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Graph Centrality Measures and the robustness of cooperation

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***Abstract*—Previous research shows that for structured populations located on a graph, one of the most important attributes that determines whether a cooperative community is robust is the topology of the graph. However, even in a graph that is highly robust with respect to cooperation, "weak points" may still exist which will allow defection to spread quickly in the community. Previous work shows that the transitivity and the average degree are related to the robustness of cooperation in the entire graph. In addition to considering the cooperation level across the entire graph, whether an individual in the graph will allow the spread of defection is an important research question in its own right. In this work, we are trying to identify both the "weak" individuals and the "robust" ones. We measure the centrality in the graph together with the degree, the local clustering coefficient, the betweenness, the closeness, the degree eigenvector, and a few newly designed centrality measures such as "clustering eigenvector centrality". The results show that for graphs that have a fixed number of vertices and edges, there are both robust individuals and weak individuals and that the higher the transitivity of the graph, the more robust the individuals are in the graph. However, although some of the graph centrality measures may indicate whether a vertex is robust or not, the prediction is still quite unstable.**

I. INTRODUCTION

It has been shown that the graph topology plays an important role in the emergence of cooperation in evolutionary games [1]. Many researchers study how specific graph attributes can influence the emergence of cooperation by comparing the evolutionary results over graphs with different attributes. It is worth mentioning that even in the same graph, an individual player's neighbourhood structure can still influence significantly the behaviour of the evolution. Different neighbourhoods will perform differently with regards to the invasion of defection. Due to the high payoff of defection, a defector usually spreads quickly in graphs with high degree. Certain structural features of an individual's neighbourhood may halt the spread of defection and increase the robustness of cooperation for the entire graph, which has been considered as an important feature in some experimental data [2]. Understanding the individual's ability to stop the spread of defection will help to find the "weak" points of a graph, and it may be very useful in terms of virus protection, or advertising etc.

It is commonly believed that the hubs and communities in the graph have an important influence on cooperation [3], [4], [5], [6], [7]. Most research suggests that heterogeneous populations and hubs better support cooperation and there are also experiments that show that hubs and heterogeneous populations can guarantee to be of benefit to cooperation [8], [5]. Generally hubs and communities only consider the degree of one or many individuals; there may also be many other centrality measures of the individual that may also influence the emergence of cooperation. To fully understand cooperation on graphs, other centrality measures, for example, the local clustering coefficient, closeness, betweenness and eigenvector centrality, are also worthy of consideration.

In this research, we explore the influence of different centrality measures on the behaviour of a population playing an evolutionary prisoner's dilemma, through identifying both "robust" and "weak" individuals in the graphs. After exploring a series of different graphs, we find that the graph transitivity can actually determine the number of highly robust individuals in low average degree graphs. We believe that individuals placed on nodes with a higher local clustering coefficient values but with lower degree values, lower betweenness scores and closeness individuals are more unlikely to spread defection.

Considering whether an individual is robust may also depend on its neighbour's centrality; in our experiments we do not fully explore this aspect; future work will explore this concept and with the aid of machine learning approaches, we hope to study and find the critical combination of the graph centralities that can predict the robustness of the individual (or "game centrality" [2]) in a graph.

Section 2 will introduce the related research, and section 3 will introduce some graph centrality measures, as well as some discussion on the expected performance of these measures. Also in section 3, we'll analyse some simple structures, and introduce some new measures of centrality that we believe may be important to the robustness of an individual vertex. Experimental results are shown in section 4, and section 5 concludes the contribution of this work, and suggests some future work.

II. BACKGROUND

Previous research has shown that the emergence of cooperation on different graphs can differ considerably [1]. It is commonly recognized that scale-free networks are more beneficial for the emergence of cooperation [9], [10], [11], [12]. Small-world networks and lattice grids can also provide reliable cooperation under certain constraints [13], [14], [15], [16], [17], [18]. Research has shown that the high level of cooperation on scale-free networks is actually promoted via the presence of specific attributes of the scale-free network, for example, the power-law of the degree distribution. Experiments show that increasing the heterogeneity of the population can normally promote the emergence of cooperation [19], [4], [8], [6], but exceptions also exist—for example when one averages the payoff garnered through game interactions [6] and also under weak selection [5]. Poncela et al. found that besides the power law distribution of the node degree, the node to node correlation also plays an important role in the performance of cooperation in a scale-free network [20]. Furthermore, small community-like clusters are also suitable for cooperation to survive, as cooperation is more robust in graphs that have high transitivity (or clustering coefficient) but low average degree [21].

