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Primary Study on Feedback Controlled Differential Evolution

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***Abstract*—The primary study on feedback controlled Differential Evolution (FCDE) is presented. FCDE is a novel framework of DE with an automatic parameter adjustment mechanism, which controls its search situation (evaluation index) to be a promising situation (reference index) by the error feedback. Its adjustment mechanism consists of three parts: Estimator, Referencer, and Controller. Estimator calculates an evaluation index which quantitatively measures the search situation about the population diversity. Referencer generates a reference index being the ideal target of the evaluation index. Controller operates the DE parameters every generation to make the evaluation index follow the reference index. Further, this paper actually realizes a FCDE method using a typical DE by designing the three parts. The effectiveness of the proposed method is confirmed through computational experiment from viewpoint of the controllability and performance.**

I. Introduction

Differential Evolution (DE), which was introduced by R. Storn and K. V. Price in 1995, is one of the powerful and efficient multipoint mataheuristis for global optimization over continuous space [1], [2]. DE has been improved or extended and widely applied in many scientific and engineering fields so far [3]. The reason why DE attracts a lot of researchers and practitioners is that it has the following virtues compared to other methods [3]:

- The structure of DE is more straightforward to program and implement.
- The search performance of DE for various problems is generally better from many research results and competitions.
- The number of setting parameter of DE is only three: the scaling factor and the crossover rate and the population size.

However, its powerful and efficient performance depends on an appropriate parameter setting corresponding to the targeted problem. That is, many trial and error for an appropriate parameter setting are imposed on users, if they do not have their adjusting knowledge or cannot know structure of the objective function in advance.

From this backdrop, a lot of automatic parameter adjusting methods have been developed so far [3]. Examples of recent parameter automatic adjusting methods are as follows. J. Brest et al. proposed jDE which adjusts the crossover rate and the scaling factor every individual [4]. In this method, each parameter is updated by choosing, with a constant probability, between a new value generated based on a fixed uniform randomness and its inherited value. A. K. Qin et.al proposed SADE that switches mutation strategies and adjusts the scaling factor and the crossover rate every individual [5]. In this method, the crossover rates are randomly adjusted by Gaussian distribution being updated based on the successful parameter values, and the scaling factors are Gaussian randomization. J. Zhang and A. C. Sanderson proposed JADE that has new mutation strategy and adjusts the scaling factor and the crossover rate every individual [6]. In this method, the scaling factors are randomly adjusted based on Cauchy distribution getting updated using history of the successful parameter values, and the crossover rates are randomly adjusted based on Gaussian distribution getting updated also with successful ones.

This paper is concerned with a new parameter automatic adjustment for DE different from the conventional methods. First, a novel framework of DE with an automatic parameter adjustment mechanism, which controls its search situation (evaluation index) to be a promising situation (reference index) by the error feedback. Its adjustment mechanism consists of three parts: Estimator, Referencer, and Controller. Estimator calculates an evaluation index which quantitatively measures the search situation about the population diversity. Referencer generates a reference index being the ideal target of the evaluation index. Controller operates the DE parameters every generation to make the evaluation index follow the reference index. Secondary, a FCDE method using a typical DE called DE/best/1 is actually realized by designing the three parts. The effectiveness of the proposed method is confirmed through computational experiment from viewpoint of the controllability and performance.

II. Outline of Differential Evolution

DE is a direct multipoint search method for continuous global optimization problems which uses evolutionary operation: differential mutation, crossover, and selection. The m individuals that are solution candidates at the g-th generation are represented by $x_i(g) \in \mathbf{R}^n,\ i = 1, \cdots, m$. The termination criterion of its search is generally defined as the generation time g reaches the maximum generation $g_{\max}$ arbitrarily given by users. There are several kinds of DE due to differences in mutation and crossover, and they are notated as DE/X/Y/Z where X is a type of mutation, Y is the number of differential vectors, and Z is a type of crossover.

This paper throughout handles a typical DE called DE/best/1/bin for unconstrained minimization problems for the

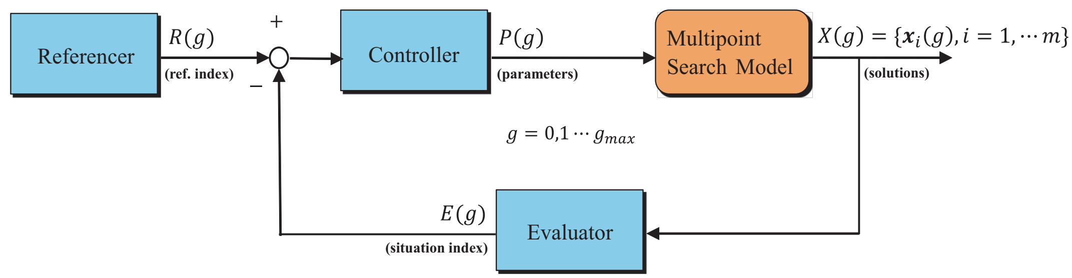


Fig. 1. Framework of Feedback Controlled Multipoint Search (FCMS)

objective function $f : \mathbf{R}^n \to \mathbf{R}$,

$$\underset{x}{\text{Minimize}}\ f(x), \quad x \in \mathbf{R}^n.$$

The details of the evolutionary operation used in DE/best/1/bin are as follows.

Mutation: The mutation operation generates the mutation vectors $v_i(g), i = 1, \cdots, m$ based on differential vectors made of the individuals. The mutation operation of best/1 uses the best individual in the population and one differential vector as follows:

$$v_i(g) = x_b(g) + F(x_{r_{i1}}(g) - x_{r_{i2}}(g)),\ i = 1, \cdots, m \quad (1)$$

where $F \in \mathbf{R}_+$ is a parameter called scaling factor, $x_b(g)$, $b = \arg\min_i f(x_i(g))$, $i = 1, \cdots, m$ is the best in the population until the g-th generation, $x_{r_{i1}}(g)$ and $x_{r_{i2}}(g)$ are different vectors chosen randomly from the population such that $i \neq b \neq r_{i1} \neq r_{i2}$.

