2014 IEEE Congress on Evolutionary Computation (CEC)
July 6-11, 2014, Beijing, China

A Novel Evaluation Function for LT Codes Degree Distribution Optimization

Pei-Chuan Tsai
Department of Computer Science
National Chiao Tung University
HsinChu City, TAIWAN
Email: pctsai@nclab.tw

Chih-Ming Chen
Department of Computer Science
National Chiao Tung University
HsinChu City, TAIWAN
Email: ccming@nclab.tw

Ying-ping Chen
Department of Computer Science
National Chiao Tung University
HsinChu City, TAIWAN
Email: ypchen@cs.nctu.edu.tw

***Abstract*—Luby transform (LT) codes implements an important property called ratelessness, meaning a fixed code rate is unnecessary and LT codes can complete the transmission without channel status. The property is advantageous to transmit over certain environments such as broadcasting in heterogeneous networks or transmitting data over unknown channels. For this reason, improving LT codes is a crucial research issue in recent years. The performance of LT codes is decided by the code length and a probability mass function, called degree distribution, used in the encoding process. To improve the performance of LT codes, many studies proposed to optimize the degree distribution by using methods in evolutionary computation. One of the key steps in the evolutionary process is to evaluate decision variables for comparing the fitness of each individual. In the optimization of LT codes, it needs to repeatedly simulate the encoding/decoding process with a given distribution and evaluate the performance over a sufficient number of runs. Hence, a lot of computational resource is necessary for the optimization of LT codes. In this paper, we propose a heuristic function to evaluate the performance of LT codes. The evaluation function estimates the expected fraction of unsolved symbols with the specified code length, reception overhead, and degree distribution. Based on the proposed function, a huge number of evaluations is possible for searching for better degree distributions. We first verify the practicality of the proposed function and then employ it in a multi-objective evolutionary algorithm to investigate the trade-off of LT codes between the computational cost and decoding performance.**

I. Introduction

In the field of channel coding, Luby transform (LT) codes [1] are the first practical implementation of digital fountain codes [2] which have an important property called ratelessness. Ratelessness means unlimited number of codewords can be generated on the fly without a fixed code rate. The encoding process of LT codes first divides the message into serval packages which are also named input symbols if the length of the packages is only one bit. To generate each codeword or called output symbol, an integer d is first sampled from a probability mass function and then d input symbols are uniformly randomly chosen to composite the output symbol by Xor-sum operator. The integer d denotes the degree of the output symbol, and hence, the probability mass function is called *degree distribution*. Any receiver who is interested in receiving the message can reconstruct it after collecting a sufficient amount of the output symbols regardless of the generated order. The performance of LT codes depends only on the code length k and the adopted degree distribution $\Omega(x)$. In the proposal of LT codes, a degree distribution form called *robust soliton distribution* was introduced with the performance bound proven by theoretical analysis. When $k + \mathcal{O}(\ln^2(k/\delta)\sqrt{k})$ output symbols are collected by the receiver, the probability of successfully recovering the original message is at least $1-\delta$, where δ is a parameter of the robust soliton distribution.

Based on LT codes, lots of variants were proposed to achieve better decoding performance or less computational cost. The most successful variant is Raptor codes proposed in [3]. Raptor codes have a two-layer coding mechanism in which a pre-code is concatenated in front of an inner code. In the coding scheme, the pre-code usually adopts block codes with a high rate to encode the original message first. LT codes then serve as inner codes at the second layer and take the result of pre-code as message to encode. On the receiver side, the inner code, i.e., the LT codes, would decode first. After a sufficient fraction of input symbols are solved, the outer code would recover the remaining unsolved input symbols to complete the recovery. While it is unnecessary to recover all the input symbols, LT codes can adopted degree distributions with a lower average degree. A lower average degree means less computational cost is required for the coding process. LT codes serve in Raptor codes with such special degree distribution are therefore called weakened LT codes since it is not able to recover all the input symbols.

