[12].

In MAs, the population-based EAs are the fundamental framework which is mainly responsible for the global search. Various effective EAs have been designed and adopted in MAs to guarantee its effectiveness. Local search is also an indispensable component in MAs. Therefore, various novel LS methods have presented or adopted in recent years. In [13], an adaptive MA (AMA) is presented which utilizes the composite benefits of differential evolution (DE) for global search and Q-learning for local refinement. In AMA, four variants of DE, including the state-of-the-art self-adaptive DE algorithm, are selected to enhance the global search. Q-learning is treated as an effective LS method when embedded into the outstanding variants of DE. To solve multi-objective optimization problems, a new LS strategy, the Hill Climber with Sidestep (HCS), is proposed for multi-objective memetic algorithm (MOEA) in [14]. The HCS is a novel iterative search procedure, which has the ability to move toward and along the (local) Pareto set utilizing the geometry of the directional cones of the optimization problems. In [15], the opposition-based learning (OBL) strategy is proposed as the LS method to form MAs with Differential Evolution. It is observed that the above works on MAs pay more attention to the refinement among EA and LS methods. Few works is on arranging the computing resources of EA and LSs in MA to improve the performance of MAs.

The concept of Local Search Depth (LSD) was first proposed by Hart [17], when studied the parameterization of MAs. LSD means the available computing resources for LS during each generation cycle. In this paper, the parameter of LSD is used to arrange the computing resources between Global Search (GS) and Local Search (LS). Most MAs use fixed value of LSD. However, for different problems and search stages, the values of LSD need to be changed to fit different landscapes. Obviously, fixed value of LSD is not a good choice and the value of LSD is often hard to adjust manually. In this paper, an Adaptive Local Search Depth (ALSD) strategy is proposed to adjust the value of LSD automatically according to the performance of GS and LS on various fitness landscapes. The performances of GS and LS are measured by the Average Fitness Increment (AFI) [16] in each iteration. According to the comparison of AFI, the value of LSD is adjusted so as to achieve an appropriate distribution of computing resources for GS and LS. In this paper, LSD is measured by the number of Fitness Evaluations (FEs). If the AFI of LS outperforms that of GS, the FEs for LS will be increased in next search cycle, otherwise decreased. In MA-ALSD, the Differential Search Algorithm (DSA) [21], [22] is adopted as the Global Search operator, and Solis and Wets' algorithm (SW), a randomized hill-climber algorithm with selfadapted search size [26], as the Local Search operator. To testify the performance of MA-ALSD for LSGO problems, it has been applied to the LSGO test suite issued in CEC'2012 [18] and obtained superior performance compared with the state-of-the-art algorithms [19], [20], [27].

The rest of this paper is organized as follows: In section II, the algorithm MA-ALSD, including the proposed Adaptive Local Search Depth strategy, is described. Experimental results and analysis are shown in section III. Finally, the paper is concluded in section IV.

II. MEMETIC ALGORITHM WITH ADAPTIVE LOCAL SEARCH DEAPTH

In this section, the ALSD strategy is proposed and is embedded into MA to form MA-ALSD for LSGO problems. According to AFI, ALSD is able to distribute the computing resources for LS dynamically, so as to obtain a good balance between global and local search. In the framework of MA-ALSD, Differential Search Algorithm (DSA) [21], [22] is selected as the global search, and Solis and Wets' (SW) as the local search.

A. The Framework of MAs

MAs attempt to combine EAs and LS, so as to make full use of the exploration ability of EAs and the exploitation ability of LS. So far, various MAs have been proposed to solve optimization problems from various application domains. In the framework of MAs, the tradeoff between global and local search is the pressing problem. Distributing computing resources dynamically between GS and LS in each iteration is a promising way to solve this problem [23], [24], [29], [30]. In this paper, ALSD is proposed to achieve the goal.

B. Differential Search Algorithm

Differential Search Algorithm (DSA) [21] is a new and effective evolutionary algorithm for solving real-valued numerical optimization problems, which is inspired by the Brownian-like random-walk movement and the migration of organisms.

DSA is a population-based algorithm and the initial population $\mathrm{pop}^{(0)}$ is randomly generated by using (1).

$$\mathrm{pop}^{(0)} = lb + rand\,(NP, D) * (ub - lb) \quad (1)$$

In (1), function $rand(NP, D)$ returns a matrix with random value between 0 and 1, NP and D stand for population size and the dimensionality of the problem respectively, lb and ub denote the lower and upper bounds of the candidate solutions in each dimensionality.