Crossover: The crossover operation generates trial vectors $u_i(g), i = 1, \cdots, m$ from $x_i(g)$ and $v_i(g)$. The typical crossover type is binomial crossover defined as follows:

$$u_{i,j}(g) = \begin{cases} v_{i,j}(g) & \text{if } p_{i,j}(g) \leq C \text{ or } j = q_i(g) \\ x_{i,j}(g) & \text{otherwise} \end{cases}, \quad (2)$$

$i = 1, \cdots, m$, $j = 1, \cdots, n$, where $C \in [0, 1] \subset \mathbf{R}$ is a crossover rate, and $p_j(g)$ is a random real number in $[0, 1] \subset \mathbf{R}$, and $q_i(g)$ is a random integer number in $[1, n] \subset \mathbf{N}$.

Selection: The selection operation compares the objective function values of $x_i(g)$ and $u_i(g)$ and selects the superior one as follows:

$$x_i(g+1) = \begin{cases} u_i(g) & \text{if } f(u_i) \leq f(x_i) \\ x_i(g) & \text{otherwise} \end{cases}, \quad (3)$$

$i = 1, \cdots, m$.

III. Framework of Feedback Controlled Differential Evolution (FCDE)

A. Diversification and Intensification

It is well known that considering a balance between diversification and intensification for search points is important to improve search performance of multipoint metaheuristics [8], [9].

- Diversification is a strategy which aims to prevent search points from remaining local area and to search for better solutions by searching wide region (global search),
- Intensification is a strategy which aims to search for better solutions intensively by searching local area around a good solution (local search).

That is, although diversification and intensification are opposite strategies, only diversification cannot find deeply better solutions, and only intensification cannot find widely better solutions. Therefore, realization of an appropriate balance corresponding to the targeted problem is important for better search.

Some of parameter adjustments in multipoint metaheuristics are designed by considering such a balance between them [9], however most of them seem to be intuitively designed. Therefore, from objectivity viewpoint, it is necessary to consider the following points:

- To define a measure to quantitatively evaluate a balance between them (evaluation index).
- To define a target of its measured value concerning an appropriate balance for better performance (reference index).

B. FCDE in FCMS

In accordance with the above discussion, we establish a novel framework for multipoint metaheuristics, which introduces an evaluation index and controls it to a reference index by the error feedback. This framework is called "Feedback Controlled Multipoint Search (FCMS)" and its image is as shown in Fig.1. This framework regards the generation g as discrete time, the multipoint search models as controlled dynamical systems, the search point population $X(g) = [x_1(g)^\top, \cdots, x_m(g)^\top]^\top \in \mathbf{R}^{mn}$ as state of the dynamical system and setting parameters $P(g) \in \mathbf{R}^d$ as control inputs into the system. In addition, the following three mechanisms are connected to the dynamical system to make feedback loop:

- Evaluator: This calculates an evaluation index $E(X(g)) \in \mathbf{R}$ which quantitatively measures a balance between the diversification and intensification for search points.

- Referencer: This generates a promising reference index $R(g) \in \mathbf{R}$ for $E(X(g))$ for good performance.

- Controller: This operates the control inputs $P(g)$ to make $E(g)$ follow $R(g)$.

This framework intends that "Controller" adjusts the parameters $P(g)$ to make $E(g)$ of "Evaluator" follow $R(g)$ of "Referencer" by the error feedback. Therefore, its automatic parameter adjustment mechanism consists of "Evaluator", 'Referencer" and "Controller". By defining and designing the three parts appropriately, the automatic parameter adjustment which can realize the promising balance can be expected. Note that, in order to achieve superior search, it is important to design the three parts so that they cooperate with each other, based on characteristics of the targeted problem and the adopted multipoint search method.

This paper applies FCMS to DE and call the resultant framework as "Feedback Controlled Differential Evolution (FCDE)". That is, in Fig.1, "Multipoint Search Model" is replaced with "Differential Evolution" and input $P(g)$ is defined as $P(g) = [F(g)\ C(g)]^\top$.

IV. Realization of FCDE

A. Policy

In this section, for simplification, we consider a case that the adjusted parameter is only F (i.e. $P(g) = [F(g)\ C]^\top$ and C is a constant.) . To actually realize a method in FCDE, we first stand the following control policy.

[Policy] Control an evaluation index which measures an average distance between individuals and the best individual such that it follows a reference index which gradually decreases from an initial one when $g = 0$ to a certain small one ε when its termination $g = g_{\max}$.

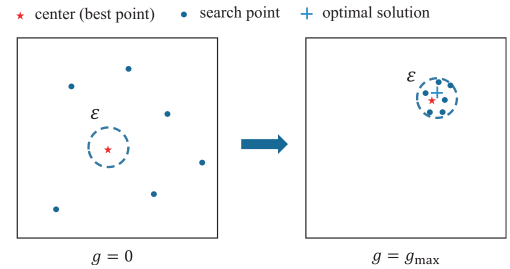
The image of this policy is shown in Fig. 2. Its validity can be proved in comparison with other inappropriate balances as shown in Fig. 3 as follows:

- Fig.3(a) shows a situation where the population are still dispersed when $g = g_{\max}$, which means that the intensification strategy that searches around the best individual cannot be realized sufficiently well (weak intensification).

- Fig.3(b) shows a situation where the population converge at subspace within ε from the best individual when $g = g_{\max}/2$, which means that diversification cannot be realized sufficiently well because the half-generation times persist (strong intensification).

