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An Improved Ant Colony Algorithm for Winner Determination in Multi-Attribute Combinatorial Reverse Auction

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***Abstract*—This paper considers the problem of one buyer procuring multi-items from multiple potential suppliers in the electronic reverse auction, where each supplier can bid on combinations of items. From the perspective of the buyer, by considering multi-attributes of each item, a winner determination problem (WDP) of multi-items single-unit combinatorial reverse auctions was described and a bi-objective programming model was established. According to the characteristics of the model, an equivalent single-objective programming model was obtained. As the problem is NP-hard, an improved ant colony (IAC) algorithm considering the dynamic transition strategy and the Max-Min pheromone strategy is proposed for the problem. Experimental results show the effectiveness of the improved algorithm.**

***Keywords*—reverse auction; winner determination problem; ant colony algorithm; dynamic transition strategy; Max-Min pheromone strategy**

I. INTRODUCTION

Combinatorial auctions are auctions that a bidder can place a single bid on a set of different items. With combinatorial auctions, bidders can avoid the risk of only obtaining a subset of items that is not worth as much as a complete set [1]. Therefore, combinatorial auctions leading to more efficient allocations than traditional multi-item auctions have been widely suggested to allocate items that are substitutable or complementary. However, price-only auctions tend to lead to dire consequences for the buyer in a procurement setting [2], [3]. Hence multi-attribute combinatorial reverse auction (MACRA) is becoming prevalent in procurement and has successfully been used in industrial activities for both services and goods. As an important decision problem in the MACRA, the winner determination problem (WDP) forms the focus of this paper.

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Sandholm [1] introduced the existing algorithms of WDP in combinatorial auctions, including enumeration algorithms, dynamic programming approaches, polynomial-time approximation algorithms, etc., and proposed a novel search algorithm based on branch and bound for the problem. However, the traditional branch and bound algorithm only solves a small scale WDP in the acceptable computation time. To improve the efficiency of the algorithm, a sophisticated optimal algorithm called combinatorial auction branch on bids (CABOB) [4] was proposed to solve the basic WDP in combinatorial auctions. CABOB is a depth-first branch-and-bound tree search algorithm that branches on bids. However, the integer programming formulation of the WDP was not tight. After obtaining several properties of the traditional formulation of the WDP, a set packing formulation of the problem was presented and a branch-and-cut algorithm was implemented [5]. After that, a particular combinatorial auction setting where the valuation functions of the bidders are known to be complement free was considered and several approximation algorithms were proposed for the WDP [6]. However, the former studies only provided decision support for the auctioneer. Hsieh [7] considered a WDP with a decision support tool to assist bidders' decisions, and designed a heuristic algorithm to find a feasible solution in a multi-round combinatorial reverse auction. However, only price is considered in such settings. It is also important for buyers to consider other quantitative or qualitative attributes, such as delivery time, quality grade of the product, etc., to form long-term relationships with potential suppliers in a procurement auction [8], [9].

Multi-attribute reverse auction (MARA) extends the traditional reverse auction to allow negotiation over price and non-price attributes and is becoming prevalent in procurement. The mathematical study of WDP of MARA started in [2], where a linear programming model is formulated to describe the problem. Then, Cheng [9] formulated a WDP model in a sealed-bid MARA setting and solved the problem with the basic "Technique for Order Preference by Similarity to Ideal Solution" (TOPSIS) method. However, only quantitative attributes were considered in his work. Both of quantitative and qualitative attributes are combined with the WDP in a MARA setting in [10], using a fuzzy logic and interval arithmetic-based TOPSIS (FTOPSIS) method. However, only single item is considered in such settings.

We consider a revised WDP in this paper, where price and non-price attributes are combined in an MACRA setting. Based on existing multi-attribute decision making method, suppliers' scores of non-price attributes can be firstly obtained. Then, a bi-objective programming model that minimizes the total cost and maximizes the scores of selected suppliers is established to describe the WDP. To solve the problem, an ant colony algorithm with the dynamic transition strategy and the Max-Min pheromone strategy is proposed based on the model. Finally, numerical examples show the effectiveness of the proposed algorithm.

