2014 IEEE Congress on Evolutionary Computation (CEC)
July 6-11, 2014, Beijing, China

DMOPSO: Dual Multi-Objective Particle Swarm Optimization

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***Abstract*—Since multi-objective optimization algorithms (MOEAs) have to find exponentially increasing number of nondominated solutions with the increasing number of objectives, it is necessary to discriminate more meaningful ones from the other nondominated solutions by additionally incorporating user preference into the algorithms. This paper proposes dual multi-objective particle swarm optimization (DMOSPO) by introducing secondary objectives of maximizing both user preference and diversity to the nondominated solutions obtained for primary objectives. The proposed DMOSPO can induce the balanced exploration of the particles in terms of user preference and diversity through the dual-stage of nondominated sorting such that it can generate preferable and diverse nondominated solutions. To demonstrate the effectiveness of the proposed DMOPSO, empirical comparisons with other state-of-the-art algorithms are carried out for benchmark functions. Experimental results show that DMOPSO is competitive with the other compared algorithms and properly reflects the user's preference in the optimization process while maintaining the diversity and solution quality.**

***Index Terms*—Multi-objective Evolutionary Algorithm, Multi-objective Particle Swarm Optimization, Dual-stage dominance check, Crowding distance, User preference**

I. Introduction

The problems in engineering usually require the process for optimizing some parameters with respect to multiple objectives, some of which conflict with each other. The multi-objective optimization is to obtain a set of Pareto-optimal solutions because there is no unique solution for the trade-offs between the objectives. Since it is impossible to calculate Pareto-optimal front explicitly, multi-objective evolutionary algorithms (MOEAs) have been developed to find the solutions as closely as possible to the Pareto-optimal front. Along with this issue of proximity to Pareto-optimal front, diversity preservation is another issue to maintain Pareto-optimal solutions as diverse a distribution as possible in multi-objective optimization. Through MOEAs, various nondominated solutions can be obtained, which are proximate to Pareto-optimal front in objective space and none of which can be directly compared with each other [1]–[16].

However, with the increasing number of objectives, the percentage of nondominated solutions increases. In fact, over 90% of the possible solutions are nondominated solutions if the number of the objectives is over 10 [17]. In other words, just finding nondominated solutions cannot be meaningful. Thus, recently, the approaches of incorporating user preference into the optimization process have been studied for discriminating the meaningful nondominated solutions from the others [18]–[20]. In these approaches, with the preferences of users, a global evaluation value of each nondominated solution is figured out and relatively more meaningful solutions are selected out in each iteration of the algorithm. In addition, since the solutions may be converged prematurely if the preferences are considered too much even though the preferences are well-defined, the solution diversity is also controlled heuristically. However, a clear criterion for the balance between user preference and diversity has not been studied yet.

In this paper, we propose dual multi-objective particle swarm optimization (DMOPSO) with the concept of dual-stage optimization by introducing secondary objectives in addition to primary objectives. In the proposed DMOPSO, each particle chooses its global best attractor (GBest) randomly from the GBest candidate pool that is extracted from the archive through the dual-stage of nondominated sorting with respect to primary and secondary objectives. Primary objectives are the given objectives of the problem and as secondary objectives, global evaluation value (GEval) with respect to user preference and crowding distance (CD) are employed. Through the dual-stage of nondominated sorting, more preferable as well as less crowded particles are selected as the GBests and the balanced exploration of the particles is induced in terms of user preference and diversity. To demonstrate the effectiveness of the proposed DMOPSO, it is empirically compared with NSGA-II, MQEA, MOPSO, and MOPSO-PS for DTLZ functions which are test problems for multi-objective optimization.

The remainder of this paper is organized as follows. Section II explains the proposed DMOPSO. In Section III, experimental results are discussed. Finally, conclusion follows in Section IV.

II. Dual Multi-Objective Particle Swarm Optimization

In this section, canonical particle swarm optimization (PSO) is briefly reviewed and dual multi-objective particle swarm optimization (DMOPSO) is proposed.

A. Brief summary of canonical PSO

A pseudo code of canonical PSO is presented in Algorithm 1 and each step is described in the following:

1. Initialize swarm

At first, the velocity and position of particles in a population are randomly initialized on D-dimensional space. A population is a set of N particles which have their own velocity and

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This work was supported by the Technology Innovation Program, 10045252, Development of robot task intelligence technology, funded by the Ministry of Trade, industry & Energy (MOTIE, Korea).