The robust soliton distribution is developed based on an asymptotic analysis. However, the optimum degree distribution is still unknown for either the LT codes or weakened LT codes for a finite code length. To improve the performance of LT codes, many studies utilized methods in evolutionary computation to search for better degree distributions. In order to employ methods in evolutionary computation, repeatedly seeking and evaluating degree distributions are necessities. All the known evaluation approaches for LT codes degree distribution are very costly. Due to the high computational cost, the size of code length k or the iterations taken in evolutionary algorithms are quite limited. Observing the fact, we propose a heuristic evaluation function for LT codes decoding process in this paper. The proposed algorithm estimates the expected fraction of unsolved symbols with the specified code length, reception overhead, and degree distribution. With the help of our proposal, researchers can then easily evaluate the performance of degree distributions without high computational cost.

The remainder of the paper is organized as follows. Section II introduces the related work of LT codes optimization. In section III, the proposed function is introduced in details. For demonstrating the usage of the evaluation function, we first verify its practicability by comparing the evaluation results with simulations. Then the proposed function is employed in multi-objective optimization to assist to design degree distributions for weakened LT codes in section IV. Finally, section V concludes the paper.

II. Related Work

Methods in evolutionary computation have long been developed for search and optimization problems. Searching for good degree distributions is naturally associated with evolutionary algorithms. Hyytiä made the first attempt in 2006 to employ heuristic search algorithms to optimize the degree distribution for LT codes with code length 100 [4]. After that, our previous work [5], [6] made the use of evolutionary algorithms on the optimization of degree distributions, in which the maximum code length is one thousand. Ref. [7] focused on maximizing the intermediate recovery rate by using multi-objective optimization algorithms. The code length in the optimization is still no more than one thousand. In these studies, real coding simulation was implemented to evaluate the performance of a particular degree distribution. The approach is feasible but not efficient because a large number of simulation runs are required to obtain precise results. The study [8] provides another solution to evaluate a degree distribution for the failure probability of full recovery. The method is a dynamic programming algorithm that exhaustively records the possible states of the decoding process. Hence, the computational cost of the method is still high $\mathcal{O}(k^3 \log^2(k))$. Although another method [9] with a lower computational complexity, $\mathcal{O}(k^2 \log(k))$, was introduced, such an expensive cost still cannot be accepted for most real-world applications of which the code length is usually larger than ten thousand. Since methods in evolutionary computation usually require a lot of evaluations to search the decision space, the evaluation of degree distributions is therefore the most computationally costly part. For the reason, an efficient evaluation function for degree distributions is in need. To handle this issue, we propose an evaluation function based on the result of the And-Or tree analysis for LT codes.

III. Evaluation Function

To enhance the efficiency of evolutionary algorithms applied to the degree distribution optimization, a fast objective function is necessary. This section presents the proposed evaluation function which can evaluate the fraction of unsolved symbols in LT codes decoding. The evaluation function is developed based on a study of random process called And-Or tree analysis [10]. The approach solved the problem of message passing in tree structure that is quite similar to the iterative decoding process in LT codes. Researchers found that the approach can be adopted to analyze the asymptotic performance of LT codes. In other words, the analysis is valid while the code length is infinite. However, almost all the real-world applications of rateless codes are limited in the scope of a finite code length. The prediction of analysis results does not match the real situation, especially for a small or medium message code length (e.g., $k < 10000$). We adopt the same idea of And-Or tree analysis and propose a heuristic function to evaluate the packet error rate of finite length LT codes.

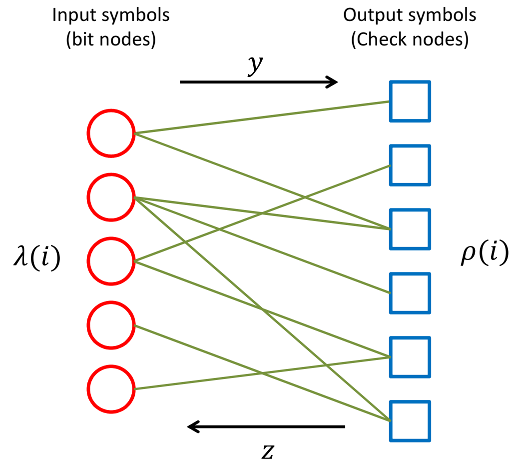

Fig. 1. The bipartite graph represents the coding structure of LT codes. $\lambda(x)$ and $\rho(x)$ denotes the edge degree distribution in the graph. y and z are the fraction of covered nodes while the message passes to the opposite side.