For population $\mathrm{pop}^{(G)}$, a new population $\mathrm{stopover}^{(G)}$ is achieved by mutation operator, which is defined as follows:

$$\begin{aligned} stopover^{(G)} = pop^{(G)} + (R * map) * \\ (direction^{(G)} - pop^{(G)}) \end{aligned} \quad (2)$$

In (2), $direction^{(G)}$ is obtained by randomly changing the permutation of individuals in $\mathrm{pop}^{(G)}$. Map is a randomly generated {0, 1} matrix of NP×D for selecting the dimensions of each individual for mutation. R denotes the step size of each mutation, whose value is produced by a gamma-random number generator.

The selection operation in DSA is to choose the better individuals between $\mathrm{stopover}^{(G)}$ and $\mathrm{pop}^{(G)}$ greedily so as to form the new population $\mathrm{pop}^{(G+1)}$, just like Differential Evolution (DE) [25].

C. Solis and Wets' Algorithm

Solis and Wets' algorithm (SW) [26] is selected as the Local search, whose performance has been verified in many large scale optimization algorithm [27], [28].The classic SW is a randomized hill-climber with an adaptive and scalable step size. SW starts with a single point x and a deviation d is generated from a normal distribution, whose standard deviation is given by a parameter σ and expectation is 0. If either $x+d$ or $x-d$ is better, x is replaced by the better point and a record of success is made. Otherwise x remains the same and a record of failure is made. When the record of success exceeds the threshold (MaxSuccesses), σ is increased to obtain a larger step size and reset the record of success. Similarly, when the record of failure exceeds the threshold (MaxFailures), σ is decreased to obtain a smaller step size and reset the record of failure.

D. Adaptive Local Search Depth and MA-ALSD

As we know, local search is the vital component in the framework of MAs. The concept of Local Search Depth (LSD) was proposed with the question "How long should the local search be run?" in [17]. It is defined as the running time of LS in each iteration of MAs framework. In this paper, LSD is measured by the number of Fitness Evaluations (FEs) and it's used to arrange computing resources in MAs framework. According to empirical experience, one tend to achieve better results by adjusting the value of LSD according to the fitness landscape of task undergoing. For different problems and search stages, MAs needs different LSD to balance the ability of exploration and exploitation. Obviously, it's impossible to adjust LSD manually during the run of algorithm. Therefore, an Adaptive Local Search Depth (ALSD) strategy is needed. To adaptively adjust LSD, we need a method to evaluate the performance of GS and LS. The AFI (Average Fitness Increment) is adopted as the evaluation method, whose definition of AFI is as follow:

$$\mathrm{AFI} = \frac{\Delta\, Fitness}{\Delta\, FEs} \quad (3)$$

In (3), $\Delta Fitness$ is the fitness increment after conducting GS or LS during each search cycle, and ΔFEs is the consumption of the number of FEs.

After each search cycle, a comparison will be made between GS and LS on AFI. If the AFI of LS outperforms that of GS, the number of FEs for LS will be increased in next search cycle, otherwise the number of FEs for LS will be decreased. The adjustment of FEs is completed by adding or subtracting a predetermined value. In each search cycle, the best individual is selected for LS operation. The ALSD strategy is embedded into MA framework together with DSA and SW to form our algorithm MA-ALSD, whose pseudo code is shown in Fig .1.

The pseudo code of MA-ALSD
Initialize the population pop(G) with NP individuals and evaluate the fitness
Set up parameters for DSA and SW
Initialize the ALSD: Max_FEs_SW=400
while FEs< = Max_FEs-NP
Apply DSA as the global search
Generate the matrix of map and step size R= 1/gamrnd(1,0.5)
Perform mutation operator to generate $stopover^{(G)}$
Select the better individuals greedily from pop and stopover to form the new population $pop^{(G+1)}$
Return the best individual gbest
Apply SW to gbest
while FEs_SW <= Max_FEs_SW-2
Update gbest with Solis and Wets' Algorithm
Adjust Success, Failures and deviation
end while
Adjust Max_FEs_SW
Calculate AFI_L and AFI_G
if AFI_L>AFI_G
Increase Max_FEs_SW by k
else
Decrease Max_FEs_SW by k
end if
end while
Save gbest

Fig. 1. Pseudo code of MA-ALSD

III. EXPERIMENTAL STUDIES A

To examine the performance of MA-ALSD, the specific test-suite is selected, proposed in the special session on Large Scale Continuous Global Optimization at CEC'2012. The test-suit with twenty benchmark functions can be classified to five types of functions as follows:

1. Separable Functions (3)
   (a) F1: Shifted Elliptic Function
   (b) F2: Shifted Rastrigin's Function
   (c) F3: Shifted Ackley's Function
2. Single-group m-nonseparable Functions (5)
   (a) F4: Single-group Shifted and m-rotated Elliptic Function
   (b) F5: Single-group Shifted and m-rotated Rastrigin's Function
   (c) F6: Single-group Shifted and m-rotated Ackley's Function
   (d) F7: Single-group Shifted m-dimensional Schwefel's Problem 1.2
   (e) F8: Single-group Shifted m-dimensional Rosenbrock's Function
3. $\frac{D}{2m}$ group m-nonseparable Functions (5)
   (a) F9: $\frac{D}{2m}$-group Shifted and m-rotated Elliptic Function
   (b) F10: $\frac{D}{2m}$-group Shifted and m-rotated Rastrigin's Function
   (c) F11: $\frac{D}{2m}$-group Shifted and m-rotated Ackley's Function
   (d) F12: $\frac{D}{2m}$-group Shifted m-dimensional Schwefel's Problem 1.2
   (e) F13: $\frac{D}{2m}$-group Shifted m-dimensional Rosenbrock's Function
4. $\frac{D}{m}$-group m-nonseparable Functions (5)
   (a) F14: $\frac{D}{2m}$-group Shifted and m-rotated Elliptic Function
   (b) F15: $\frac{D}{2m}$-group Shifted and m-rotated Rastrigin's Function
   (c) F16: $\frac{D}{2m}$-group Shifted and m-rotated Ackley's Function
   (d) F17: $\frac{D}{2m}$-group Shifted m-dimensional Schwefel's Problem 1.2
   (e) F18: $\frac{D}{2m}$-group Shifted m-dimensional Rosenbrock's Function
5. Fully-nonseparable Functions (2)
   (a) F19: Shifted Schwefel's Problem 1.2
   (b) F20: Shifted Rosenbrock's Function

A. Comparison with Fixed LSD

To testify the effectiveness of the ALSD strategy, a performance comparison is made between Adaptive LSD (ALSD) and fixed LSD (FLSD). The depth value of FLSD is set 400, which also works as the initial value of ALSD. The results of MA-ALSD with FEs=3·106 and dimension=1000 are compared with MA-FLSD, as is presented in Table I. For all separable and fully-nonseparable functions, MA-ALSD outperforms MA-FLSD. On most of single-group m-nonseparable and D/m-group m-nonseparable functions, MA-ALSD achieves better results. On F5, F10, F11, the results of MA-ALSD are similar to those of MA-FLSD. As to F1, F7, F12, F17, MA-ALSD has obvious advantage than MA-FLSD. All above show the effectiveness of ALSD.

B. Results of MA-ALSD

The algorithm runs 25 times on each test function independently and the best, worst, median, mean and standard derivation of MA-ALSD are computed. The dimension of each function is 1000 and the values are recorded with FEs=$1.2{\cdot}10^5$, $6{\cdot}10^5$, and $3{\cdot}10^6$, as shown in Table II. According to the results, we can make the following conclusions:

- For most of the functions, the differences between mean and median value are very small, which shows that our algorithm has strong robustness.
- As to F1, F6, F7, F9, F12, F13, F14, and F17, the performance of MA-ALSD achieves obvious improvement, with the increase of FEs. Therefore, if these functions have more computing resource, they will obtain better performance.
- For nonseparable functions F6, F7, F12, and F17, MA-ALSD achieves pretty good results.

C. Comparison with Other Algorithms

The results of MA-ALSD with FEs=$3{\cdot}10^6$ are compared with the reference LSGO algorithms DECC-G, MLCC and MA-SW-Chains, which are shown in Table III and Table IV. According to the two tables, we can obtain the following characteristics:

- MA-ALSD achieves the best results among nonseparable functions except F11, F16, and F18. DECC-G is so suitable for solving F11 and F16 that no other algorithms have achieved better solutions on these two functions.
- As to F6, F7, F12, and F17, MA-ALSD has the significant advantage compared with DECC-G and MLCC.
- For all functions of single-group m-nonseparable and fully-nonseparable, MA-ALSD obtains the best results, which implies that our algorithm is fit to solve this category of functions.
- For non-separable problems, MA-ALSD obtains the similar performance compared with MA-SW-Chains, which also uses MA framework.