Algorithm 1 Particle Swarm Optimization

1) Initialize swarm
2) Update swarm

```
for each particle do
    Evaluate the objective function
end for
for each particle do
    Update ^p x_k^t and ^g x_k^t
    Update velocity and position
end for
```

3) Repeat

Algorithm 2 Global Evaluation

- M: The number of objectives
- N: The number of particles
- $O = \{o_1, o_2, \cdots, o_M\}$: A set of objectives
- $P(O)$: A power set of O
- $h_i(\mathbf{x}_k)$: Partial evaluation value of $\mathbf{x}_k$ over o_i
- $e(\mathbf{x}_k)$: Global evaluation value of the k-th particle

1) Fuzzy measure identification

```
for each A ∈ P(O) do
    Calculate fuzzy measure g(A)
end for
```

2) Global evaluation of particles

```
for k = 1, 2, ..., N do
    for i = 1, 2, ..., M do
        Calculate h_i(x_k)
    end for
end for
for k = 1, 2, ..., N do
    e(x_k) = ∫ g ∘ h (Choquet fuzzy integral)
end for
```

position. The velocity $\mathbf{v}_k$ and position $\mathbf{x}_k$ of the k-th particle, p_k, $k = 1, 2, \ldots, N$, are the D-dimensional vectors as follows:

$$\mathbf{v}_k \in \mathbb{R}^D, \mathbf{x}_k \in \mathbb{R}^D.$$

The personal best position and the global best position are also initialized.

2. Update swarm

After initialization process, objective function value of each particle at generation t, $f(\mathbf{x}_k^t)$, $k = 1, 2, \ldots, N$, is evaluated, where

$$f(\mathbf{x}_k^t) : \mathbb{R}^D \rightarrow \mathbb{R}.$$

After function evaluation, the personal best position of each particle, ${}^p\mathbf{x}_k^t$, $k = 1, 2, \ldots, N$, is updated. The personal best position of p_k is defined as the position where the fitness value is the largest all over the past positions of p_k. After that, the global best position of each particle, ${}^g\mathbf{x}_k^t$, $k = 1, 2, \ldots, N$, is updated. The global best position of p_k is defined as the best position of the personal best positions of p_k's neighbors. Finally, the velocity and position of each particle are updated as follows:

$$\begin{cases} \mathbf{v}_k^t = & w \cdot \mathbf{v}_k^{t-1} + c \cdot \{\phi_k^{1,t}({}^p\mathbf{x}_k^{t-1} - \mathbf{x}_k^{t-1}) \\ & +\phi_k^{2,t}({}^g\mathbf{x}_k^{t-1} - \mathbf{x}_k^{t-1})\} \\ \mathbf{x}_k^t = & \mathbf{x}_k^{t-1} + \mathbf{v}_k^t \end{cases} \quad (1)$$

where w and c are constants and $\phi_k^{1,t}$ and $\phi_k^{2,t}$ are random real values uniformly distributed in $[0, 1]$. $\mathbf{v}_k^t$ and $\mathbf{x}_k^t$ represent the velocity and position of the k-th particle at generation t, respectively. New random values are generated for each particle and generation.

3. Repeat

Repeat Step 2 until a termination condition is met.

B. DMOPSO

The word 'dual' implies that there are two stages of nondominated sorting in DMOPSO. In the first stage, the dominated particles with respect to primary objectives are culled out where primary objectives are the given objectives of the problem. This is called primary objectives-based nondominated sorting. And then, one more nondominated sorting is conducted with respect to secondary objectives, called secondary objectives-based nondominated sorting. Global evaluation value (GEval) and crowding distance (CD) are adopted as secondary objectives to induce the balanced exploration of the particles in terms of user preference and diversity. To reflect the user's preference in each iteration of the optimization process, the GEval of every particle is used, which represents the quality of the particle according to the user's preference [18], [20]. The user's preference or the degree of consideration for each objective is represented by using the fuzzy measure. The GEval of a particle is calculated by the fuzzy integral that integrates its partial evaluation valuee (PEvals) with respect to the degree of consideration. In addition, for diversity, the archive is sorted by CD and the archive members with relatively long CDs are selected as attractors or elites. The CD of a point is an estimate of the size of the largest cuboid enclosing the point without including any other points [21]. Algorithms 2 and 3 show the overall procedures for calculating GEval and CD, respectively.