A. And-Or Tree Analysis

The coding structure of LT codes can be represented as a bipartite graph as shown in Figure 1. Input symbols and output symbols are denoted as circle and square nodes. We first define the left degree of an edge in the graph to denote the remaining degree of the left node after eliminating the edge. Let $\lambda(x)$ and $\rho(x)$ be the edge degree distribution respectively for the left and right nodes. According to the And-Or tree analysis, if y_t is the fraction of input symbols that still covered at decoding iteration t, $1 - y_t$ of edges will pass the message to the right side and the fraction of unknown output symbols can be evaluated by the function,

$$z_t = 1 - \rho(1 - y_t) \ . \tag{1}$$

Oppositely, the fraction of covered input symbols in next iteration can be calculated as the form,

$$y_{t+1} = \lambda(z_t) \ . \tag{2}$$

In LT codes, the degree of an output symbol is sampled from the given degree distribution, $\Omega(x)$, and $\rho(x) = \Omega'(x)/\Omega'(1)$. At the sender side, the input symbols are unformly randomly chosen to join the encoding process and hence the degree distribution of input symbols can be described by Poisson distribution with parameter α. α is the average degree of the distribution, $\Omega(x)$. Assume that the amount of received output symbols are $\gamma \cdot k$. α will be $\gamma \cdot \Omega'(1)$. The degree distribution of input symbols is $\Pi(x) = (e^{-\alpha}\alpha^x)/x!$ and then $\lambda(x) = \Pi'(x)/\Pi'(1) = e^{\alpha(x-1)}$. Finally, the iterative evaluation function is given as

$$\begin{aligned} y_{t+1} =& \lambda(1-\rho(1-y_t)) \\ =& \lambda(1-\frac{\Omega'(1-y_t)}{\Omega'(1)}) \\ =& e^{-\alpha(\frac{\Omega'(1-y_t)}{\Omega'(1)})} \\ =& e^{-\gamma\cdot\Omega'(1-y_t)} , \end{aligned} \tag{3}$$

in which γ is the reception ratio and y_0 is 1. When $t \to \infty$, the iterative function will converge and solve the asymptotic performance of LT codes for a given degree distribution, $\Omega(x)$, and a particular proportion of reception, γ.

The function iteratively estimates the fraction of covered symbols in each side of the graph. For each iteration, if $y_{t+1} < y_t$, it means that some symbols are newly solved and the decoding process can keep going and reduce the covered symbols. Therefore, a good degree distribution of LT codes is required to satisfy the condition as much as possible for $y \in [0, 1]$. Ref. [7] employs the function to study the optimum intermediate recovery rate of LT codes. The evaluation result is only valid for asymptotic case although it works well. Evaluating degree distributions for finite length LT codes is still in need.

B. Proposed Evaluation Function

In the decoding process of LT codes, the input symbols which are newly solved will be pushed in a waiting queue called *ripple*. The input symbols in ripple can be used to unpack the output symbols with degree more than one for seeking new elements of the ripple. The decoding fails when the ripple become empty and there still are covered input symbols. Observing the analysis in the previous section, Equation (3) iteratively estimates the fraction of covered symbols. In other words, $y_t - y_{t+1}$ is the additional fraction of the ripple at iteration t. For a full recovery, it is expected that $y_t - y_{t+1} > 0$ until the unsolved symbols are finished, i.e., $y_{t+1} = 0$. Moreover, the expected size of ripple should be sufficiently large to resist the variance because the encoding process of LT codes is stochastic. The proposal [3] of Raptor codes recommends a practical threshold based on the theory of random walk. For a high probability to complete the decoding, the expected ripple size should be greater than $\sqrt{k}$. As a consequence, the condition can be formulated as