IV. CONCLUSION

In this paper, an Adaptive Local Search Depth strategy is proposed to adjust the Local Search Depth dynamically during the optimization process according to the comparison of Average Fitness Increment between Local Search and Global Search. Then a Memetic Algorithm with ALSD, MA-ALSD, is presented. In MA-ALSD, DSA is mainly responsible for global search, and Solis and Wets' Algorithm is selected for local search. The effectiveness of ALSD and MA-ALSD are testified via experiments on the LSGO test suite issued in CEC'2012.

V. ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant No. 61071024 and 61331015.

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TABLE I

COMPARISON WITH MA-FLSD, FEs=3.0e6, LSD=400

		F1	F2	F3	F4	F5	F6	F7
	Best	6.46e+05	3.49e+03	1.04e+01	5.78e+11	7.47e+07	1.75e+01	6.34e+05
	Median	7.99e+05	3.96e+03	1.19e+01	7.75e+11	**1.30e+08**	1.79e+01	9.10e+05
MA-FLSD	Worst	1.40e+06	4.24e+03	1.33e+01	9.94e+11	2.48e+08	1.88e+01	1.25e+06
	Mean	8.97e+05	3.93e+03	1.19e+01	7.80e+11	**1.43e+08**	1.80e+01	9.20e+05
	Std	2.30e+05	1.98e+02	8.16e-01	1.20e+10	4.96e+07	3.32e-01	1.56e+05
	Best	5.26e-15	4.26e+02	1.08e+00	5.02e+10	7.56e+07	1.47e+00	3.79e-02
	Median	**1.85e-12**	**5.00e+02**	**1.84e+00**	**5.49e+11**	1.93e+08	**1.77e+00**	**6.73e-01**
MA-ALSD	Worst	3.86e-05	6.50e+02	1.64e+01	6.67e+11	2.59e+08	2.26e+00	4.69e+01
	Mean	**3.01e-06**	**5.06e+02**	**2.50e+00**	**5.01e+11**	1.82e+08	**1.79e+00**	**8.76e+00**
	Std	8.81e-06	4.80e+01	2.98e+00	1.71e+11	5.85e+07	1.76e-01	1.51e+01
		F8	F9	F10	F11	F12	F13	F14
	Best	1.52e+07	8.43e+07	4.73e+03	1.99e+02	2.10e+01	7.00e+02	1.13e+08
	Median	3.46e+07	1.19e+08	**5.70e+03**	**2.02e+02**	2.17e+01	4.98e+03	1.31e+08
MA- FLSD	Worst	1.36e+08	1.41e+08	6.93e+03	2.19e+02	2.26e+01	3.76e+04	1.47e+08
	Mean	4.83e+07	1.18e+08	**5.75e+03**	**2.06e+02**	2.17e+01	7.88e+03	1.29e+08
	Std	3.32e+07	1.31e+07	5.64e+02	7.44e+00	6.12e-01	8.08e+03	1.09e+07
	Best	3.23e+00	3.59e+07	4.35e+03	2.18e+02	2.95e-07	2.96e+02	4.54e+07
	Median	**2.80e+07**	**5.56e+07**	6.22e+03	2.18e+02	**1.73e-06**	**8.30e+02**	**5.72e+07**
MA-ALSD	Worst	9.36e+07	6.80e+07	1.19e+04	2.19e+02	3.01e-06	1.96e+03	6.71e+07
	Mean	**3.19e+07**	**5.49e+07**	6.53e+03	2.18e+02	**1.71e-06**	**8.47e+02**	**5.68e+07**
	Std	2.57e+07	6.78e+06	1.74e+03	1.92e-01	6.13e-07	3.93e+02	5.84e+06
		F15	F16	F17	F18	F19	F20	
	Best	6.92e+03	3.96e+02	4.98e+02	2.74e+03	3.13e+05	1.04e+03	
	Median	7.67e+03	**3.97e+02**	5.48e+02	2.53e+04	3.54e+05	1.19e+03	
MA-FLSD	Worst	1.42e+04	3.97e+02	5.63e+02	4.58e+04	5.25e+05	1.82e+03	
	Mean	**7.90e+03**	**3.97e+02**	5.32e+02	2.46e+04	3.59e+05	1.26e+03	
	Std	1.36e+03	2.88e-01	2.88e+01	1.50e+04	4.46e+04	1.89e+02	
	Best	6.81e+03	3.96e+02	5.29e-02	1.33e+03	1.06e+05	1.00e+03	
	Median	**7.56e+03**	**3.97e+02**	**7.56e-02**	**1.61e+04**	**1.27e+05**	**1.07e+03**	
MA-ALSD	Worst	1.49e+04	3.97e+02	9.93e-02	5.60e+04	1.57e+05	1.68e+03	
	Mean	8.92e+03	**3.97e+02**	**7.62e-02**	**1.86e+04**	**1.26e+05**	**1.12e+03**	
	Std	2.75e+03	2.54e-01	1.26e-02	1.39e+04	1.08e+04	1.61e+02	