Fig. 1 shows the flow diagram of DMOPSO, where $\mathbf{A}_t$ is the external archive, $\mathbf{P}_t$ is the population, $\mathbf{G}_t$ is the GBest candidate pool, ${}^p\mathbf{x}_t^k$ is the personal best attractor (PBest) position of the k-th particle, and ${}^g\mathbf{x}_t^k$ is the GBest position of the k-th particle at generation t. In DMOPSO, each particle chooses its global best attractor (GBest) randomly from the GBest candidate pool, extracted from the archive after the secondary objectives-based nondominated sorting. The secondary objectives-based nondominated sorting is done by the following three steps: i) calculate GEval and CD of every archive member. ii) set GEval and CD as secondary objective variables, and perform nondominated sorting for all the archive members with respect to the secondary objectives. iii) gather the particles in the first tier of the sorted archive, i.e. hyper-nondominated particles, to the GBest candidate

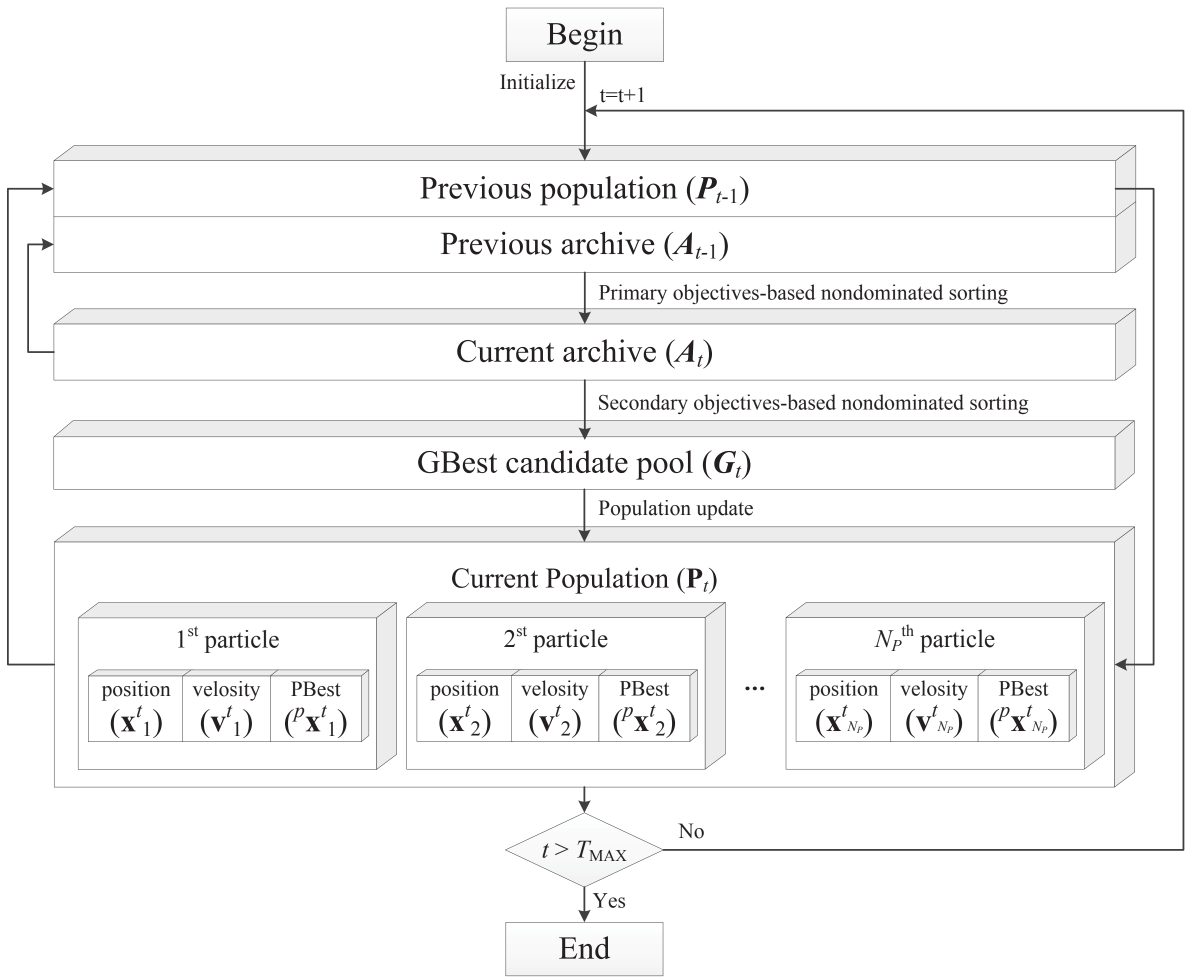

Fig. 1: The flow diagram of DMOPSO.

pool. The overall procedure of DMOPSO is summarized as Algorithm 4 and each step of the algorithm is described in the following.