$$y_t - y_{t+1} > \sqrt{\frac{1}{k}} . \tag{4}$$

According to Equation (4), we develop Algorithm 1 to evaluate degree distributions for LT codes with code length k and proportion of received symbols γ. The algorithm iteratively calculates the unsolved fraction of input symbols y_t until the inequality cannot hold true. The evaluation procedure is quite straightforward and faster than any known evaluation approach to the best of our limited knowledge. The experiment and optimization results presented in next section demonstrate the feasibility of our proposed evaluation algorithm.

Algorithm 1 Degree Distribution Evaluation Function

Input: code length k, degree distribution $\Omega(x)$, reception ratio γ;
Output: fraction of unsolved input symbols y;

```
procedure DD-EVAL(k , Ω(x) , γ)
    y ← 0;
    while true do
        y_new ← exp(−γ · Ω′(1 − y));
        if y − y_new < ·√(1/k) then
            return y;
        end if
        y ← y_new;
    end while
end procedure
```

IV. Experiments

In the section, there are two experiments including single objective and multiple objective optimization on degree distributions of LT codes. In the first experiment, the proposed algorithm is adopted as the evaluation function to minimize the error probability of unsolved symbols when a fixed ratio of output symbols is received. The expected error probability of optimized degree distributions are then compared with simulation results to verify the validity of our proposed evaluation function. The second experiment is for studying the application of the proposed method. Once the degree distribution of LT codes can be rapidly evaluated, a large number of evaluations is possible for applying multi-objective optimization on LT codes. Therefore, we introduce a multi-objective algorithm to solve the requirement of building Raptor codes. The single and multiple objective algorithms employed in the paper respectively are *covariance matrix adaption evolution strategy* (CMA-ES) [11], [12], [13] and *multi-objective evolutionary algorithm based on decomposition* (MOEA/D)[14], [15], [16]. The reason to choose the two well known evolutionary algorithms are not only that their performance has been confirmed in the literature but also that the source codes of them are publicly available [17], [18].

A. Single Objective

The first experiment is to minimize the fraction of unsolved symbols by using CMA-ES with the proposed evaluation function. CMA-ES belongs to the family of evolution strategies which has an important evolutionary mechanism called *self-adaptation*. The mechanism allows the algorithmic parameters to evolve with individuals such that the user can focus on the target problem without the need to address the tuning of parameters. CMA-ES naturally inherits the characteristic, and hence, it is adopted in our experiment with the default settings except that the maximum number of evaluations is 10^5. Our first experiment is to minimize the ratio of unsolved input symbols while a fixed number of output symbols are

TABLE I

THE TABLE LISTS THE OPTIMIZATION RESULTS FOR MINIMIZING THE ERROR PROBABILITY y. NOTE THAT THE FIRST TWO INSTANCES WITH $\gamma \leq 0.824$ MATCH THE THEORETICAL OPTIMUM.

Opt.	#1	#2	#3	#4	#5	#6	#7
γ	0.7	0.8	0.9	0.95	1.0	1.02	1.05
p_1	1.0	0.0156	0.0357	0.0113	0.0461	0.0273	0.0401
p_2	0	0.9844	0.4003	0.5652	0.3744	0.4159	0.3693
p_3	0	0	0.5641	0.0966	0.3744	0.3328	0.3278
p_5	0	0	0	0.3263	0	0	0.0811
p_8	0	0	0	0.0007	0.2052	0.1629	0.0362
p_{13}	0	0	0	0	0	0.0611	0.1110
p_{21}	0	0	0	0	0	0	0
p_{34}	0	0	0	0	0	0	0.0345
p_{55}	0	0	0	0	0	0	0
p_{89}	0	0	0	0	0	0	0
p_{144}	0	0	0	0	0	0	0
p_{233}	0	0	0	0	0	0	0
p_{377}	0	0	0	0	0	0	0
p_{611}	0	0	0	0	0	0	0
p_{990}	0	0	0	0	0	0	0
y	0.4966	0.3747	0.2383	0.1398	0.0871	0.0545	0.0216

received. Let y denotes the fraction of unsolved input symbols that $y = \#(\text{unsolved input symbols})/\#(\text{input symbols})$ and the ratio of received output symbols is $\gamma = \#(\text{received symbols})/\#(\text{input symbols})$, which has been introduced in section III. Therefore, the proposed algorithm evaluates y for arbitrary degree distributions when code length k and received ratio γ are given.