The organization of the paper is as follows. Section 2 presents the problem and the model of WDP of MACRA. Then an improved ant colony algorithm is presented in Section 3. Section 4 shows the effectiveness of the proposed algorithm by numerical examples. Finally, Section 5 concludes this paper.

II. Problem Description

In an MACRA, one buyer seeks to procure different items from multiple potential suppliers. It is assumed that a single unit of each item is needed by the buyer. Suppliers are allowed to place a single bid on a set of several distinct items. Each bid includes a total quoted price of the packaged items and non-price attributes of each item in the package. It is also assumed that the buyer uses a sealed-bid reverse auction format to run the procurement. After the auction process, the buyer determines the winner and allocates the items to the winning suppliers.

A. Notations and the Model

The notations used in the representation of the WDP of MACRA are defined as follows.

Decision variables

x_i : 1 if supplier i is been selected by the buyer ($i = 1,\dots,n$); 0 otherwise

Parameters

b_{ij} : 1 if supplier i provides item j ($j = 1,\dots,m$); 0 otherwise

p_i : the total bid price of supplier i for all his bidding items

a_{ijk} : the value of i -th supplier j -th item for the k -th attribute ($k = 1,\dots,K$), such as delivery time, product quality, etc.

p_s : switching cost if a new supplier is selected

p_p : the penalty of delay delivery time

d_{ij} : the due date of supplier i for item j

D_j : the due date of the buyer's requirement of item j

M : the maximized number of items allowed in a single package for each supplier

Let $[y]^+ = \max\{y,0\}$. Using the notations defined above, the mathematical model of the WDP of MACRA can be constructed as follows.

$$\min \sum_{i=1}^{n} p_i x_i + p_s \sum_{i=1}^{n} x_i + p_p \sum_{i=1}^{n}\sum_{j=1}^{m} \left[d_{ij} - D_j \right]^+ b_{ij} x_i \quad (1)$$

$$\max \sum_{i}^{n}\sum_{j=1}^{m}\sum_{k=1}^{K} a_{ijk} b_{ij} x_i \quad (2)$$

s.t.

$$\sum_{i=1}^{n} b_{ij} x_i = 1 \quad j = 1,\dots,m \quad (3)$$

$$\sum_{j=1}^{m} b_{ij} x_i \le M \quad i = 1,\dots,n \quad (4)$$

$$x_i \in \{0,1\} \quad i = 1,\dots,n \quad (5)$$

In the formulation, Equation (1) minimizes the procurement cost. Equation (2) maximizes the score of the selected suppliers' attributes. Equation (3) ensures that the buyer procures all the items required. Equation (4) ensures that each supplier wins M items at most. Equation (5) defines, x_i , as a 0-1 decision variable.

B. Pre-processing Procedures

Some pre-processing procedures of the bi-objective programming are presented before the improved ant colony algorithm. The bi-objective model can be converted to a single-objective model as follows, which is *NP*-hard [1], [11].

$$\min \sum_{i=1}^{n} p_i x_i + p_s \sum_{i=1}^{n} x_i + p_p \sum_{i=1}^{n}\sum_{j=1}^{m} \left[d_{ij} - D_j \right]^+ b_{ij} x_i - \delta \sum_{i}^{n}\sum_{j=1}^{m}\sum_{k=1}^{K} a_{ijk} b_{ij} x_i \quad (6)$$

s.t.

$$\sum_{i=1}^{n} b_{ij} x_i = 1 \quad j = 1,\dots,m \quad (7)$$

$$\sum_{j=1}^{m} b_{ij} x_i \le M \quad i = 1,\dots,n \quad (8)$$

$$x_i \in \{0,1\} \quad i = 1,\dots,n \quad (9)$$

where δ is a parameter to ensure that the magnitude of (1) and (2) is the same. Equation (6) is the objective function, i.e., minimizing the total procurement cost of the buyer. Equation (7) and Equation (8) constrain the procurement requirement and the maximized items of one bid, respectively. Equation (9) constrains the decision variable.