As heterogeneous graphs (graphs which have both high and low degree nodes) have shown their ability to maintain cooperation in comparison to homogeneous graphs in the more general situation [19], [4], [8], [6], research in the domain has turned to exploring which graph attributes (both local and global) influence the emergence of cooperation. Community structure and even individual behaviour have also been well researched [22], [23], [24]. One emerging opinion is that cooperators on hubs and in highly connected communities are more likely to survive [3], [7]. In order to attempt to understand how an individual player can influence cooperation on the entire graph, features such as the initial population, centrality of the graph etc. have all been considered as potential features that can contribute to the emergence and robustness of cooperation. There are even new centrality measures that have been defined such as Banzhaf, Shapley-Shubik, effort and satisfaction centrality [24]. The spread of defection from an individual player has also been considered as a measure of graph centrality (or called "Game Centrality") [25], [26], [2], and this measure has shown it successful on measuring several real world networks [2].

The notion of game centrality has "proved to be an important measure to predict the importance of nodes in the integration and regulation of complex systems" [2]. However, unlike other graph centrality measures such as the clustering coefficient and betweenness etc., the game centrality is not a measure that can be directly related to the topology of the graph. As the spread of defection in complex graphs has not been fully understood, currently we can only measure the game centrality by running the evolutionary social dilemma on the specific graph. Although Simoko showed some correlations between the game centrality and degree and betweenness [2], it is unable to be used to predict the spread of defection from any individual on a graph. Also, for graphs that have different structures, the correlation between the graph centrality and game centrality may vary, which makes predicting the outcome even more difficult.

The centrality measures that we have used in this paper include:

- Degree Centrality, or node degree refers to the number of connections of a node to other nodes in the graph.
- The Clustering Coefficient measures the proportion of a node's neighbours connected to each other [27].
- "Closeness" is measured by taking the inverse of the "farness", which is the sum of the shortest distance from a node to all other nodes in the graph [28], [29].
- The betweenness centrality for node v_i is the number of shortest paths between any two nodes v_j and v_k (where $j, k \neq i$) in graph G that have pass node v_i. It is first introduced by Freeman to demonstrate the importance of a node, in terms of whether it occupied a position on the shortest path of others [30].
- The eigenvector centrality measure the importance of a node based on its connections to high-scoring nodes in the entire graph. It is the eigenvector of the largest eigenvalue of the adjacency matrix of the graph.

In this paper, we measure the spread of defection from each individual over graphs with a wide range of average degree values and transitivity values; we observe several graph centrality measures and try to predict the spread of defection from an individual in the complex graph. In addition to considering the degree, clustering coefficient, closeness, betweenness, and degree eigenvector centrality, we identify potential candidates for correlation involving the combination of several measures.

III. Robustness of Graph and Robustness of Individual

A. Cooperation of the graph and the invasion of individuals

If a node has high ability to stop defection, that node is considered to be a "robust node", and a "robust graph" refers to the graph was able to prevent the successful invasion of defectors. One measure of whether a graph is robust to the spread of cooperation is to examine the cooperation rate during an evolutionary run. It is true that different initial populations have an influence on the final result as a defector in different positions of the graph will influence the final cooperation rate of the evolution. The cooperation rate can be examined for a graph with different randomly initialized populations and the average calculated over several runs. Alternatively one can use a set of pre-designed patterns as initial populations.

Considering running an evolutionary simulation where players interact via a 2 player game with payoff matrix: $\begin{pmatrix} 1 & 0 \\ \beta & \alpha \end{pmatrix}$ in a complex graph. A defector who wishes to invade a group of cooperators, even in a strict learning environment (where the player will adopt the strategy of the most successful neighbour for the next generation), will need to gain a payoff score of more than 1 (which is the reward payoff obtained among mutual cooperators). As we know, in these 2 player games, the equilibrium is to defect. Hence, between each two individuals in the graph, the defector can always win the game against its direct cooperative neighbour. However, on a graph, both the defector and the cooperator will also be playing with other players. Since the defector is more likely to invade its cooperative neighbours, it may obtain a smaller *overall* payoff than cooperators who receive more rewards for mutual cooperation from interacting with fellow cooperators. Furthermore, for a mutator (i.e. a cooperator changed to a defector) in a fully cooperative population, the mutator will definitely obtain a better payoff due to all of its adjacent neighbours being cooperators (which means it will receive the temptation payoff in every game it plays). So, in a higher-degree graph, a single defector usually can invade most individuals due to the high connectivity in the graph. For this reason, in our research, the experiments are undertaken on graphs with a low average degree; this increases the likelihood that there will exist cooperative communities which can successfully avoid the invasion of defectors.