1. Initialize $\mathbf{P}_0$ and $\mathbf{A}_0$

A population is a set of N particles, which have their own position and velocity. The position $\mathbf{x}^k$ and velocity $\mathbf{v}^k$ of the k-th particle p^k, $k = 1, 2, \ldots, N$, are the D-dimensional vectors as follows:

$$\mathbf{v}^k \in \mathbb{R}^D, \mathbf{x}^k \in \mathbb{R}^D.$$

For every particle in the population, the position is randomly initialized in a D-dimensional space and the velocity is initially set to $\mathbf{0}$. The M-objective function of each particle $f(\mathbf{x}_t^k)$ is evaluated, which is defined as

$$f(\mathbf{x}_t^k) : \mathbb{R}^D \rightarrow \mathbb{R}^M$$

and the PBest position of each particle is set to be the position of itself. The external archive is also initialized as a null set.

2. Update $\mathbf{A}_t$

The current archive $\mathbf{A}_t$ is updated by uniting the previous archive $\mathbf{A}_{t-1}$ and the previous population $\mathbf{P}_{t-1}$ and conducting primary objectives-based nondominated sorting. For each particle in $\mathbf{A}_t$, the GEval and CD are calculated.

3. Extract $\mathbf{G}_t$

At first, the particles in $\mathbf{A}_t$ are sorted by secondary objectives-based nondominated sorting for GEval and CD. And then, the particles of the first tier are stored into $\mathbf{G}_t$. Since the particles in $\mathbf{G}_t$ are mutually nondominating and no particles in $\mathbf{G}_t$ are dominated by $\mathbf{x}_{t-1}^k$, every particle in $\mathbf{G}_t$ can be a candidate for the GBest of each particle in $\mathbf{P}_{t-1}$. Moreover, the particles in $\mathbf{G}_t$ are nondominated with respect to GEvals and CDs and this means that they are less crowded and more preferable than the others. Thus, by choosing ${}^g\mathbf{x}_t^k$ from $\mathbf{G}_t$, the particles are guided by the user's preference while maintaining diversity.

4. Update particles

For each particle, ${}^g\mathbf{x}_t^k$ is randomly chosen from $\mathbf{G}_t$. The velocity and position of each particle are updated by (1). After that, $f(\mathbf{x}_t^k)$ for every particle in the updated $\mathbf{P}_{t-1}$ is evaluated. Finally, ${}^p\mathbf{x}_t^k$ is updated. ${}^p\mathbf{x}_t^k = \mathbf{x}_t^k$ if $\mathbf{x}_t^k$ weakly dominates ${}^p\mathbf{x}_{t-1}^k$ or they are mutually non-dominating. Otherwise, ${}^p\mathbf{x}_t^k = {}^p\mathbf{x}_{t-1}^k$.

Algorithm 3 Crowding distance

- M: The number of objectives
- N: The number of particles
- $CD(p_k)$: CD of the k-th particle
- $f_i(\mathbf{x}_k)$: The i-th objective function value of $\mathbf{x}_k$

1) Initialization

for $k = 1, 2, \ldots, N$ **do**
  $CD(\mathbf{x}_k) = 0$
end for

2) Computation of the CD of each particle

for $i = 1, 2, \ldots, M$ **do**
  for $k = 1, 2, \ldots, N$ **do**
    Calculate $f_i(\mathbf{x}_k)$
  end for
  Sort the particles according to $f_i(\mathbf{x}_k)$
  $CD(p_1) = CD(p_1) + 100$ (large value)
  $CD(p_N) = CD(p_N) + 100$ (large value)
  for $k = 2, 3, \ldots, N-1$ **do**
    $CD(p_k) = CD(p_k) + (f_i(\mathbf{x}_{k+1}) - f_i(\mathbf{x}_{k-1}))$
  end for
end for

5. Go back to Step 2 and repeat

Go back to Step 2 and repeat until a termination condition is met.