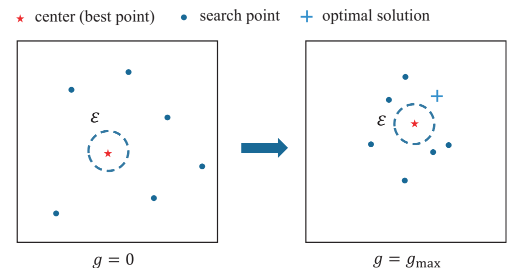
B. Evaluator

In accordance with Policy, to measure an average distance between individuals and the best, we introduce the following evaluation index:

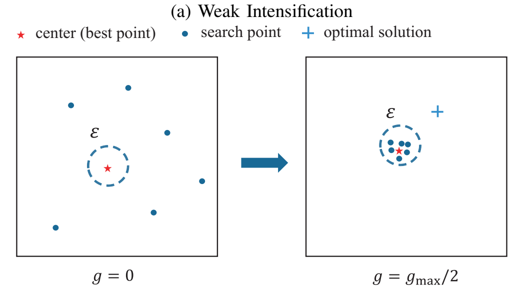


(a) Appropriate Intensification

Fig. 2. Image of Promising Balance from Diversification to Intensification



(a) Weak Intensification



(b) Strong Intensification

Fig. 3. Image of Unpromising Balance from Diversification to Intensification

$$
\begin{aligned}
E(g) &= \frac{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{i,j}(g) - x_{b(g),j}(g)|\right)/mn}{B/mn} \\
&= \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{i,j}(g) - x_{b(g),j}(g)|}{B}
\end{aligned}
\tag{4}
$$

where

$$B = \sum_{i=1}^{m}\sum_{j=1}^{n}|x_{i,j}(0) - x_{b(0),j}(0)|,$$

and $E(0) = 1$. This definition is based on the mean of 1-norm of difference vectors between $x_{b(g)}(g)$ and $x_i(g)$, relative to $x_i(0)$.

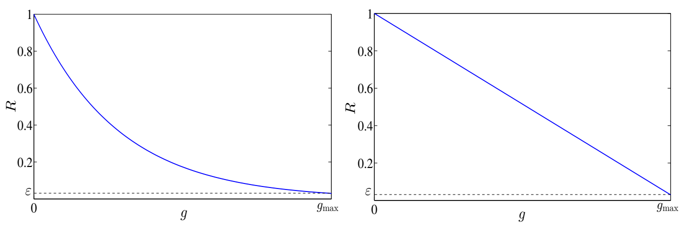
This index can quantitatively evaluate a balance between diversification and intensification for the population during the search process. That is, if the index is moving near 1, the search points tend to be under diversification around the initial population; if it is getting close to 0, they tend to be under intensification toward the best individual.

C. Referencer

In accordance with Policy, to generate a promising reference index $R(g)$ for the above $E(g)$, we introduce the

TABLE I. TEST FUNCTIONS

Name	Definition	Optimal solution $\boldsymbol{x}^\star$	$f(\boldsymbol{x}^\star)$	Initial region
1. Sphere	$f(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2$	$(0, 0, \cdots, 0)$	0	$\mathbf{S} = [-5.0\ 5.0]^n$
2. Rosenbrock	$f(\boldsymbol{x}) = \sum_{i=1}^{n-1} 100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2$	$(1, 1, \cdots, 1)$	0	$\mathbf{S} = [-2.0\ 2.0]^n$
3. Bohachevsky	$f(\boldsymbol{x}) = \sum_{i=1}^{n-1} \{x_i^2 + 2x_{i+1}^2 - 0.3\cos(3\pi x_i) - 0.4\cos(4\pi x_{i+1}) + 0.7\}$	$(0, 0, \cdots, 0)$	0	$\mathbf{S} = [-5.0\ 5.0]^n$
4. Rastrigin	$f(\boldsymbol{x}) = \sum_{i=1}^{n} (x_i^2 - 10\cos 2\pi x_i + 10)$	$(0, 0, \cdots, 0)$	0	$\mathbf{S} = [-5.0\ 5.0]^n$
5. Levy	$f(\boldsymbol{x}) = \pi/n[\sum_{i=1}^{n-1}\{(x_i - 1)^2(1 + 10\sin^2(\pi x_{i+1}))\} + 10\sin^2(\pi x_1) + (x_n - 1)^2]$	$(1, 1, \cdots, 1)$	0	$\mathbf{S} = [-5.0\ 5.0]^n$
6. Ackely	$f(\boldsymbol{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i\right) + 20 + \exp(1)$	$(0, 0, \cdots, 0)$	0	$\mathbf{S} = [-5.0\ 5.0]^n$
7. 2^n minima	$f(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^4 - 16x_i^2 + 5x_i$	$(0, 0, \cdots, 0)$	$-78n$	$\mathbf{S} = [-5.0\ 5.0]^n$
8. Six-hump	$f(\boldsymbol{x}) = \sum_{i=1}^{n/2}(4 - 2.1x_i^2 + 1/3x_i^4)x_i^2 + x_i x_{i+n/2} + (-4 + 4x_{i+n/2}^2)x_{i+n/2}^2$	$(0.09, 0.07, 0.09, \cdots, 0.07)$	$-0.52n$	$\mathbf{S} = [-2.0\ 2.0]^n$



(a) Ref1: Exponential curve (b) Ref2: Straight line

Fig. 4. Referencer

following two types.

$$\text{Ref1}: \quad R(g+1) = \sqrt[g_{\max}]{\varepsilon}\, R(g), \ \ R(0) = 1 \tag{5}$$

which generates an exponential curve, as shown in Fig.4 (a).

$$\text{Ref2}: \quad R(g+1) = R(g) - (R(0) - \varepsilon)/g_{\max}, \ \ R(0) = 1 \tag{6}$$

which generates a straight line, as shown in Fig.4 (b). Both are functions of the maximum generation $g_{\max}$ and a small $\varepsilon > 0$, and converge to $R(g_{\max}) = \varepsilon$ at the search termination $g = g_{\max}$ from $R(0) = 1$.

D. Controller

Here we aim to design a simple control law for F to make $E(g)$ follow $R(g)$ by the following steps.

First, we calculate an expected value of $E(g+1)$ from (4) as follows. The j-th element of the mutation vector $v_i(g)$ in (1) is

$$v_{i,j}(g) = x_{b(g),j}(g) + F\xi_{r_i,j}(g), \tag{7}$$

where $\xi_{r_i,j}(g) = x_{r_{i1},j}(g) - x_{r_{i2},j}(g)$ and $b(g) = \arg\min_i f(x_i(g)),\ i = 1, 2, \cdots, m$. Therefore, expectation of the j-th element of the trial vector $u_i(g)$ is

$$\begin{aligned}\mathrm{E}[u_{i,j}(g)] &= C'v_{i,j}(g) + (1 - C')x_{i,j}(g) \\ &= C'x_{b(g),j}(g) + C'F\xi_{r_i,j}(g) + (1 - C')x_{i,j}(g)\end{aligned} \tag{8}$$

where $\mathrm{E}[\cdot]$ represents an expected value of a random variable and $C' = C + 1/n - C/n = C(1 - 1/n) + 1/n$ is the actual crossover rate based on (2).