The degree distribution $\Omega(x)$ can be represented in the form of generating function $\sum_{i=1}^{k} p_i \cdot x^i$ in which the coefficients, p_i, are the decision variables in the optimization. If all the coefficients are considered, the dimensions of the problem will equal to code length k. The optimization becomes impractical when the targets are LT codes with a large code length, e.g., one thousand. To solve the difficulty, free degree distributions are replaced with sparse degree distributions of which only partial degrees are considered to have nonzero probabilities. Sparse degree are commonly used to design good distributions for less complexity [3]. Our previous work [19] has provided a general approach to define a subset of degrees and form a sparse degree distribution which can well approximate full degrees. According to [19], degree subset $S = \{1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 611, 990\}$ with size $|S| = 15$ is adopted in the experiment, and then the optimization problem can be formulated as follows,

$$\text{Decision variables} : (p_1, p_2, ..., p_i) \text{ for } i \in S$$
$$\text{Objective} : \Omega(x) = \sum_{i \in S} p_i \cdot x^i$$
$$\min(\text{DD-EVAL}(k, \Omega(x), \gamma))$$

The optimization results for code length $k = 10000$, and a series of reception ratio is given in Table I. There are many zero entry in each degree distribution confirms that using appropriately selected sparse degrees is sufficient to compose a good degree distribution. The evaluation values for these optimized degree distributions are given in the bottom of the table. The author would like to note that the theoretical optimum solution for the range, $y \in [\frac{1}{3}, 1]$, has been found [20]. Ref. [20] proved that the optimum degree distributions for the two region of y. In the first region, $y \in [\frac{1}{2}, 1]$, the optimum degree distribution only have probability on degree one and the required reception ratio is $\gamma = -log(y)$. For the second region, $y \in [\frac{1}{3}, \frac{1}{2}]$, degree distribution with all probability on degree two is the optimum for $\gamma = \frac{-log(y)}{2(1-y)}$. The first two experiment instances are $\gamma = 0.7$ and $\gamma = 0.8$ whose optimization results pretty match the prediction of the theory. For the other instances, we make the real encoding/decoding simulation, as used in previous studies, to verify the soundness of the proposed evaluation function. Figure 2 displays the average results over 1000 simulation runs, in which the scale focus on $\gamma \geq 0.9$ because the two instances out of the scope have been confirmed by the known optimum. To compare

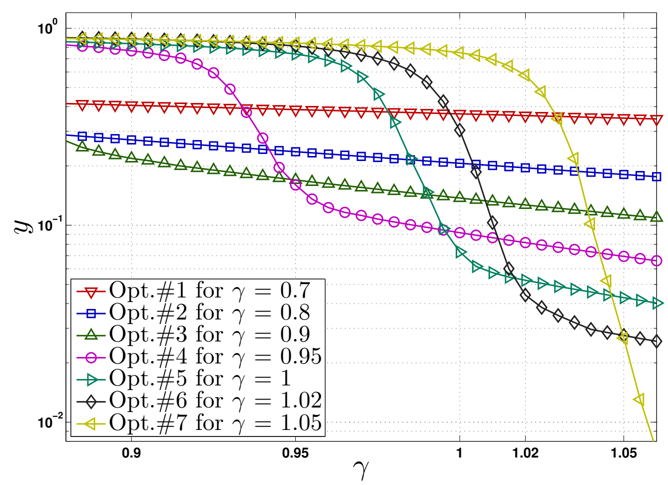

Fig. 2. The figure shows the simulation results for each optimization case in Table I. The results are average over 1000 independent runs and the code length k us 10000.