III. Experimental Results

At first, the proposed DMOPSO was compared with NSGA-II [4], MQEA [5], [6], MOPSO [13], and MOPSO-PS [20] through DTLZ functions [22] to demonstrate its effectiveness. Note that for fair comparison, the preference degrees in both DMOPSO and MOPSO-PS were set to $f_1 : f_2 : f_3 : f_4 : f_5 = 1 : 1 : 1 : 1 : 1$. In addition, to see how the preference degrees effect on the result, DMOPSO was repeatedly tested by varying the preference degree configurations.

The parameters used in the analysis are given in Table I. Every DTLZ function is a minimization problem and the number of objectives was set to 5. The number of variables of each DTLZ function was set to 9 for DTLZ1, 16 for DTLZ2 - DTLZ6, and 26 for the DTLZ7 function.

Two performance metrics, the size of dominated space and diversity, were employed to evaluate the performance of the algorithms. The size of dominated space, $\mathcal{S}$ is defined by the hypervolume of nondominated solutions [1]. The reference point to calculate $\mathcal{S}$ was set to (10.0, 10.0, 10.0, 10.0, 10.0). The quality of the obtained solution set is proportional to the hypervolume. Diversity, $\mathcal{D}$ is for evaluating the spread of nondominated solutions [23], which is defined as follows:

$$\mathcal{D} = \frac{\sum_{k=1}^{n}(f_k^{(max)} - f_k^{(min)})}{\sqrt{\frac{1}{|N_0|}\sum_{i=1}^{|N_0|}(d_i - \bar{d})^2}} \qquad (2)$$

where N_0 is the set of nondominated solutions, d_i is the minimal distance between the i-th solution and the nearest neighbor, and $\bar{d}$ is the mean value of all d_i. $f_k^{(max)}$ and $f_k^{(min)}$ represent the maximum and minimum objective function values of the k-th objective, respectively.

Algorithm 4 DMOPSO

- $\mathbf{P}_t$: The population at iteration t
- $\mathbf{A}_t$: The external archive at iteration t
- $\mathbf{G}_t$: GBest candidate pool at iteration t
- $N_P/N_A/N_G$: The number of particles in $\mathbf{P}_t/\mathbf{A}_t/\mathbf{G}_t$
- D: The dimension of the search space
- $rand(L, U)$: Random integer value between L and U
- $\mathbf{v}_t^k$: The velocity of the k-th particle at iteration t
- $\mathbf{x}_t^k$: The position of the k-th particle at iteration t
- $\mathbf{f}(\mathbf{x}_t^k)$: The objective function of $\mathbf{x}_t^k$
- ${}^{g}\mathbf{x}_t^k$: GBest position of the k-th particle at iteration t
- ${}^{p}\mathbf{x}_t^k$: PBest position of the k-th particle at iteration t

1) Initialize $\mathbf{P}_0$ and $\mathbf{A}_0$

$t = 0$
for $k = 1, 2, \ldots, N_P$ **do**
  $\mathbf{x}_t^k$ = random vector $\in \mathbb{R}^D$
  $\mathbf{v}_t^k = \mathbf{0}$
  Evaluate $f(\mathbf{x}_t^k)$
  ${}^{p}\mathbf{x}_t^k = \mathbf{x}_t^k$
end for
$\mathbf{A}_t$ = Null set

2) Update $\mathbf{A}_t$

$t = t + 1$
$\mathbf{A}_t = \mathbf{A}_{t-1} \cup \mathbf{P}_{t-1}$
Cull the dominated particles of $\mathbf{A}_t$
for $k = 1, 2, \ldots, N_A$ **do**
  Evaluate the GEval of each particle in $\mathbf{A}_t$
  Calculate the CD of each particle in $\mathbf{A}_t$
end for

3) Extract $\mathbf{G}_t$

Perform the secondary objectives-based nondominated sorting
Take the first tier as $\mathbf{G}_t$

4) Update particles

for $k = 1, 2, \ldots, N_P$ **do**
  $r = rand(0, N_G)$
  ${}^{g}\mathbf{x}_t^k$ = the position of the r-th particle in $\mathbf{G}_t$
  Update $\mathbf{v}_t^k$ and $\mathbf{x}_t^k$ by (1)
  Evaluate $f(\mathbf{x}_t^k)$
  Update ${}^{p}\mathbf{x}_t^k$
end for

5) Go back to Step 2 and repeat

To verify that the comparison result is statistically significant, statistical hypothesis testing was used [24]. For every pair of comparison, X_1 and X_2, a null hypotheses, $\mathcal{H}_0$ was defined as $\bar{X}_1 - \bar{X}_2 = 0$ which means that there is no significant difference between X_1 and X_2, where $\bar{X}$ is the mean value of X. Through the 50 pairs of sample data, Welch's t-value and the corresponding p-value were calculated [25]. The null hypothesis was rejected and the corresponding