Assuming a success probability of the selection of $u_i(g)$ to be $\mu(g) \in [0, 1] \subset \mathbf{R}$, expectation of the j-th element of $x_i(g+1)$ is

$$\begin{aligned}\mathrm{E}[x_{i,j}(g+1)] &= \mu(g)\mathrm{E}[u_{i,j}(g)] + (1 - \mu(g))x_{i,j}(g) \\ &= \mu(g)C'x_{b(g),j}(g) + \mu(g)C'F\xi_{r_i,j}(g) \\ &\quad + (1 - \mu(g)C')x_{i,j}(g).\end{aligned} \tag{9}$$

Based on (9), expectation of the j-th element of $x_{b(g+1)}(g+1)$, $b(g+1) = \arg\min_i f(x_i(g+1))$ is

$$\begin{aligned}&\mathrm{E}[x_{b(g+1),j}(g+1)] \\ &\quad = \mu(g)C'x_{b(g),j}(g) + \mu(g)C'F\xi_{r_{b(g+1)},j}(g) \\ &\qquad + (1 - \mu(g)C')x_{b(g+1),j}(g).\end{aligned} \tag{10}$$

From (9) and (10), we can derive the following equation.

$$\begin{aligned}&\left|\mathrm{E}[x_{i,j}(g+1) - x_{b(g+1),j}(g+1)]\right| \\ &\quad = \big|(1 - \mu(g)C')(x_{i,j}(g) - x_{b(g+1),j}(g)) \\ &\qquad + \mu(g)C'F(\xi_{r_i,j}(g) - \xi_{r_{b(g+1)},j}(g))\big|.\end{aligned} \tag{11}$$

Therefore, from (4) and (11), expectation of $E(g+1)$ is

$$\begin{aligned}&\mathrm{E}[E(g+1)] \\ &\ = \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}\left|\mathrm{E}[x_{i,j}(g+1) - x_{b(g+1),j}(g+1)]\right|/B \\ &\ = \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}\big|(1 - \mu(g)C')(x_{i,j}(g) - x_{b(g+1),j}(g)) \\ &\quad + \mu(g)C'F(\xi_{r_i,j}(g) - \xi_{r_{b(g+1)},j}(g))\big|/B.\end{aligned} \tag{12}$$

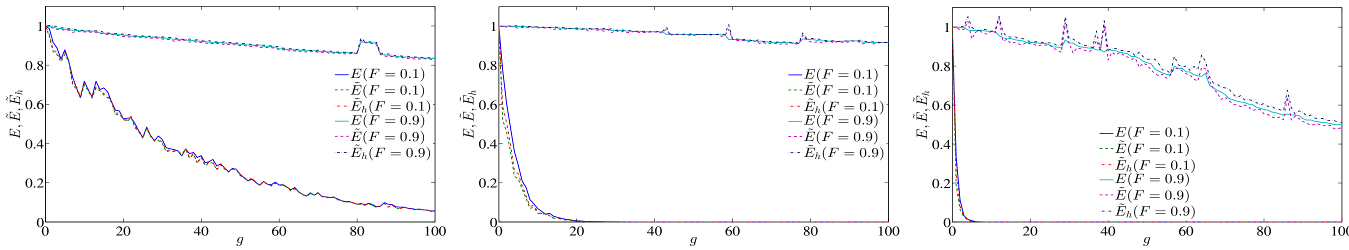
Further, because $(1 - \mu(g)C') \geq 0$, $\mu(g)C'F \geq 0$, using triangle inequality, the following inequality holds:

$$\mathrm{E}[E(g+1)] \leq (1 - \mu(g)C')E(g)' + \mu(g)C'F\Theta'(g) \tag{13}$$

where

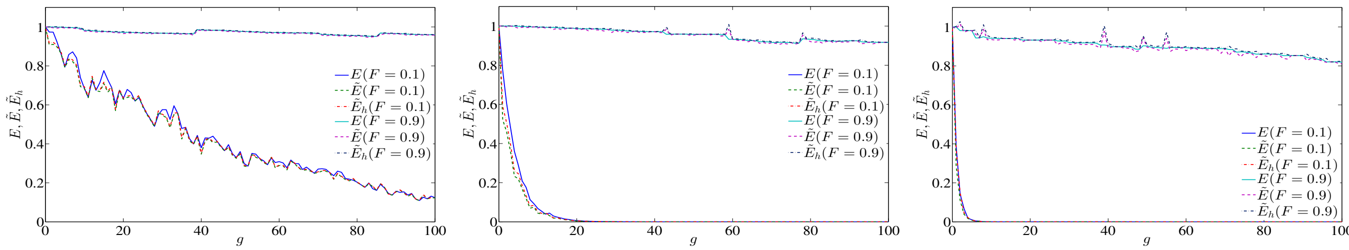
$$\begin{aligned}E(g)' &= \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}|(x_{i,j}(g) - x_{b(g+1),j}(g))|/B, \\ \Theta(g)' &= \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}|(\xi_{r_i,j}(g) - \xi_{r_{b(g+1)},j}(g))|/B.\end{aligned}$$

Secondary, we investigate relations between E, $\widetilde{E} := \mathrm{E}[E(g+1)]$ and the right side of (13) : $\widetilde{E}_{\mathrm{h}} := (1 - \mu(g)C')E(g)' + \mu(g)C'F\Theta'(g)$ through various numerical experiments. The used test functions are Sphere, Rastrigin and 2^n-minima which are shown in Table I. The number of the individuals is set $m = 100$, the dimension is $n = 100$ and the maximum generation is $g_{\max} = 100$ and 1000. Note that $\mu(g)$ is calculated based on the actual successful selection rate of $u_i(g),\ i = 1, \cdots, m$. Some of results are shown in Fig.5-7. It is observed that the error between E and $\widetilde{E}$ is quite small, the error between $\widetilde{E}$ and $\widetilde{E}_{\mathrm{h}}$ is also quite small, and the inequality of (13) holds. Although we cannot all data due to the page