Previous research [21] showed that graph transitivity is a key to whether or not the cooperation level of the graph is high. However, this result is obtained using simulations over multiple runs over graphs with different transitivity values. In these simulations, one randomly chosen cooperator is changed to a defector and the effect is observed. Despite the general trend showing that graph transitivity is a key feature, there still exists exceptions where even in a very robust graph, defection may still invade a huge proportion of the graph. We posit that this is because those unusual cases involve nodes that exhibit some particular feature, which causes the outcome to differ from the majority outcome. In an effort to understand these features and phenomena, research on the individual node's robustness (or lack thereof) is necessary.

B. Invasion of a defector

Before we start discussing the spread of a defector in a graph, we need to know how a single defector can invade its neighbourhood. In our research, we adopt the mechanism whereby each player learns from her best performing neighbour, which means the player will always learn from the neighbour with the highest payoff. Given a payoff matrix $\begin{pmatrix}1 & 0\\ \beta & \alpha\end{pmatrix}$, a defector will never invade a cooperator while it satisfies the following condition: $\frac{n\times\beta+(N-n)\times\alpha}{N} < \frac{m}{M}$, where N and M are the total number of neighbours of the defector and the cooperator respectively, n and m are the number of cooperative neighbours of the defector and cooperator respectively. To satisfy this condition, n must be much smaller than N.

In a fully cooperative graph, mutating one player from cooperation to defection in the initial generation, is equivalent to mutating all of its neighbours at the same time, as all of its neighbours will have learned to defect after just one generation. A high-degree graph is usually very poor at preventing the invasion of defection. This is the reason we restrict our experiments to low average-degree graphs.

This feature may in turn also stop the spread of defection in a highly clustered neighbourhood as a high local clustering coefficient of the first mutated node will spread defection causing a community of defectors to exist, which means they will receive a much lower payoff in the next generation as these connected defectors will receive the punishment payoff. So, we hypothesise that individual nodes with a higher clustering coefficient but a lower degree are more likely to stop the spread of defection.

C. Centrality measures that may influence the spread of defection

Considering the above analysis, in our experiments, in addition to the common centrality of the graph, the degree, the clustering coefficient, the betweenness, the closeness and the degree eigenvector centrality, we will also consider some combinations of these measures to see if they directly influence the ability of an individual to spread defection.

As we proposed earlier, a higher clustering coefficient but lower degree individual may be more likely to stop the spread of defection. So, we also consider $\frac{local\_clustering\_coefficient}{local\_degree}$ in the experiments. Furthermore, as the ability of an individual to spread defection may also be influenced by the individual's second and even third degree connections, we consider a measure for the clustering coefficient that takes this phenomenon into account: we consider the eigenvector of the degree. To calculate this, we first build a node-by-node matrix, M, where each cell $M_{i,j}$ has the value $\frac{common\_neighbours\_of\_node\_i\_and\_j}{total\_neighbours\_of\_node\_i\_and\_j}$. The eigenvector of this matrix can represent the centrality of the clustering of the corresponding individuals in the graph. We call this centrality measure the *clustering eigenvector centrality*.

As we consider $\frac{local\_clustering\_coefficient}{local\_degree}$, we similarly considered the value of $\frac{degree\_eigenvector}{clustering\_eigenvector}$ as a potential measure.

Since the spread of defection is very complex and may controlled by more than two node centrality measures, we do not expect any of the measures to fully provide a perfect indication as to whether defection will spread or not. However, it may be give us a hint of how the graph centrality can influence an individual node's robustness with respect to cooperation. We hope this could provide a step towards developing a suitable prediction algorithm.