TABLE I: The parameter settings of the algorithms

Algorithms	Parameters	Values
NSGA-II	Population size (N)	100
	No. of generations	3000
	Mutation probability (p_m)	0.1
MQEA	Global population size ($n \cdot s$)	100
	No. of generations	3000
	Sub-population size (n)	25
	No. of sub-populations (s)	4
	No. of multiple observations	10
	The rotation angle ($\Delta\theta$)	0.23π
MOPSO, MOPSO-PS, DMOPSO	Population size (N)	100
	No. of generations	3000
	Max. archive size	500
	Inertia weight (w)	$1/(2 \cdot \log 2)$
	Cognitive/Social parameter (c)	$0.5 + \log 2$

alternative hypothesis, $\mathcal{H}_1$ is claimed if the calculated p-value is below the significance level of 0.05. Note that in this paper, the normality of every sample data set was verified through Jarque-Bera test [26]. Table II and III show the pairwise hypothesis testing results on the both $\mathcal{S}$ and $\mathcal{D}$, respectively. As shown in Table II, in $\mathcal{S}$, DMOPSO outperformed the other algorithms except MOPSO-PS. $\mathcal{S}$ of DMOPSO was similar or slightly better than that of MOPSO-PS. In case of $\mathcal{D}$, DMOPSO showed better $\mathcal{D}$ than MQEA, MOPSO, and MOPSO-PS. And, there was no choice between DMOPSO and NSGA-II; for DTLZ1, DTLZ3, and DTLZ6, $\mathcal{D}$ of DMOPSO was significantly larger than that of NSGA-II while the opposite was true for DTLZ2, DTLZ5, and DTLZ7. The canonical MOEAs including NSGA-II, MQEA, and NSGA-II use dominance check for proximity to Pareto optimal front and crowding distance control for diversity. However, as mentioned above, if the percentage of nondominated solutions increases with the increasing number of objectives, it is hard to improve the proximity with dominance check. Thus, DMOPSO and MOPSO-PS showed better $\mathcal{S}$s than the others due to the additional exploitation movement through incorporating user preference into them. Moreover, DMOPSO showed better $\mathcal{D}$ than MOPSO-PS because the balance between diversity and user preference was efficiently managed by the dual-stage optimization in DMOPSO whereas it was handled heuristically in MOPSO-PS.

To see the effect of preference degrees on DMOPSO, it was repeatedly tested with various preference degree configurations. In addition to the preference degree configuration of 1:1:1:1:1 for the five objectives, 2:1:2:1:2 and 10:1:10:1:10 were used. Since f_1, f_3, and f_5 were more considered at every generation, DMOPSO could obtain the optimized solutions that were more focused on those more preferred objectives. Table IV shows the average objective function values over 50 runs. As the table shows, the average values of f_1, f_3, and f_5 of DMOPSO decrease with the increasing preference degrees. This means that the preference degrees had a pronounced effect on the final solutions. However, the effect was not highly sensitive to the magnitude of the preference degrees. The hypothesis testing result also indicates that there was no significant difference among the metrics of the three configurations as shown in Table V. It is a distinctive advantage of DMOPSO that its $\mathcal{S}$ and $\mathcal{D}$ were maintained, even though the preferred objectives were considered more. Note that when the conventional utility function method, like the weighted sum method, is used for considering the user's preference, the weights need to be set very carefully in order to obtain the solutions optimized not only for preferred objectives but also for the other objectives to a certain level. On the other hand, DMOPSO can solve this problem more efficiently by employing the fuzzy measure representing the interactions between the objectives and the user's preference for them.

TABLE II: The hypothesis testing on $\mathcal{S}$ of various algorithms

	$\mathcal{H}_0 : \bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	2.754 (0.041)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} > 0$
DTLZ2	3.414 (0.001)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} > 0$
DTLZ3	4.039 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} > 0$
DTLZ4	3.476 (0.001)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} > 0$
DTLZ5	3.138 (0.003)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} > 0$
DTLZ6	6.086 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} > 0$
DTLZ7	3.469 (0.001)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{NSGA-II}} > 0$