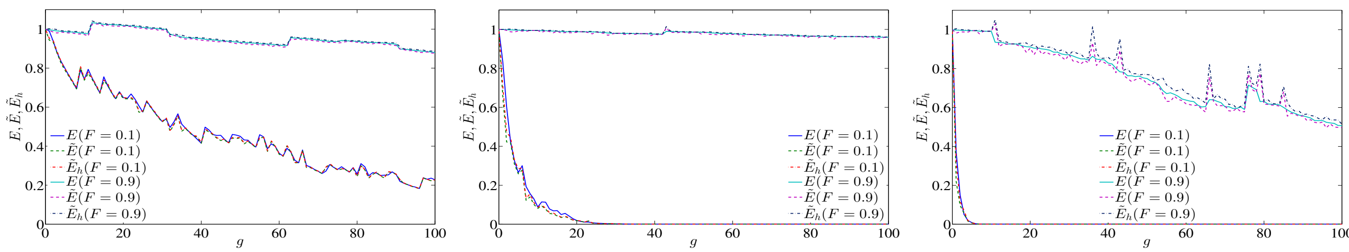
(a) $C = 0.1$ (b) $C = 0.5$ (c) $C = 0.9$

Fig. 5. Trajectories of E, $\widetilde{E}$, and $\widetilde{E}_{\mathrm{h}}$ for Sphere ($m = 100, n = 100$)



(a) $C = 0.1$ (b) $C = 0.5$ (c) $C = 0.9$

Fig. 6. Trajectories of E, $\widetilde{E}$, and $\widetilde{E}_{\mathrm{h}}$ for 2^n-minima ($m = 100, n = 100$)



(a) $C = 0.1$ (b) $C = 0.5$ (c) $C = 0.9$

Fig. 7. Trajectories of E, $\widetilde{E}$, and $\widetilde{E}_{\mathrm{h}}$ for Rastrigin ($m = 100, n = 100$)

limitation, let us report that the same tendency for all data was observed.

Finally, considering the numerous computational results, $\mathrm{E}[E(g+1)]$ can be approximated based on $\widetilde{E}_{\mathrm{h}}$ as follows:

$$\mathrm{E}[E(g+1)] \approx (1-\mu(g)C')E(g)' + \mu(g)C'\Theta(g)'F. \quad (14)$$

We use this model in order to analyze the effect of the parameter F and derive its adjusting law. Based on (14), since $\mu(g)C'\Theta(g)' \geq 0$, assuming $\mu(g) \neq 0, \Theta(g)' \neq 0$, we can analyze the influence of F on $E(g+1)$ as follows:

- F and $E(g+1)$ are strong positive correlation.

Using this relation, we can propose a simple parameter adjusting law using the error feedback between $R(g)$ and $E(g)$ and a gain $K_{\mathrm{F}} > 0$ as follows:

$$F(g+1) = F(g) + K_{\mathrm{F}}(R(g) - E(g)) \quad (15)$$

where its domain is $F_{\min} \leq F(g) \leq F_{\max}$, and if the adjusted valus are outside of its domain, they are modified such that $F(g+1) = F_{\max}$ (if $F(g+1) > F_{\max}$) and $F(g+1) = F_{\min}$ (if $F(g+1) < F_{\min}$).

This law increases $F(g)$ when $R(g) > E(g)$ to increase $E(g)$ and decreases $F(g)$ when $R(g) < E(g)$ to decrease $E(g)$ depending on the error degree, and then enable $E(g+1)$ follow $R(g)$.

E. Algorithm

By designing the three mechanism from the former subsections, a method in FCDE has been realized. The algorithm in case of using Ref1 is described as follows:

Algorithm (Proposed Method in FCDE)

Step 0: Set the number of individuals m, the maximum iteration $g_{\max}$, the feedback gain K_{F}, the parameter ε, the maximum $F_{\max}$, the minimum g$F_{\min}$ and the crossover rate C.

Step 1: Set the initial position of individuals $x_i(0) \in \mathbf{S}, i = 1,\ 2,\ \cdots,\ m$, randomly. Set the initial iteration $g = 0$, the initial scaling factor $F(0)$, and $R(0) = 1$.

Step 2: Calculate $F(g+1)$ based on (15).

Step 3: Generate $v_i(g), i = 1,\ \cdots,\ m$, from

$$v_i(g) = x_b(g) + F(g+1)(x_{r_{i1}}(g) - x_{r_{i2}}(g)).$$

Step 4: Generate $u_i(g), i = 1,\ \cdots,\ m$, from (2).

Step 5: Determine $x_i(g+1), i = 1,\ \cdots,\ m$, from (3).

Step 6: Calculate $E(g+1)$ from (4).

Step 7: Generate $R(g+1)$ from (5).

Step 8: Terminate the whole search if $g = g_{\max}$, otherwise set $g := g + 1$ and return to Step 2.

V. Computational Experiment

To verify the effectiveness of the proposed method, we conduct computational experiment for various problems.

A. Experiment Conditions

The 8 test functions with different structures in Table I are used. Their structures are classified as follows: function 1 - 3: unimodal, function 4 - 6: globally unimodal, and function 7 - 8: globally multimodal. The regions of initial search points $\mathbf{S}$ are defined every function in Table I.

To verify whether the proposed method is achieved for different $g_{\max}$, we set $g_{\max} = 100$ and 1000.

To verify whether the proposed method is achieved for various dimensions of functions, we set $m = 100$ and n as follows: $n = 30$ which means in case of dimensions are fewer than search points, $n = 100$ which means in case of dimensions are equal to search points, and $n = 300$ which means in case of dimensions are more than search points.

The proposed methods are used as PRM1 which uses (4), (5), (15) and PRM2 which uses (4), (6), (15). The setting parameters in PRMs are $K_{\mathrm{F}} = 1.0$, $F_{\max} \approx \infty$, $F_{\min} = 0$, and $(F(0), C) = \{(0.5, 0.1), (0.5, 0.9), (0.1, 0.1), (0.9, 0.9)\}$.