TABLE IV.

COMPARISON WITH MA-SW-CHAINS, FEs=3.0e6

		F1	F2	F3	F4	F5	F6	F7
	Best	3.18e-15	7.04e+02	3.34e-13	3.04e+11	2.89e+07	8.13e-07	3.35e-03
	Median	**1.50e-14**	7.90e+02	**6.11e-13**	**3.54e+11**	2.31e+08	**1.60e+00**	9.04e+01
MA-SW-Chains	Worst	8.15e-14	9.37e+02	1.58e-12	3.97e+11	2.90e+08	1.16e+06	2.68e+02
	Mean	**2.10e-14**	8.10e+02	**7.28e-13**	**3.53e+11**	1.68e+08	8.14e+04	1.03e+02
	Std	1.99e-14	5.88e+01	3.40e-13	3.12e+10	1.04e+08	2.84e+05	8.70e+01
	Best	5.26e-15	4.26e+02	1.08e+00	5.02e+10	7.56e+07	1.47e+00	3.79e-02
	Median	1.85e-12	**5.00e+02**	1.84e+00	5.49e+11	**1.93e+08**	1.77e+00	**6.73e-01**
MA-ALSD	Worst	3.86e-05	6.50e+02	1.64e+01	6.67e+11	2.59e+08	2.26e+00	4.69e+01
	Mean	3.01e-06	**5.06e+02**	2.50e+00	5.01e+11	**1.82e+08**	**1.79e+00**	**8.76e+00**
	Std	8.81e-06	4.80e+01	2.98e+00	1.71e+11	5.85e+07	1.76e-01	1.51e+01
		F8	F9	F10	F11	F12	F13	F14
	Best	1.54e+06	1.19e+07	1.81e+03	2.74e+01	2.65e-06	3.86e+02	2.79e+07
	Median	**3.43e+06**	**1.40e+07**	**2.07e+03**	**3.75e+01**	3.50e-06	1.07e+03	**3.09e+07**
MA-SW-Chains	Worst	1.80e+08	1.62e+07	2.28e+03	5.11e+01	4.98e-06	2.92e+03	3.67e+07
	Mean	**1.41e+07**	**1.41e+07**	**2.07e+03**	**3.80e+01**	3.62e-06	1.25e+03	**3.11e+07**
	Std	3.68e+07	1.15e+06	1.44e+02	7.35e+00	5.92e-07	5.72e+02	1.93e+06
	Best	3.23e+00	3.59e+07	4.35e+03	2.18e+02	2.95e-07	2.96e+02	4.54e+07
	Median	2.80e+07	5.56e+07	6.22e+03	2.18e+02	**1.73e-06**	**8.30e+02**	5.72e+07
MA-ALSD	Worst	9.36e+07	6.80e+07	1.19e+04	2.19e+02	3.01e-06	1.96e+03	6.71e+07
	Mean	3.19e+07	5.49e+07	6.53e+03	2.18e+02	**1.71e-06**	**8.47e+02**	5.68e+07
	Std	2.57e+07	6.78e+06	1.74e+03	1.92e-01	6.13e-07	3.93e+02	5.84e+06
		F15	F16	F17	F18	F19	F20	
	Best	2.56e+03	8.51e+01	1.04e+00	7.83e+02	2.49e+05	9.25e+02	
	Median	**2.72e+03**	**9.44e+01**	1.26e+00	**1.19e+03**	2.85e+05	**1.06e+03**	
MA-SW-Chains	Worst	2.96e+03	1.24e+02	1.63e+00	2.55e+03	3.32e+05	1.21e+03	
	Mean	**2.74e+03**	**9.98e+01**	1.24e+00	**1.30e+03**	2.85e+05	**1.07e+03**	
	Std	1.22e+02	1.40e+00	1.25e-01	4.36e+02	1.78e+04	7.29e+01	
	Best	6.81e+03	3.96e+02	5.29e-02	1.33e+03	1.06e+05	1.00e+03	
	Median	7.56e+03	3.97e+02	**7.56e-02**	1.61e+04	**1.27e+05**	**1.07e+03**	
MA-ALSD	Worst	1.49e+04	3.97e+02	9.93e-02	5.60e+04	1.57e+05	1.68e+03	
	Mean	8.92e+03	3.97e+02	**7.62e-02**	1.86e+04	**1.26e+05**	**1.12e+03**	
	Std	2.75e+03	2.54e-01	1.26e-02	1.39e+04	1.08e+04	1.61e+02	