III. Improved Ant Colony Algorithm

Because of its robustness and distributed computation, the ant colony algorithm has been widely applied to various fields [12], [13], [14]. In this paper, an improved ant colony (IAC) algorithm is proposed to solve the WDP of MACRA. We improve the traditional ant colony algorithm using two strategies as follows:

(a) To improve the optimization capability of the ant and to prevent prematurity of the algorithm, the Max-Min pheromone strategy is used to control the pheromone.

(b) To improve the searching capability of globally optimal solution and the searching speed, the dynamic transition strategy is also incorporated into the algorithm.

By combining the strategies with traditional ant colony algorithm, an IAC algorithm is briefly described.

A. Real Number Encoding

According to the characteristics of the WDP of MACRA, real numbers are used to encode potential suppliers in the auction, where each ant has to be assigned an initial supplier. One possibility is to assign each ant a random initial supplier, e.g., we assign a random number chosen according to a uniform distribution over the set of suppliers $\{i=1,\ldots,n\}$.

B. Fitness Function

A fitness function is usually designed based on the objective function. As a result, the objective function of an ant is defined by

$$f=-\sum_{i=1}^{n}E_i x_i \tag{10}$$

where $E_i=p_i+p_s+p_p\sum_{j=1}^{m}\left[d_{ij}-D_j\right]^+ b_{ij}-\delta\sum_{j=1}^{m}\sum_{k=1}^{K}a_{ijk}b_{ij}$ denotes the virtue cost of supplier i.

C. Dynamic Transition Strategy

The dynamic transition strategy is defined by combining the traditional selection strategy with the dynamic transition parameter, which changes as the different times of selected suppliers and the number of iterations.

$$q_i=\frac{\tau_i^{\alpha}\cdot\eta_i^{\beta}\cdot\chi_i\cdot z_i}{\sum_{i=1}^{n}\tau_i^{\alpha}\cdot\eta_i^{\beta}\cdot\chi_i\cdot z_i},\quad i=1,\ldots,n \tag{11}$$

$$\chi_i=\frac{L\cdot t}{L\cdot t+g_i\cdot(\eta_i/\max\eta)},\quad i=1,\ldots,n \tag{12}$$

Equations (11) and (12) are the dynamic transition strategy and dynamic transition parameter, respectively. τ_i is the pheromone trail of supplier i and is equally to other $n-1$ suppliers at the initial time. η_i is the heuristic information, where $\eta_i=1/E_i$, E_i is the virtue cost of supplier i. z_i is 1 if supplier i is not visit by the ant, otherwise z_i equals 1. α and β denote the relative importance of pheromone trail and heuristic information, respectively, with interval $[0,5]$. L and t represent the number of ants and the number of iterations, respectively. g_i is the number of choosing supplier i. $\max\eta$ is the maximum value of the heuristic information.

D. Pheromone Trails Update Rule

After all the ants have constructed their tours, i.e., selecting potential suppliers to provide all the items required by the buyer, the pheromone trails are updated. This is done by first lowering the pheromone value on all suppliers by a constant factor, and then adding pheromone to the suppliers the ants have crossed in their tours. Pheromone evaporation is implemented by

$$\tau_i\leftarrow(1-\rho)\cdot\tau_i,\quad i=1,\ldots,n \tag{13}$$

where $0.1<\rho<0.99$ is the pheromone evaporation rate. The parameter ρ is used to avoid unlimited accumulation of the pheromone trails and it enables the algorithm to forget bad decisions previously taken.