IV. Experiment results

The experiments have been undertaken on a series of graphs with 1000 vertices and an average degree of 5. The graphs that have been generated for the experiment can be guaranteed to provide an even distribution of transitivity values. The experiments start with a fully cooperative population, and the player strategy updates are synchronized. At the beginning of each experiment, one individual is mutated to be a defector, and we then record the cooperation rate after 100 generations. For each graph we run the experiment 1000 times so that every individual is mutated once for an independent run in order to find which node will allow defection spread more widely.

We use 10 different graphs for each transitivity value from 0.1 to 0.8 and these have been generated by the algorithm introduced in the paper [21]. We have also tested the graph with higher average degree values (6, 7, 8, 9 and 10) with each transitivity value from 0.1 to 0.8 as well. Although the payoff matrix of the game is important to the invasion, however, a node in a robust community is always being robust against invasion in compare with nodes that are in the position which are easy to be invaded, so we decide to use a 2 player prisoner's dilemma game with a payoff matrix $\begin{pmatrix}1 & 0\\ 0 & 1.61\end{pmatrix}$. The temptation payoff is set to 1.61, in order to prevent has a cooperator that has the same payoff as a defector (for example, a defector with 5 cooperate neighbours will have payoff 8.05, that is slightly higher than a cooperator which having 8 cooperate neighbours).

We run each evolutionary simulation on each graph. The evolution converged very quickly and the cooperation rate reaches a relatively stable state usually within 50 generations. We recorded the cooperation rate of each independent run after 100 generations and used this as a measure of the robustness of cooperation for the corresponding individual.

We ranked the cooperation rate after 100 generations for each individual as a defector in each graph, and then we plotted several centrality measures of the individual, and attempted to identify if one (or a combination of a few) graph centrality measure can capture whether the individual will spread defection widely into the graph. We also explore for different graphs whether the graph's attributes can affect the overall performance of the individual in the graph.

A sample plot of the cooperation rate for a graph with transitivity 0.5 (Graph 5.1) is shown below (Fig. 1):

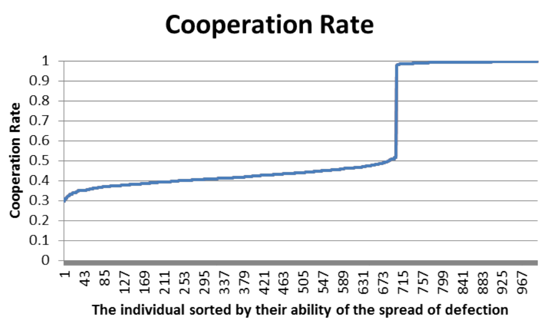

Fig. 1. The cooperation rate of each individual in graph with transitivity value 0.5

It is interesting to see that there are both "robust" and "weak" individuals in the same graph, with a quite clear separation (we can see that there is a quick transition from 0.52 to 0.98). We can see there is a transition that identifies the number of "robust" individuals and "weak" individuals with 705 individuals that will spread defection to more than half of the graph while others almost didn't spread defection at all.

This proportion varies in relation to the overall transitivity of the graph. The average rate of highly robust individuals averaged over 10 randomly generated graphs with transitivity values ranging from 0.1 to 0.6 (Fig. 2) shows that, for graphs which has 1000 vertices and 2500 edges, the higher the transitivity of the graph, the more robust individuals present in the graph. However, in the graphs with transitivity 0.7 and 0.8, there are hardly any individuals that allowed defection to spread very quickly; in other words, almost all individuals in such graphs are very robust. The overall comparison of graphs with different transitivity scores is shown in Figure 3.

From Figures 2 and 3, we can see that both the transitivity and the number of edges can determine the number of highly robust individuals in the graph. This supports previous results [21]. The individuals have quite different performance in each graph. Some of them spread defection over almost the entire graph, while others did not spread defection at all. In the graphs with high levels of transitivity, some individuals even reverted back to cooperation subsequent to their mutation to

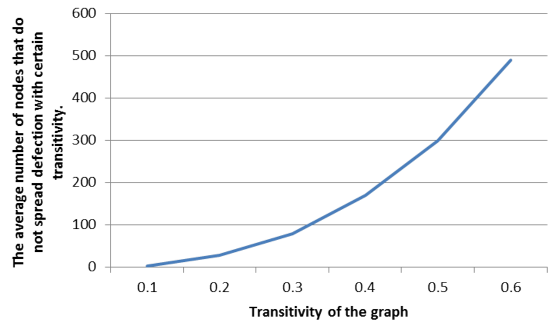

Fig. 2. The average number of individuals that can not spread defection in the graph with specific values of transitivity.