TABLE II.
EXPERIMENTAL RESULTS WITH MA-ALSD

		F1	F2	F3	F4	F5	F6	F7
FEs=1.2e5	Best	4.28e+08	7.21e+03	1.66e+01	1.36e+12	1.61e+08	3.89e+01	3.55e+06
	Median	5.43e+08	7.79e+03	1.97e+01	2.83e+12	4.60e+08	1.96e+07	4.99e+06
	Worst	8.42e+08	8.41e+03	1.99e+01	6.81e+12	6.02e+08	1.98e+07	1.09e+07
	Mean	5.48e+08	7.78e+03	1.95e+01	3.03e+12	4.13e+08	1.39e+07	5.08e+06
	Std	9.82e+07	2.73e+02	1.01e+00	1.09e+12	1.54e+08	8.30e+06	1.34e+06
FEs=6.0e5	Best	8.81e+04	1.45e+03	5.84e+00	7.34e+11	1.02e+08	7.56e+00	2.12e+04
	Median	7.68e+07	1.64e+03	1.99e+01	1.03e+12	1.68e+08	1.19e+01	1.03e+05
	Worst	1.02e+08	1.89e+03	1.99e+01	1.53e+12	4.09e+08	2.86e+06	2.87e+06
	Mean	4.80e+07	1.66e+03	1.68e+01	1.05e+12	1.94e+08	1.23e+05	4.35e+05
	Std	4.34e+07	9.79e+01	5.50e+00	1.73e+11	8.53e+07	5.71e+05	7.12e+05
FEs=3.0e6	Best	5.26e-15	4.26e+02	1.08e+00	5.02e+10	7.56e+07	1.47e+00	3.79e-02
	Median	1.85e-12	5.00e+02	1.84e+00	5.49e+11	1.93e+08	1.77e+00	6.73e-01
	Worst	3.86e-05	6.50e+02	1.64e+01	6.67e+11	2.59e+08	2.26e+00	4.69e+01
	Mean	3.01e-06	5.06e+02	2.50e+00	5.01e+11	1.82e+08	1.79e+00	8.76e+00
	Std	8.81e-06	4.80e+01	2.98e+00	1.71e+11	5.85e+07	1.76e-01	1.51e+01
		F8	F9	F10	F11	F12	F13	F14
FEs=1.2e5	Best	4.32e+07	1.04e+09	1.35e+04	2.19e+02	1.62e+05	1.14e+06	1.35e+09
	Median	4.74e+07	1.42e+09	1.47e+04	2.20e+02	2.16e+05	1.66e+06	1.60e+09
	Worst	6.84e+09	1.63e+09	1.52e+04	2.20e+02	2.68e+05	6.70e+06	2.04e+09
	Mean	4.73e+08	1.36e+09	1.46e+04	2.20e+02	2.18e+05	2.14e+06	1.61e+09
	Std	1.37e+09	1.49e+08	4.62e+02	2.24e-01	2.60e+04	1.19e+06	1.45e+08
FEs=6.0e5	Best	1.71e+07	2.21e+08	5.73e+03	2.18e+02	6.13e+02	3.67e+03	2.98e+08
	Median	4.43e+07	3.04e+08	1.43e+04	2.18e+02	8.25e+02	1.18e+04	3.25e+08
	Worst	7.77e+08	3.71e+08	1.50e+04	2.19e+02	1.14e+03	3.14e+06	3.96e+08
	Mean	1.28e+08	3.00e+08	1.25e+04	2.18e+02	8.40e+02	2.98e+05	3.33e+08
	Std	1.92e+08	3.67e+07	3.50e+03	2.72e-01	1.44e+02	6.98e+05	2.89e+07
FEs=3.0e6	Best	3.23e+00	3.59e+07	4.35e+03	2.18e+02	2.95e-07	2.96e+02	4.54e+07
	Median	2.80e+07	5.56e+07	6.22e+03	2.18e+02	1.73e-06	8.30e+02	5.72e+07
	Worst	9.36e+07	6.80e+07	1.19e+04	2.19e+02	3.01e-06	1.96e+03	6.71e+07
	Mean	3.19e+07	5.49e+07	6.53e+03	2.18e+02	1.71e-06	8.47e+02	5.68e+07
	Std	2.57e+07	6.78e+06	1.74e+03	1.92e-01	6.13e-07	3.93e+02	5.84e+06
		F15	F16	F17	F18	F19	F20	
FEs=1.2e5	Best	1.40e+04	3.97e+02	9.03e+05	1.98e+05	3.48e+06	2.47e+03	
	Median	1.48e+04	3.97e+02	1.10e+06	8.21e+05	4.12e+06	5.00e+03	
	Worst	1.52e+04	3.97e+02	1.27e+06	2.08e+06	5.29e+06	6.36e+04	
	Mean	1.48e+04	3.97e+02	1.11e+06	9.39e+05	4.12e+06	8.27e+03	
	Std	2.65e+02	2.27e-01	1.14e+05	5.01e+05	4.44e+05	1.19e+04	
FEs=6.0e5	Best	1.40e+04	3.96e+02	3.21e+04	1.12e+04	1.21e+06	1.03e+03	
	Median	1.47e+04	3.97e+02	4.04e+04	4.68e+04	1.43e+06	1.30e+03	
	Worst	1.55e+04	3.97e+02	4.93e+04	7.88e+04	1.96e+06	4.94e+03	
	Mean	1.47e+04	3.97e+02	4.00e+04	4.41e+04	1.47e+06	1.74e+03	
	Std	3.63e+02	2.73e-01	4.53e+03	1.73e+04	1.69e+05	1.08e+03	
FEs=3.0e6	Best	6.81e+03	3.96e+02	5.29e-02	1.33e+03	1.06e+05	1.00e+03	
	Median	7.56e+03	3.97e+02	7.56e-02	1.61e+04	1.27e+05	1.07e+03	
	Worst	1.49e+04	3.97e+02	9.93e-02	5.60e+04	1.57e+05	1.68e+03	
	Mean	8.92e+03	3.97e+02	7.62e-02	1.86e+04	1.26e+05	1.12e+03	
	Std	2.75e+03	2.54e-01	1.26e-02	1.39e+04	1.08e+04	1.61e+02	