After evaporation, all ants deposit pheromone on the suppliers they have crossed in their tour:

$$\tau_i\leftarrow(1-\rho)\cdot\tau_i+\sum_{l=1}^{L}\Delta\tau_i^l,\quad i=1,\ldots,n \tag{14}$$

where $\Delta\tau_i^l$ is the amount of pheromone ant l deposits on the suppliers it has chosen. It is defined by

$$\Delta\tau_i^l=\begin{cases}1/h_l, & \text{if supplier } i \text{ is chosen by ant } l\\ 0, & \text{otherwise}\end{cases},\quad i=1,\ldots,n \tag{15}$$

where h_l is the currently best solution that ant l has searched.

E. Max-Min Pheromone Strategy

The maximum and minimum pheromone trail values are assumed to be $\tau_{\max}$ and $\tau_{\min}$, respectively. The pheromone trail value of each ant is limited to the interval $[\tau_{\min},\tau_{\max}]$, which is defined by

$$\tau_i=\begin{cases}\tau_{\max}, & \text{if } \tau_i>\tau_{\max}\\ \tau_{\min}, & \text{if } \tau_i>\tau_{\min}\end{cases},\quad i=1,\ldots,n \tag{16}$$

If there is no better solution after a constant iteration, then the pheromone trail value is defined by

$$\tau_i=\begin{cases}\tau_{\min}, & \text{if } \tau_i=\tau_{\max}\\ \tau_{\max}, & \text{if } \tau_i=\tau_{\min}\end{cases},\quad i=1,\ldots,n \tag{17}$$

The Max-Min pheromone strategy can improve the optimization capability of the ant and prevent prematurity of the algorithm.

F. The Termination Rule

After running a constant loops, i.e., NG, stop the process and output the best solution.

G. The Process of IAC Algorithm

Step 1: Initialize parameters of the IAC algorithm, including the maximum loops, NG, the maximum iteration of the same local solution, T, the heuristic information, η_i, and parameters, α, β, ρ, $\tau_{\min}$, $\tau_{\max}$.

Step 2: After assigning each ant a random number of supplier set $\{i=1,\ldots,n\}$, based on the pheromone trail values, calculate the dynamic transition probability using (11)-(12). According to the dynamic transition strategy, the ant chooses one supplier with the maximum probability. Check whether the selected supplier is enforced by the constraints. If not, delete the supplier form the set of selected supplier, label the supplier as the visited one; otherwise, go to visit the next supplier that the ant has not visited based on the dynamic transition probability. Thus, a feasible solution can be searched, that is all items required by the buyer are selected.

Step 3: Calculate the fitness function using (10). Store the currently best solution and the currently global solution by comparing all the fitness values of feasible solutions constructed by the ants.

Step 4: If the currently global solution is the same for T continuous iterations, exchange the maximum and minimum pheromone trail values using (16)-(17).

Step 5: Update the pheromone trail values using (13)-(15) for the next iteration.

Step 6: Check whether the maximum allowable number of iterations, NG, is reached. If not, go to Step 2; otherwise, stop the process and output the best solution.

H. Enumeration Algorithm with Backtracking (EAB)

Enumeration algorithm firstly constructs all the feasible solutions of the problem, an optimal solution then can be obtained by comparing all the feasible solutions. Although enumeration algorithms always find an optimal solution, they also need much more time, especially solving a large scale problem. Therefore, enumeration algorithms cannot be extensively used in practice.

Backtracking is a simple algorithm for finding an optimal solution among all solutions systematically, that incrementally builds candidates to the solutions, and abandons each partial candidate violating the constraints, then backtracks to the last candidate.

Considering the characteristic of the model, the main process of an enumeration algorithm with backtracking (EAB) is shown as follows:

Step 1 *:* Initialize the decision variables, $x_i=0$, $i=1,\ldots,n$, and the parameter $i=1$.

Step 2: Search candidate suppliers sequentially to build feasible solutions using the depth-first tree algorithm, $x_i=1$.