	$\mathcal{H}_0 : \bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	30.026 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} > 0$
DTLZ2	3.592 (0.001)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} > 0$
DTLZ3	3.212 (0.002)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} > 0$
DTLZ4	6.048 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} > 0$
DTLZ5	7.490 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} > 0$
DTLZ6	4.847 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} > 0$
DTLZ7	9.028 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MQEA}} > 0$

	$\mathcal{H}_0 : \bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	3.932 (0.001)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO}} > 0$
DTLZ2	7.085 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO}} > 0$
DTLZ3	-0.252 (0.802)	NO	N/A
DTLZ4	5.212 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO}} > 0$
DTLZ5	5.319 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO}} > 0$
DTLZ6	3.629 (0.001)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO}} > 0$
DTLZ7	3.223 (0.002)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO}} > 0$

	$\mathcal{H}_0 : \bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO-PS}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	1.688 (0.098)	NO	N/A
DTLZ2	1.153 (0.254)	NO	N/A
DTLZ3	-0.980 (0.332)	NO	N/A
DTLZ4	3.959 (0.000)	YES	$\bar{\mathcal{S}}_{\text{DMOPSO}} - \bar{\mathcal{S}}_{\text{MOPSO-PS}} > 0$
DTLZ5	0.962 (0.340)	NO	N/A
DTLZ6	0.840 (0.405)	NO	N/A
DTLZ7	-1.584 (0.120)	NO	N/A

IV. Conclusions

In this paper, dual multi-objective particle swarm optimization (DMOPSO) was proposed by introducing secondary objectives of maximizing user preference and diversity to the nondominated solutions obtained for primary objectives. The most important advantage of the DMOPSO was that both the user's preference and the crowding distance could be incorporated into the optimization process through the

TABLE III: The hypothesis testing on $\mathcal{D}$ of various algorithms

	$\mathcal{H}_0 : \overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{NSGA-II}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	30.818 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{NSGA-II}} > 0$
DTLZ2	-18.612 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{NSGA-II}} < 0$
DTLZ3	25.012 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{NSGA-II}} > 0$
DTLZ4	0.662 (0.511)	NO	N/A
DTLZ5	-10.041 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{NSGA-II}} < 0$
DTLZ6	34.610 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{NSGA-II}} > 0$
DTLZ7	-48.817 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{NSGA-II}} < 0$

	$\mathcal{H}_0 : \overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MQEA}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	44.586 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MQEA}} > 0$
DTLZ2	10.418 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MQEA}} > 0$
DTLZ3	22.815 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MQEA}} > 0$
DTLZ4	3.290 (0.002)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MQEA}} > 0$
DTLZ5	1.284 (0.205)	NO	N/A
DTLZ6	34.127 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MQEA}} > 0$
DTLZ7	3.565 (0.001)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MQEA}} > 0$

	$\mathcal{H}_0 : \overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	-1.304 (0.198)	NO	N/A
DTLZ2	-0.627 (0.534)	NO	N/A
DTLZ3	-0.459 (0.648)	NO	N/A
DTLZ4	-0.356 (0.723)	NO	N/A
DTLZ5	3.841 (0.001)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO}} > 0$
DTLZ6	6.152 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO}} > 0$
DTLZ7	17.126 (0.000)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO}} > 0$

	$\mathcal{H}_0 : \overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO-PS}} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	1.071 (0.289)	NO	N/A
DTLZ2	2.733 (0.009)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO-PS}} > 0$
DTLZ3	0.775 (0.442)	NO	N/A
DTLZ4	0.601 (0.551)	NO	N/A
DTLZ5	0.793 (0.432)	NO	N/A
DTLZ6	2.218 (0.031)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO-PS}} > 0$
DTLZ7	2.156 (0.036)	YES	$\overline{\mathcal{D}}_{\text{DMOPSO}} - \overline{\mathcal{D}}_{\text{MOPSO-PS}} > 0$

TABLE IV: Average objective function values by DMOPSO with each preference degree