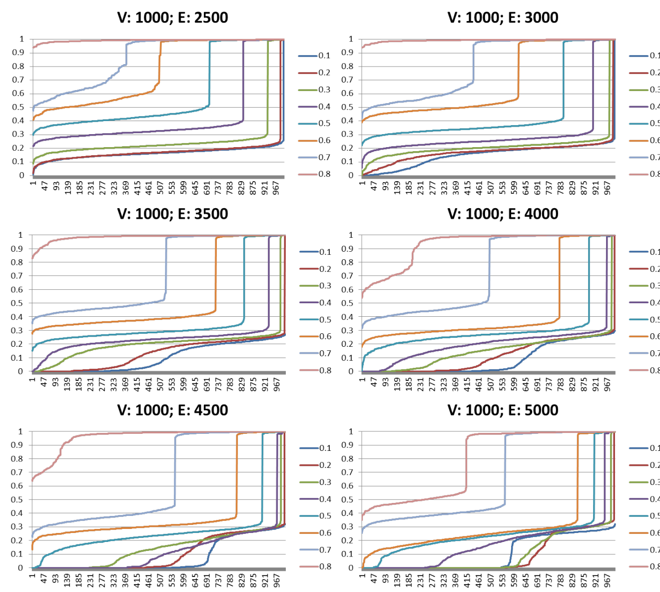

Fig. 3. Cooperation rates after 100 generations for the graphs with different transitivity and average degree. The number of edges is specified above the graphs; the colour of the lines defines the transitivity; the x-axis plots the index of the node that have been set to defect; the cooperation rate is plotted on the y-axis.

defection. To understand this phenomenon, we need to explore the centrality of each individual.

In order to observe whether there are any graph centrality measures that can identify the robustness of cooperation (or the spread of defectors) for each individual in the graph, we consider the following measures: the local clustering coefficient, local degree, betweenness centrality, closeness centrality and eigenvector centrality.

We plot the cooperation rate for each individual in each graph based on the rank for each different graph centrality measures. We include some sample plots in Figure 4. These correspond to the graph illustrated in Figure 1. Other graphs demonstrate similar features.

From the experiments, we can see that there is a clear distinction for each centrality measure between the individuals leading to a low cooperation rate and those leading to a high cooperation rate. Although the results are quite noisy, there is