Step 3: Check whether the currently selected candidate violating the constraints. If not, use the depth-first tree algorithm to search the next candidate supplier, otherwise, abandon the current candidate, and backtrack to the last candidate. Thus, a feasible solution is found.

Step 4: Calculate the virtual cost function of the selected suppliers, store the currently best solution.

Step 5: Check whether the enumeration tree is finished. If not, go to Step 3; otherwise $i=i+1$, go to Step 2.

Step 6: The currently best solution is the globally optimal solution, output the optimal solution and stop the process.

IV. NUMERICAL EXAMPLES

To illustrate the effectiveness of the proposed IAC algorithm, three numerical examples are calculated with the comparison of an EAB. Example 1 assumes that the buyer procures 10 different items using a first price sealed-bid auction, where 30 potential suppliers bid competitively with maximized 3 items in a single package at most. Example 2 assumes that the buyer procures 30 different items in a reverse auction system with 100 competitive suppliers, and 10 items are allowed in a single package. In example 3, one buyer and 150 potential suppliers are in the electronic reverse auction system, where the buyer procures 40 different items and allows 8 items in a single package.

Usually, the best combination of the algorithm parameters has to be set to get a better solution. In this paper, we use example 2 to test the best parameters of the IAC algorithm, i.e., the population size, L, the number of iteration, NG, importance of pheromone trails, α, the importance of heuristic information, β, the maximum pheromone trail value, $\tau_{\max}$, the minimum pheromone trail value, $\tau_{\min}$, and the pheromone evaporation rate, ρ. The results show that it is better to set $L=50$, $NG=50$, $\alpha=0.9$, $\beta=0.7$, $\tau_{\max}=0.7$, $\tau_{\min}=0.4$, and $\rho=0.3$ for further analysis because the optimization value is 14748, and the deviation is minimum.

After obtaining the best combination of the parameters, the IAC, the traditional ant colony (AC) algorithm and the EAB are used to compute the numerical examples, respectively. The results are shown in Table I, where the best solution (BS), the worst solution (WS), the mean value relative to the iteration number (Mean) and the computation time (Time) are used to investigate the performance of the improved algorithm.

TABLE I. THE COMPARISON OF IAC, AC AND EAB FOR WDP IN DIFFERENT SCALE PROBLEMS

Alg[a]	Ex[b]	L	NG	BS	WS	Mean	Time
EAB	1	–[c]	–	5902.7	5902.7	5902.7	<1s
	2	–	–	14178	14178	14178	3s
	3	–	–	–	–	–	–
AC	1	20	30	5902.7	5902.7	5902.7	<1s
	2	50	50	14748	14748	14748	4s
	3	75	1000	4012.7	4012.7	4012.7	433s
IAC	1	20	30	5902.7	5902.7	5902.7	<1s
	2	50	50	14748	14748	14748	9s
	3	75	1000	3811.2	3870.4	3823.9	832s

a. "Alg" represents the algorithm in this paper

b. "Ex" represents the number of the examples mentioned before

c. "—" represents the non-existing parameters or non-available values of the EA method

From Table 1, we see that, for the small scale problem (example 1), the EAB, the AC and the IAC methods can find the optimal solution. For the mid-scale problem (example 2), the EAB method can find the optimal solution in less time, while the AC and the IAC algorithms can find a satisfactory solution with a deviation of 0.04 from the optimal solution in a short time. This is because suppliers are allowed to bid more items in a single package in example 2 and the EAB can prune the branch of infeasible solutions effectively. For the large scale problem (example 3), the EAB method cannot find a solution in limited time, while the AC and IAC algorithms can find a satisfactory solution in a short time. This is because example 3 has the most combinations compared to the other examples and the EAB method is unable to solve the large scale problem effectively. Moreover, the IAC algorithm provides a better solution compared to the AC algorithm due to the premature convergence of the AC algorithm. By embedding the dynamic transition strategy and Max-Min pheromone strategies, the algorithm can search more areas in the solution space, deviate from the local optimal solution, improve the ability of global search and increase the probability of searching a better solution. Therefore, the IAC method is effective for the buyer to find an optimal solution for a small scale WDP of MACRA and a satisfactory solution for a mid-scale and large scale problems in a short time.