TABLE II
THE TABLE LISTS THE OPTIMIZATION RESULT OF SEVERAL INDIVIDUALS BY MOEA/D AND "RAPTOR" IS THE OPTIMIZED DEGREE DISTRIBUTION FROM THE LITERATURE [3]

indiv.	#1	#10	#20	#30	#40	Raptor
p_1	0.04197	0.04602	0.00750	0.00594	0.00686	0.00797
p_2	0.35548	0.35736	0.46647	0.49613	0.51276	0.49357
p_3	0.32341	0.32481	0.24211	0.20844	0.11037	0.16622
p_4	0.06613	0.06164	0.00820	0.00035	0.01781	0.07265
p_5	0.01622	0.02109	0.00793	0.00270	0.18379	0.08256
p_8	0.07170	0.06553	0.20583	0.27056	0.16164	0.05606
p_9	0.00432	0.00300	0.01420	0.00891	0.00651	0.03723
p_{18}	0.06333	0.07465	0.02178	0.00036	0.00002	0
p_{19}	0.01160	0.01432	0.00894	0.00450	0.00006	0.05559
p_{20}	0.01187	0.02872	0.01704	0.00201	0.00014	0
p_{65}	0.03008	0.00031	0	0	0.00001	0.02502
p_{66}	0.00361	0.00220	0	0	0.00001	0.00314
p_{67}	0.00028	0.00034	0	0.00010	0.00001	0
y	0.00163	0.00930	0.01732	0.02644	0.03641	0.00426
$\Omega'(1)$	6.49119	5.01718	4.41671	4.02172	3.71125	5.8703

the results in Table I and Figure 2, it can be found that all the degree distributions obtained in optimization show the expected error probability of input symbols. It illustrates that the proposed algorithm is valid to evaluate the degree distribution. The evaluation function is quite straightforward and its computational complexity is lower than any known LT codes evaluation function to the best of our limited knowledge.

B. Multiple Objectives

Since the proposed evaluation function is valid and efficient, the second experiment presents a multi-objective application for designing Raptor codes. Raptor codes are a two-layer coding structure consisting of a pre-coder and the inner LT codes. Many traditional block codes can serve as the pre-coder to share the loading of full recovery. Therefore, instead of solving all the input symbols, the new objective of the inner LT codes is changed to recover a fixed fraction. LT codes with degree distributions devised for the objective are called *weakened LT codes*. Weakened LT codes need not to recover all the input symbols, and hence, the average degree of the adopted distribution can be lower to reduce the coding complexity. However, weakened LT codes must ensure a high probability to solve the certain amount of input symbols. It means that we cannot choose degree distributions with an arbitrary small mean degree. Although there are some optimization results of weakened LT codes presented in the literature [3], most of them considered only a single objective. In this work, we introduce a multi-objective optimization algorithm to investigate the trade-off between the decoding performance and computational complexity. The employed multi-objective optimization algorithm is MOEA/D proposed in [14] in 2007. As CMA-ES, the performance of MOEA/D has been verified in the literature, and it was the winner of the MOEA Competition in CEC-2009. In the experiment, the two objectives are defined as

$$\text{Objective 1}: \min(\Omega'(1))$$
$$\text{Objective 2}: \min(\text{DD-EVAL}(k, \Omega(x), \gamma)) ,$$