One compared method is "ORI" that is the original DE/best/1/bin with the initial parameters $(F(0), C)$. The other compared method is "JADE" which is known as a conventional powerful adaptive DE [6], [7].

1) Results and Discussions: Table II and III show the results which are mean of the resultant function values after 25 runs from different initial search points in $\mathbf{S}$. Table II shows the results in case of $g_{\max} = 100$ and Table III shows the results in case of $g_{\max} = 1000$. In addition, to evaluate the controllability of the proposed method, "error" which is mean of every gap between $R(g)$ and $E(g)$ is shown.

Now, based on the results, let us discuss about the controllability and search performance as follows:

(1) Controllability: It is confirmed that "error" in both PRM1 and PRM2 are almost small. It seems that the proposed method has good controllability in various cases. Here, some examples of trajectories of $E(g)$, $R(g)$ and $F(g)$ are shown in Fig. 8-10, which each trajectory is obtained through one trial with the same initial search points. It is observed that $E(g)$ actually follows $R(g)$ while $F(g)$ is automatically adjusted.

(2) Performance: The following facts are observed:

- The performance of PRMs are affected by the initial parameters $(F(0), C)$. PRM with $(F(0), C) = (0.5, 1)$ is generally more powerful than other cases.
- PRM1 is stronger than PRM2 in almost cases. That means the reference index with the exponential curve is superior to one with the strait line.
- PRM1 tends to be stronger than ORI in many cases: In case of $(F(0), C) = (0.5, 0.1)$, PRM1 is stronger than ORI in both $g_{\max} = 100$ and $g_{\max} = 1000$. In case of $(F(0), C) = (0.1, 0.1)$, ORI is a little stronger than PRM in $g_{\max} = 100$, but ORI is a much weaker than PRM in $g_{\max} = 1000$. In case of $n = 300$ and $g_{\max} = 1000$, PRM tends to be stronger.
- PRM1 and PRM2 are weaker for unimodal function than JADE. But, in multimodal functions, some of PRM is stronger than JADE.

VI. Conclusions

This paper proposed the novel framework for DE with the automatic parameter adjusting mechanism called FCDE and realized a method in FCDE. The effectiveness is confirmed through computational experiments. This is the primary study on FCDE, and remains many future tasks including how to adjust the parameter C and how to define Referencer, Estimator for improving search performance.

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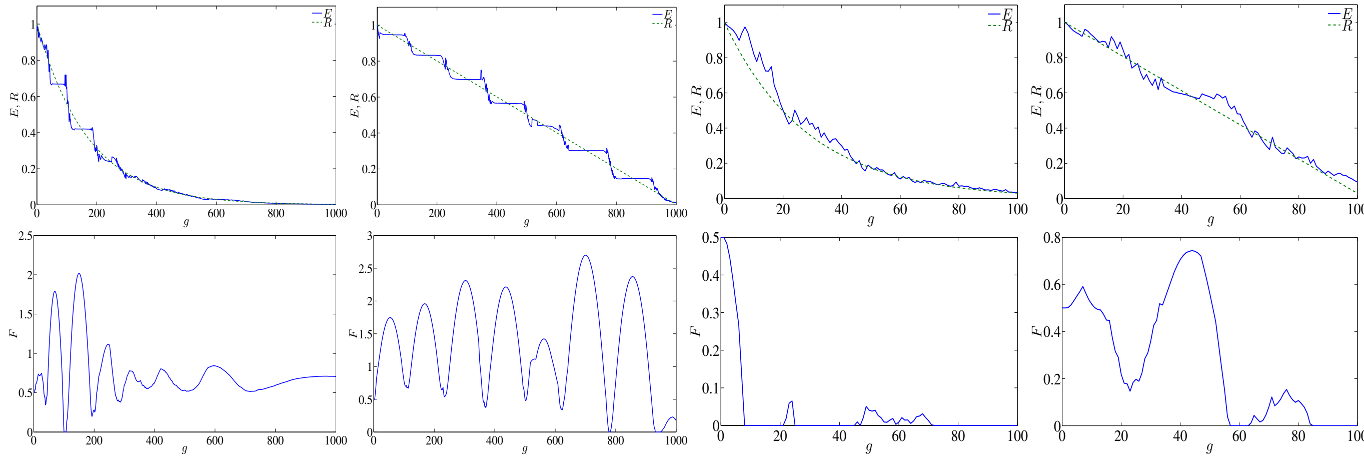
TABLE II. SIMULATION RESULTS ($g_{\max} = 100$)