	Preference degree	f_1	f_2	f_3	f_4	f_5
DTLZ1	1:1:1:1:1	0.0173	0.0180	0.0200	0.0226	0.0225
	2:1:2:1:2	0.0161	0.0204	0.0180	0.0266	0.0202
	10:1:10:1:10	0.0158	0.0208	0.0177	0.0267	0.0202
DTLZ2	1:1:1:1:1	0.0570	0.0553	0.0605	0.0691	0.0701
	2:1:2:1:2	0.0479	0.0653	0.0557	0.0781	0.0663
	10:1:10:1:10	0.0475	0.0704	0.0544	0.0789	0.0639
DTLZ3	1:1:1:1:1	0.0641	0.0581	0.0697	0.0719	0.0794
	2:1:2:1:2	0.0635	0.0699	0.0658	0.0747	0.0791
	10:1:10:1:10	0.0572	0.0759	0.0623	0.0887	0.0769
DTLZ4	1:1:1:1:1	0.0853	0.0487	0.0475	0.0474	0.0457
	2:1:2:1:2	0.0799	0.0545	0.0449	0.0523	0.0428
	10:1:10:1:10	0.0778	0.0580	0.0439	0.0553	0.0402
DTLZ5	1:1:1:1:1	0.0609	0.0607	0.1310	0.1928	0.0775
	2:1:2:1:2	0.0616	0.0625	0.1320	0.1948	0.0744
	10:1:10:1:10	0.0607	0.0647	0.1306	0.2041	0.0727
DTLZ6	1:1:1:1:1	0.0593	0.0592	0.1515	0.2284	0.0823
	2:1:2:1:2	0.0583	0.0608	0.1490	0.2219	0.0775
	10:1:10:1:10	0.0592	0.0630	0.1518	0.2696	0.0779
DTLZ7	1:1:1:1:1	0.1142	0.1140	0.1134	0.1139	1.2746
	2:1:2:1:2	0.1107	0.1179	0.1108	0.1176	1.2701
	10:1:10:1:10	0.1075	0.1213	0.1069	0.1217	1.2676

TABLE V: The hypothesis testing on $\mathcal{S}$ and $\mathcal{D}$ of DMOPSO with various preference degrees

	$\mathcal{H}_0 : \overline{\mathcal{S}}_{1:1:1:1:1} - \overline{\mathcal{S}}_{2:1:2:1:2} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	0.197 (0.845)	NO	N/A
DTLZ2	0.602 (0.550)	NO	N/A
DTLZ3	-0.112 (0.911)	NO	N/A
DTLZ4	0.527 (0.601)	NO	N/A
DTLZ5	-0.264 (0.793)	NO	N/A
DTLZ6	0.899 (0.373)	NO	N/A
DTLZ7	0.795 (0.430)	NO	N/A

	$\mathcal{H}_0 : \overline{\mathcal{S}}_{1:1:1:1:1} - \overline{\mathcal{S}}_{10:1:10:1:10} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	0.987 (0.328)	NO	N/A
DTLZ2	0.737 (0.465)	NO	N/A
DTLZ3	-0.827 (0.412)	NO	N/A
DTLZ4	0.868 (0.390)	NO	N/A
DTLZ5	0.319 (0.751)	NO	N/A
DTLZ6	-0.802 (0.426)	NO	N/A
DTLZ7	-0.758 (0.452)	NO	N/A

	$\mathcal{H}_0 : \overline{\mathcal{D}}_{1:1:1:1:1} - \overline{\mathcal{D}}_{2:1:2:1:2} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	-0.553 (0.583)	NO	N/A
DTLZ2	-1.040 (0.303)	NO	N/A
DTLZ3	0.197 (0.845)	NO	N/A
DTLZ4	0.611 (0.544)	NO	N/A
DTLZ5	0.182 (0.856)	NO	N/A
DTLZ6	1.476 (0.146)	NO	N/A
DTLZ7	0.521 (0.605)	NO	N/A

	$\mathcal{H}_0 : \overline{\mathcal{D}}_{1:1:1:1:1} - \overline{\mathcal{D}}_{10:1:10:1:10} = 0$		
	t-value (p-value)	Reject	$\mathcal{H}_1$
DTLZ1	-0.989 (0.327)	NO	N/A
DTLZ2	-0.724 (0.473)	NO	N/A
DTLZ3	-0.628 (0.533)	NO	N/A
DTLZ4	1.007 (0.319)	NO	N/A
DTLZ5	-2.304 (0.026)	YES	$\overline{\mathcal{D}}_{1:1:1:1:1} - \overline{\mathcal{D}}_{10:1:10:1:10} < 0$
DTLZ6	0.477 (0.635)	NO	N/A
DTLZ7	-1.189 (0.240)	NO	N/A

the dual-stage of nondominated sorting. The effectiveness of DMOPSO was demonstrated by comparison with NSGA-II, MQEA, MOPSO, and MOPSO-PS for the DTLZ functions. The comparison results indicated that DMOPSO is competitive with the other algorithms and properly reflects user preference in the optimization process while maintaining the diversity and solution quality.

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