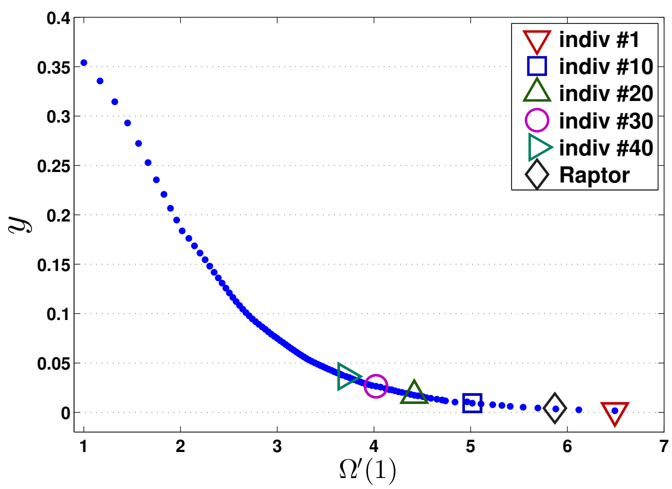

Fig. 3. The figure shows the Pareto front of multi-objective optimization results in which $k = 65536$ and $\gamma = 1.038$. The degree distribution of the five individuals with maker symbols are listed in Table II.

for a given degree distribution $\Omega(x)$. We set $k = 65536$ and minimize the two objectives for reception ratio $\gamma = 1.038$.

Figure 3 displays the optimization results, in which the Pareto front is well approximated by 100 individuals. Each individual represents a degree distribution with non-dominated objective values. The result demonstrates lots of possible solutions for Raptor codes designer to choose according to their own requirements. An optimized degree distribution from the literature [3] is given in Table II and marked as "Raptor." The degree distribution was obtained by minimizing the average degree for the same settings of k and γ in our experiment. We evaluate it with the proposed algorithm and plot it in the Figure 3. It seems like an individual in the Pareto front even though coming from other optimization. In the next, totally five individuals with different average degrees are chosen and listed in Table II. We investigate their decoding performance by real simulation again. The simulation results in Figure 4 clear

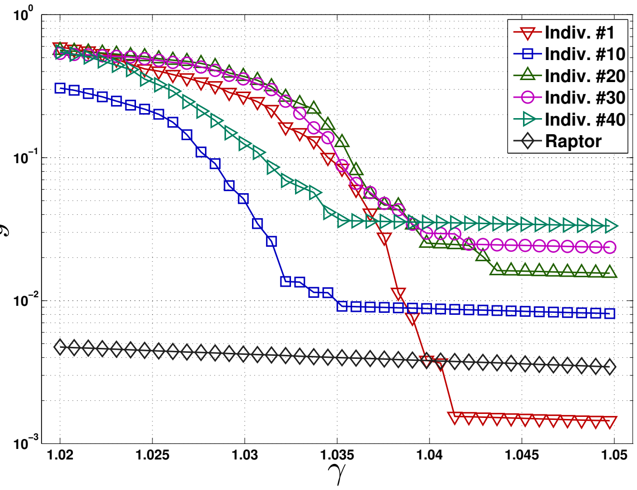

Fig. 4. The figure shows the average results over 1000 simulation runs in which $k = 65536$.

presents the different error floor of each degree distribution. Individuals with smaller average degrees perform worse on another objective of decoding performance. The efficiency of the proposed function make possible for researchers to do a huge amount of evaluations in multi-objective evolutionary algorithm for designing weakened LT codes.

V. Conclusions

Applying evolutionary algorithms to solve the optimization on LT codes degree distributions is a crucial, meaningful issue, but an efficient evaluation function was missing. The evaluation approaches in existence were so costly that the results of LT codes optimization were limited within the scale of $k \leq 1000$. To resolve this issue, this paper proposed a heuristic function to evaluate the fraction of unsolved input symbols for a given code length, reception overhead, and degree distribution. The proposed function is not only efficient but also practical. The first part of our experiment demonstrated that by optimizing a single objective of degree distributions. In the second experiment, the proposed function was applied in a multi-objective evolutionary algorithm to demonstrate its utility. The results of the multi-objective optimization help the user to choose the degree distribution to build various weakened LT codes. We believe that the evaluation function is extremely helpful when the issue of LT codes optimization is investigated.

Acknowledgments

The authors are grateful to the National Center for High-performance Computing for computer time and facilities. The work was supported in part by the National Science Council of Taiwan under Grant NSC 101-2628-E-009-024-MY3.

References

[1] M. Luby, "LT codes," in *Proceedings of the 43rd Symposium on Foundations of Computer Science*. IEEE Computer Society, 2002, pp. 271–280.