V. CONCLUSIONS

Winner determination is an important decision problem for buyers to procure goods or services. Allowing suppliers to bid combination of items in a single package tends to lead to efficient allocation compared to traditional sequential action, especially when the items are complementary. However, many researches neglect multi-attributes of the multiple items in a combinatorial auction. This paper focuses on the WDP of MACRA from the buyer's perspective. Firstly, a bi-objective programming model is constructed. Then an improved ant colony algorithm with the consideration of the dynamic transition strategy and the Max-Min pheromone strategy is proposed to solve the problem. Finally, three numerical examples are used to show the effectiveness of the IAC algorithm to solve the WDP compared with the EAB and traditional AC methods.

REFERENCES

[1] T. Sandholm, "Algorithm for optimal winner determination in combinatorial auctions," *Artificial intelligence*, vol. 135, no. 3, pp. 1-54, 2002.

[2] J. E. Teich, H. Wallenius, J. Wallenius, and A. Zaitsev, "A multi-attribute e-auction mechanism for procurement: Theoretical foundations," *European Journal of Operational Research*, vol. 175, no.1, pp. 90-100, 2006.

[3] Z.Wan, and D. R. Beil, "RFQ auctions with supplier qualification screening," *Operations Research*, vol. 57, no. 4, pp. 934-949, 2009.

[4] T. Sandholm, S. Suri, A. Gilpin, and D. Levine, "CABOB: A fast optimal algorithm for winner determination in combinatorial auctions," *Management Science*, vol. 51, no. 3, pp. 374-390, 2005.

[5] L. F. Escudero, M. Landete, and A. Marín, "A branch-and-cut algorithm for the winner determination problem," *Decision Support Systems*, vol. 46, no. 3, pp. 649-659, 2009.

[6] S. Dobzinski, N. Nisan, and M. Schapira, "Approximation algorithms for combinatorial auctions with complement-free bidders," *Mathematics of Operations Research*, vol. 35, no.1, pp. 1-13, 2010.

[7] F. S. Hsieh, "Combinatorial reverse auction based on revelation of Lagrangian multipliers," *Decision Support Systems*, vol. 48, no.2, pp. 323-330, 2010.

[8] S. D. Jap, "The impact of online reverse auction design on buyer-supplier relationships," *Journal of Marketing*, vol. 71, pp. 146-159, 2007.

[9] C. B. Cheng, "Solving a sealed-bid reverse auction problem by multiple-criterion decision-making methods," *Computers and Mathematics with Applications*, vol. 56, no.12, pp. 3261-3274, 2008.

[10] R. K. Singh, and L. Benyoucef, "Fuzzy logic and interval arithmetic-based TOPSIS method for multicriteria reverse auctions," *Service Science*, vol. 4, no. 2, pp. 101-117, 2012.

[11] M. H. Rothkopf, A. Pekeč, R. M. Harstad. "Computationally manageable combinational auctions," *Management science*, vol. 44, no. 8, pp. 1131-1147, 1998.

[12] S. R. Balseiro, I. Loiseau and J. Ramonet, "An ant colony algorithm hybridized with insertion heuristics for the time dependent vehicle routing problem with time windows," *Computers & Operations Research*, vol. 38, no. 6, pp. 954-966, 2011.

[13] O. P. Verma, P. Kumar, M. Hanmandlu, et al., "High dynamic range optimal fuzzy color image enhancement using artificial ant colony system," *Applied Soft Computing*, vol. 12, no.1, pp. 394-404, 2012.

[14] X. Liu, H. Yi, Z. Ni, "Application of ant colony optimization algorithm in process planning optimization," *Journal of Intelligent Manufacturing*, vol. 24, no. 1, pp. 1-13, 2013.