$g_{\max} = 100$		$(F(0), C) = (0.5, 0.1)$					$(F(0), C) = (0.5, 0.9)$					$(F(0), C) = (0.1, 0.1)$					$(F(0), C) = (0.9, 0.9)$					
Functions	Dim.	ORI	PRM1		PRM2		ORI	PRM1		PRM2		ORI	PRM1		PRM2		ORI	PRM1		PRM2		JADE
		$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$
1. Sphere	30	0.5	0.2	0.005	2.4	0.038	6.3	0.6	0.036	14.6	0.093	1.2	0.2	0.011	2.8	0.063	96.4	1.1	0.023	5.9	0.019	0.0
	100	119.0	65.1	0.046	51.1	0.026	103.7	61.5	0.023	131.6	0.050	51.8	61.2	0.027	65.6	0.044	650.6	70.8	0.050	147.4	0.064	2.1
	300	1,527.0	566.6	0.152	584.2	0.046	1,290.4	572.6	0.054	713.8	0.095	529.5	569.6	0.123	629.7	0.061	2,178.5	598.0	0.061	751.7	0.134	71.0
2. Rosenbrock	30	105.1	105.7	0.016	133.8	0.044	145.7	61.7	0.030	274.7	0.054	124.9	98.9	0.015	156.2	0.062	3,625.5	74.9	0.028	164.9	0.033	28.0
	100	5,732.5	1894.0	0.069	1864.2	0.033	1,950.6	1505.1	0.023	3265.5	0.033	1,547.7	1928.0	0.061	1998.8	0.044	29,655.8	1457.7	0.050	3650.0	0.087	173.1
	300	88,844.2	19277.8	0.176	20861.2	0.062	109,241.8	17626.3	0.065	22397.6	0.132	17,277.2	18124.5	0.148	21854.1	0.070	109,241.8	17109.0	0.074	32038.9	0.164	1567.4
3. Bohachevsky	30	14.0	4.7	0.007	21.4	0.040	38.5	20.3	0.035	52.1	0.076	10.3	4.2	0.011	23.4	0.061	294.3	17.6	0.024	37.8	0.020	2.4
	100	423.6	263.7	0.041	224.2	0.027	377.5	283.8	0.025	471.7	0.048	214.9	256.2	0.027	235.7	0.036	1,997.5	299.8	0.053	529.6	0.056	60.0
	300	4,755.5	1931.2	0.148	1903.4	0.045	4,101.4	1985.8	0.053	2320.3	0.097	1,772.3	1959.8	0.120	2163.5	0.065	6,716.2	1978.2	0.067	2554.6	0.143	440.3
4. Rastrigin	30	130.9	26.3	0.014	39.5	0.025	85.7	133.3	0.028	169.0	0.048	35.7	28.1	0.010	43.7	0.044	356.0	133.3	0.026	157.5	0.016	153.3
	100	1,009.8	412.7	0.084	422.0	0.045	591.9	796.2	0.023	912.0	0.047	529.4	415.1	0.066	433.3	0.058	1,574.6	826.9	0.044	908.1	0.046	794.5
	300	4,481.8	2364.6	0.179	2529.5	0.072	2,564.1	3069.0	0.037	3269.0	0.044	3,001.7	2329.2	0.163	2594.1	0.092	5,067.9	3146.2	0.066	3389.5	0.118	2799.4
5. Levy	30	2.0	0.4	0.010	1.1	0.022	4.8	3.2	0.026	10.9	0.059	1.0	0.5	0.008	1.2	0.050	56.6	3.1	0.016	7.5	0.020	0.2
	100	37.9	10.2	0.103	11.1	0.046	16.3	23.5	0.026	31.8	0.043	11.9	10.2	0.090	11.4	0.045	120.5	20.3	0.030	29.6	0.042	2.0
	300	106.5	32.1	0.192	37.3	0.082	31.0	42.4	0.021	51.9	0.045	45.0	31.1	0.175	37.5	0.096	144.6	45.8	0.060	51.8	0.088	11.4
6. Ackely	30	5.0	3.6	0.009	7.2	0.034	10.9	5.7	0.033	12.6	0.079	5.8	3.6	0.011	8.1	0.066	19.1	6.0	0.025	9.9	0.018	1.3
	100	16.8	14.1	0.048	13.6	0.027	15.7	15.1	0.024	16.8	0.043	13.8	14.2	0.034	14.0	0.040	20.7	15.5	0.043	17.2	0.048	4.9
	300	20.5	17.9	0.156	18.1	0.048	17.4	18.1	0.040	18.4	0.063	18.1	18.0	0.126	18.3	0.070	20.9	18.2	0.063	18.9	0.139	10.8
7. 2^n minima	30	-2,130.3	-2310.0	0.022	-2291.4	0.022	-1,926.6	-1969.3	0.017	-1819.9	0.031	-2,327.8	-2325.7	0.015	-2289.2	0.038	-1,024.5	-1955.1	0.045	-1832.6	0.039	-1930.1
	100	-4,022.3	-6559.5	0.110	-6533.0	0.043	-2,886.0	-5180.3	0.051	-4451.2	0.053	-6,782.1	-6554.1	0.086	-6465.3	0.055	-2,336.3	-4938.3	0.068	-4245.0	0.089	-4404.3
	300	-5,447.2	-15855.9	0.194	-14937.8	0.083	-5,042.5	-11811.8	0.065	-10647.8	0.119	-15,729.9	-15921.0	0.167	-15243.9	0.075	-5,042.5	-8629.4	0.135	-10369.1	0.144	-10229.7
8. Sixhump	30	-24.9	-25.8	0.008	-24.3	0.029	-15.8	-19.6	0.022	-13.9	0.051	-25.5	-25.8	0.010	-23.9	0.049	27.5	-19.3	0.032	-13.6	0.033	-25.9
	100	17.4	-34.0	0.082	-38.8	0.035	-12.4	-29.0	0.029	-5.3	0.030	-45.5	-33.9	0.060	-35.4	0.049	255.5	-23.8	0.052	3.2	0.079	-72.7
	300	787.7	72.3	0.179	88.9	0.069	1,016.9	57.6	0.050	103.1	0.089	56.7	65.8	0.166	84.5	0.068	1,016.9	88.3	0.073	188.3	0.149	-120.7

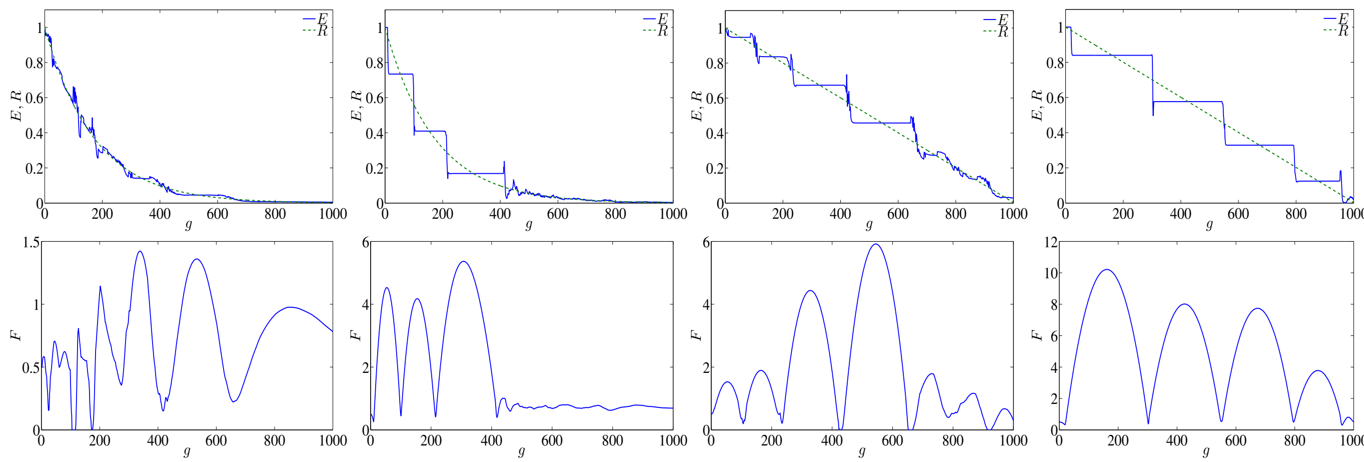
TABLE III. SIMULATION RESULTS ($g_{\max} = 1000$)