[2] J. W. Byers, M. Luby, M. Mitzenmacher, and A. Rege, "A digital fountain approach to reliable distribution of bulk data," in *Proceedings of the ACM SIGCOMM '98 conference on Applications, technologies, architectures, and protocols for computer communication*. ACM, 1998, pp. 56–67.

[3] A. Shokrollahi, "Raptor codes," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2551–2567, 2006.

[4] E. Hyytiä, T. Tirronen, and J. Virtamo, "Optimizing the degree distribution of LT codes with an importance sampling approach," in *Proceedings of the 6th InternationalWorkshop on Rare Event Simulation (RESIM 2006)*, 2006, pp. 64–73.

[5] C.-M. Chen, Y.-p. Chen, T.-C. Shen, and J. K. Zao, "On the optimization of degree distributions in LT code with covariance matrix adaptation evolution strategy," in *Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2010)*, 2010, pp. 3531–3538.

[6] ——, "Optimizing degree distributions in LT codes by using the multiobjective evolutionary algorithm based on decomposition," in *Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2010)*, 2010, pp. 3635–3642.

[7] A. Talari and N. Rahnavard, "Rateless codes with optimum intermediate performance," in *Proceedings of the Global Telecommunications Conference (GLOBECOM 2009)*, 2009, pp. 4197–4202.

[8] R. Karp, M. Luby, and A. Shokrollahi, "Finite length analysis of LT codes," in *Proceedings of the IEEE International Symposium on Information Theory 2004 (ISIT 2004)*, 2004, p. 39.

[9] E. Maneva and A. Shokrollahi, "New model for rigorous analysis of LT-codes," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT 2006)*, 2006, pp. 2677–2679.

[10] M. G. Luby, M. Mitzenmacher, and M. A. Shokrollahi, "Analysis of random processes via And-Or tree evaluation," in *Proceedings of the 9th Annual ACM-SIAM Symposium on Discrete Algorithms*, 1998, pp. 364–373.

[11] N. Hansen and A. Ostermeier, "Adapting arbitrary normal mutation distributions in evolution strategies: the covariance matrix adaptation," in *Proceedings of the IEEE International Conference on Evolutionary Computation*, 1996, pp. 312–317.

[12] A. Auger and N. Hansen, "Performance evaluation of an advanced local search evolutionary algorithm," in *Proceedings of the 2005 IEEE Congress on Evolutionary Computation (CEC 2005)*, 2005, pp. 1777–1784.

[13] ——, "A restart CMA evolution strategy with increasing population size," in *Proceedings of the 2005 IEEE Congress on Evolutionary Computation (CEC 2005)*, 2005, pp. 1769–1776.

[14] Q. Zhang and H. Li, "MOEA/D: A multi-objective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, 2007.

[15] H. Li and Q. Zhang, "Multiobjective optimization problems with complicated pareto sets, MOEA/D and NSGA-II," *Evolutionary Computation, IEEE Transactions on*, vol. 13, no. 2, pp. 284–302, 2009.

[16] Q. Zhang, W. Liu, and H. Li, "The performance of a new version of MOEA/D on CEC09 unconstrained MOP test instances," in *Proceeding of IEEE Cogress on Evolutionary Computation (CEC 2009)*, 2009, pp. 203–208.

[17] N. Hansen, "CMA-ES Source Code," http://www.lri.fr/~hansen/cmaes_inmatlab.html, Jan 2014.

[18] Q. Zhang, "MOEA/D Homepage," http://dces.essex.ac.uk/staff/zhang/webofmoead.htm, Jan 2014.

[19] P.-C. Tsai, C.-M. Chen, and Y.-p. Chen, "Sparse degrees analysis for LT codes optimization," in *Proc. IEEE Congress on Evolutionary Computation (CEC 2012)*, 2012, pp. 2463–2468.

[20] S. Sanghavi, "Intermediate performance of rateless codes," in *Proceedings of the IEEE Information Theory Workshop*, 2007, pp. 478–482.