TABLE III.
COMPARISON WITH DECC-G AND MLCC, FEs=3.0e6

		F1	F2	F3	F4	F5	F6	F7
DECC-G	Best	1.63e-07	1.25e+03	1.20e+00	7.78e+12	1.50e+08	3.89e+06	4.26e+07
	Median	2.86e-07	1.31e+03	1.39e+00	1.51e+13	2.38e+08	4.80e+06	1.07e+08
	Worst	4.84e-07	1.40e+03	1.68e+00	2.65e+13	4.12e+08	7.73e+06	6.23e+08
	Mean	2.93e-07	1.31e+03	1.39e+00	1.70e+13	2.63e+08	4.96e+06	1.63e+08
	Std	8.62e-08	3.26e+01	9.73e-02	5.37e+12	8.44e+07	8.02e+05	1.37e+08
MLCC	Best	0.00e+00	1.73e-11	1.28e-13	4.27e+12	2.15e+08	5.85e+06	4.16e+04
	Median	**0.00e+00**	**6.43e-11**	**1.46e-13**	1.03e+13	3.92e+08	1.95e+07	5.15e+05
	Worst	3.83e-26	1.09e+01	1.86e-11	1.62e+13	4.87e+08	1.98e+07	2.78e+06
	Mean	**1.53e-27**	**5.57e-01**	**9.88e-13**	9.61e+12	3.84e+08	1.62e+07	6.89e+05
	Std	7.66e-27	2.21e+00	3.70e-12	3.43e+12	6.93e+07	4.97e+06	7.37e+05
MA-ALSD	Best	5.26e-15	4.26e+02	1.08e+00	5.02e+10	7.56e+07	1.47e+00	3.79e-02
	Median	1.85e-12	5.00e+02	1.84e+00	**5.49e+11**	**1.93e+08**	**1.77e+00**	**6.73e-01**
	Worst	3.86e-05	6.50e+02	1.64e+01	6.67e+11	2.59e+08	2.26e+00	4.69e+01
	Mean	3.01e-06	5.06e+02	2.50e+00	**5.01e+11**	**1.82e+08**	**1.79e+00**	**8.76e+00**
	Std	8.81e-06	4.80e+01	2.98e+00	1.71e+11	5.85e+07	1.76e-01	1.51e+01
		F8	F9	F10	F11	F12	F13	F14
DECC-G	Best	6.37e+06	2.66e+08	1.03e+04	2.06e+01	7.78e+04	1.78e+03	6.96e+08
	Median	6.70e+07	3.18e+08	1.07e+04	**2.33e+01**	8.87e+04	3.00e+03	8.07e+08
	Worst	9.22e+07	3.87e+08	1.17e+04	2.79e+01	1.07e+05	1.66e+04	9.06e+08
	Mean	6.44e+07	3.21e+08	1.06e+04	**2.34e+01**	8.93e+04	5.12e+03	8.08e+08
	Std	2.89e+07	3.38e+07	2.95e+02	1.78e+00	6.87e+03	3.95e+03	6.07e+07
MLCC	Best	4.51e+04	8.96e+07	2.52e+03	1.96e+02	2.42e+04	1.01e+03	2.62e+08