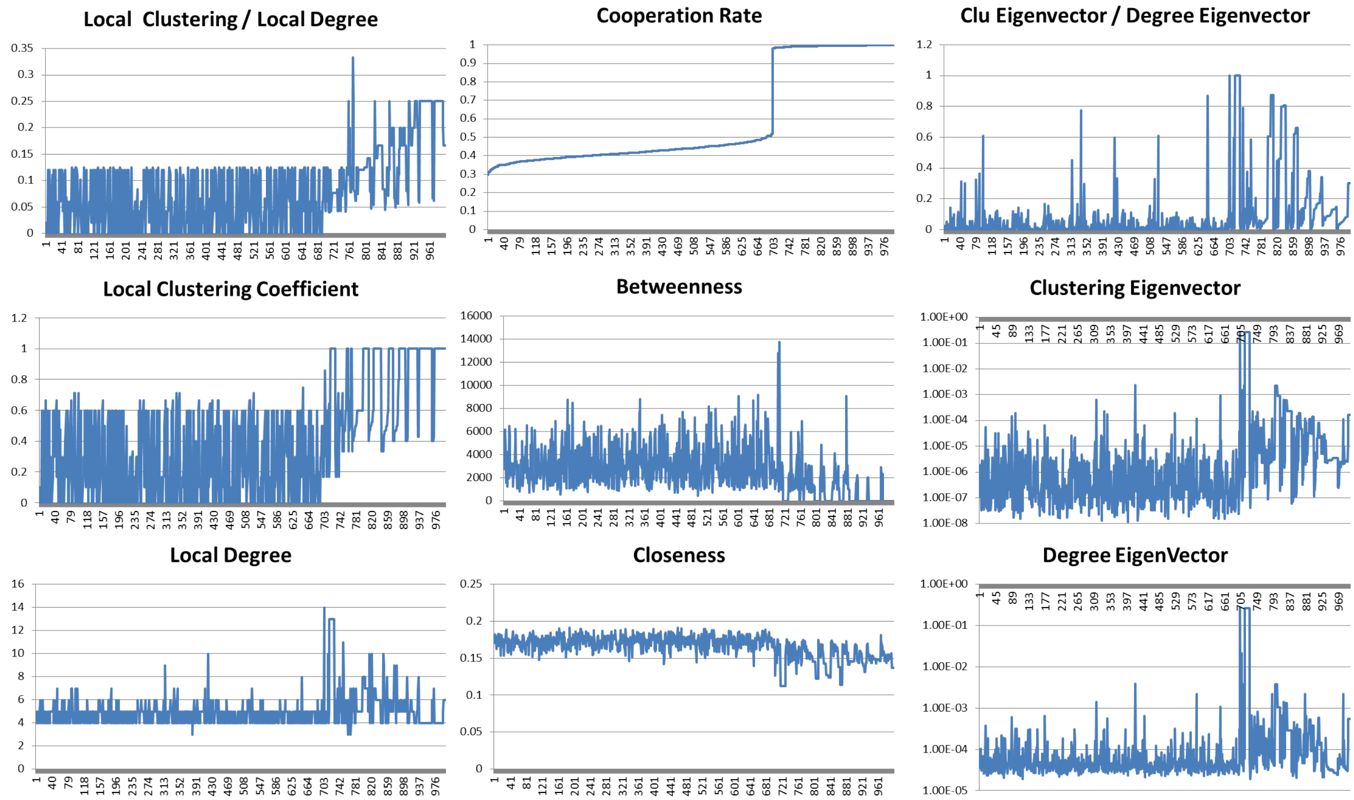

Fig. 4. Individual centrality by the order of each individual's spread of defection in graph 5.1

a clear trend that a high clustering coefficient and high degree nodes are more robust to the spread of defection. Also, the betweenness and closeness scores are, on average, much lower for those robust nodes. Surprisingly, the influence of all of the eigenvector measures is quite small with little correlation between these values and the outcomes. This is probably due to the average degree of the graph being too small in comparison to the size of the graph, so the eigenvector's absolute value is too small. Having said this, there is some correlation present with highly robust individuals having a higher probability of having a higher value of clustering eigenvector / degree eigenvector.

It is interesting to note that for individuals having a similar ability to spread defection, the variance on each centrality measure is high. There is an intermediate value of the centrality that may perform both behaviour on spread defection or not.

However, the extreme values of some centrality measures, will still make a good prediction for the robustness of individuals. For example, a node with clustering coefficient 1, and betweenness 0, will definitely not spread defection at all.

The current findings are not strong enough to make a definite prediction on the individual robustness present in a graph as it is obvious that none of the graph centrality measures exactly predicts the spread of defection of the corresponding individual perfectly on its own. However, the result gives us an indication of the effects of graph centrality on the spread of defection, as the extreme value only appears for either high or low cooperation.

V. Discussion and Future Work

By analysing each individual on a wide range of graphs, we attempt to make a connection between the graph topology and the cooperation levels present in social dilemma games from the "macro" level (the graph level) to "micro" level (the individual view). The higher transitivity levels can decrease the number of nodes with a higher game centrality [2] (the nodes that have higher ability to spread defection), which extends the research in [21]. Increasing the average degree can increase the average game centrality since it actually increases the size of the community while retaining the actual graph size. Ignoring the size of the graph, we believe that cooperation on the graph can be represented as the spread of defection on each individual. The degree to which defection spreads from the individual decides the number of individuals that a defector can invade, which will directly influence the cooperation rate for graphs that have a finite number of vertices.

On the other hand, for an individual, the likelihood of defection spreading from the node can be roughly predicted by its centrality. Despite the presence of the high level of fluctuation in the intermediate values, the extreme values of the node centrality measure always appear with extreme values of the cooperation rate. So we can guarantee that a node with clustering coefficient 1 and betweenness 0, will never spread defection, even if that node itself changes to defection either through initial mutation or invasion.

Although the experiment results show some noise, the results are quite promising. Considering that the game centrality is a useful measure in many real world data settings [2], it cannot currently be predicted on given graphs without running the evolutionary social dilemma game on that particular graph. The measurable graph centrality is more easily obtained from the graph. Future work will involve learning the relationship between potential combinations of these centrality measures and the spread of defection. This hopefully will lead to a mechanism for predicting the spread of defection.

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