	Median	4.67e+07	1.24e+08	**3.16e+03**	1.98e+02	3.47e+04	1.91e+03	3.16e+08
	Worst	9.06e+07	1.46e+08	5.90e+03	1.98e+02	4.25e+04	3.47e+03	3.77e+08
	Mean	4.38e+07	1.23e+08	**3.43e+03**	1.98e+02	3.49e+04	2.08e+03	3.16e+08
	Std	3.45e+07	1.33e+07	8.72e+02	6.98e-01	4.92e+03	7.27e+02	2.77e+07
MA-ALSD	Best	3.23e+00	3.59e+07	4.35e+03	2.18e+02	2.95e-07	2.96e+02	4.54e+07
	Median	**2.80e+07**	**5.56e+07**	**6.22e+03**	2.18e+02	**1.73e-06**	**8.30e+02**	**5.72e+07**
	Worst	9.36e+07	6.80e+07	1.19e+04	2.19e+02	3.01e-06	1.96e+03	6.71e+07
	Mean	**3.19e+07**	**5.49e+07**	**6.53e+03**	2.18e+02	**1.71e-06**	**8.47e+02**	**5.68e+07**
	Std	2.57e+07	6.78e+06	1.74e+03	1.92e-01	6.13e-07	3.93e+02	5.84e+06
		F15	F16	F17	F18	F19	F20	
DECC-G	Best	1.09e+04	5.97e+01	2.50e+05	5.61e+03	1.02e+06	3.59e+03	
	Median	1.18e+04	**7.51e+01**	2.89e+05	2.30e+04	1.11e+06	3.98e+03	
	Worst	1.39e+04	9.24e+01	3.26e+05	4.71e+04	1.20e+06	5.32e+03	
	Mean	1.22e+04	**7.66e+01**	2.87e+05	2.46e+04	1.11e+06	4.06e+03	
	Std	8.97e+02	8.14e+00	1.98e+04	1.05e+04	5.15e+04	3.66e+02	
MLCC	Best	5.30e+03	2.08e+02	1.38e+05	2.51e+03	1.21e+06	1.70e+03	
	Median	**6.89e+03**	3.95e+02	1.59e+05	**4.17e+03**	1.36e+06	2.04e+03	
	Worst	1.04e+04	3.97e+02	1.86e+05	1.62e+04	1.54e+06	2.34e+03	
	Mean	**7.11e+03**	3.76e+02	1.59e+05	**7.09e+03**	1.36e+06	2.05e+03	
	Std	1.34e+03	4.71e+01	1.43e+04	4.77e+03	7.35e+04	1.80e+02	
MA-ALSD	Best	6.81e+03	3.96e+02	5.29e-02	1.33e+03	1.06e+05	1.00e+03	
	Median	**7.56e+03**	3.97e+02	**7.56e-02**	1.61e+04	**1.27e+05**	**1.07e+03**	
	Worst	1.49e+04	3.97e+02	9.93e-02	5.60e+04	1.57e+05	1.68e+03	
	Mean	**8.92e+03**	3.97e+02	**7.62e-02**	1.86e+04	**1.26e+05**	**1.12e+03**	
	Std	2.75e+03	2.54e-01	1.26e-02	1.39e+04	1.08e+04	1.61e+02	