$g_{\max} = 1000$		$(F(0), C) = (0.5, 0.1)$					$(F(0), C) = (0.5, 0.9)$					$(F(0), C) = (0.1, 0.1)$					$(F(0), C) = (0.9, 0.9)$					
Functions	Dim.	ORI	PRM1		PRM2		ORI	PRM1		PRM2		ORI	PRM1		PRM2		ORI	PRM1		PRM2		JADE
		$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$	$f(x)$	error	$f(x)$	error	$f(x)$
1. Sphere	30	0.0	0.0	0.009	0.2	0.033	5.1	0.0	0.021	8.5	0.071	0.8	0.0	0.014	0.2	0.052	0.0	0.0	0.008	1.2	0.012	0.0
	100	0.0	0.0	0.003	1.3	0.011	97.4	0.5	0.011	89.1	0.046	25.3	0.0	0.005	1.3	0.026	650.6	0.3	0.014	77.7	0.024	0.0
	300	6.6	6.8	0.003	63.8	0.012	581.5	120.0	0.022	613.9	0.046	257.7	10.3	0.005	67.3	0.017	2,178.5	188.3	0.037	624.2	0.061	0.0
2. Rosenbrock	30	24.1	25.6	0.021	27.2	0.025	139.6	21.7	0.014	120.1	0.062	99.6	26.7	0.023	29.8	0.044	27.8	25.9	0.007	60.1	0.014	0.0
	100	100.8	143.3	0.004	290.5	0.014	1,854.8	197.6	0.010	2031.5	0.027	806.6	146.4	0.006	344.9	0.024	29,655.8	166.6	0.012	1715.7	0.028	82.3
	300	3,314.1	1923.3	0.005	2851.7	0.010	109,241.8	3242.8	0.027	17958.2	0.065	6,470.1	1815.9	0.006	3116.9	0.017	109,241.8	4781.2	0.047	20894.0	0.076	293.6
3. Bohachevsky	30	0.0	0.1	0.009	1.9	0.029	32.6	6.9	0.016	39.3	0.078	8.8	0.2	0.014	3.5	0.065	1.0	4.3	0.007	18.1	0.015	0.0
	100	0.1	0.8	0.004	22.4	0.014	364.1	47.9	0.011	366.2	0.050	130.1	0.7	0.005	20.3	0.022	1,997.5	59.5	0.013	271.2	0.023	14.7
	300	193.7	88.9	0.003	330.4	0.011	1,728.8	711.8	0.021	2050.9	0.038	895.0	97.7	0.005	333.2	0.018	6,716.2	886.7	0.039	2009.4	0.072	147.4
4. Rastrigin	30	0.3	2.5	0.007	4.1	0.016	77.0	102.6	0.022	156.3	0.042	22.2	2.9	0.009	6.7	0.037	64.4	79.0	0.014	161.4	0.021	0.0
	100	451.7	64.9	0.005	97.2	0.016	575.3	509.7	0.014	834.1	0.030	176.4	69.8	0.006	104.7	0.022	1,494.4	586.1	0.013	833.4	0.018	148.3
	300	2,703.9	668.2	0.005	956.6	0.014	2,577.7	2238.5	0.013	3099.3	0.028	990.4	694.1	0.008	972.2	0.021	5,067.9	2364.4	0.023	3232.2	0.039	1481.0
5. Levy	30	0.1	0.1	0.004	0.2	0.020	4.1	1.0	0.017	8.8	0.054	0.7	0.1	0.009	0.1	0.039	0.1	0.4	0.008	4.5	0.017	0.1
	100	0.0	0.0	0.003	0.3	0.013	15.1	8.5	0.015	27.4	0.047	3.0	0.0	0.006	0.4	0.019	39.9	7.0	0.010	24.0	0.021	0.0
	300	16.9	3.0	0.005	5.4	0.015	30.6	20.9	0.012	45.7	0.030	9.1	3.2	0.006	6.1	0.020	144.6	19.1	0.016	45.0	0.031	0.6
6. Ackely	30	0.0	0.4	0.008	2.4	0.030	10.5	1.8	0.017	10.0	0.070	5.0	0.4	0.013	3.3	0.052	1.2	1.6	0.008	6.6	0.017	0.0
	100	0.0	0.4	0.003	3.8	0.013	15.7	9.8	0.012	16.2	0.050	10.2	0.5	0.007	4.2	0.023	18.5	10.6	0.011	15.2	0.021	0.9
	300	5.5	5.6	0.004	9.0	0.010	17.4	17.3	0.013	18.1	0.037	14.7	6.3	0.005	10.3	0.019	20.9	17.5	0.025	18.2	0.042	4.3
7. 2^n minima	30	-2,347.7	-2349.9	0.008	-2322.6	0.019	-1,936.8	-2055.8	0.010	-1940.5	0.033	-2,327.6	-2349.9	0.010	-2339.3	0.027	-2,157.2	-2049.0	0.011	-1897.9	0.026	-2342.1
	100	-7,680.5	-7560.3	0.004	-7517.8	0.012	-4,522.6	-6587.0	0.011	-5419.9	0.037	-7,278.2	-7583.0	0.006	-7536.4	0.022	-2,336.3	-6567.5	0.023	-5056.5	0.035	-7399.0
	300	-11,435.3	-20969.0	0.005	-20088.4	0.011	-5,042.5	-17017.1	0.033	-10822.9	0.073	-19,493.2	-20926.9	0.006	-19848.9	0.017	-5,042.5	-16397.0	0.040	-10778.1	0.063	-17612.1
8. Sixhump	30	-26.0	-26.0	0.007	-25.9	0.027	-15.8	-20.9	0.012	-16.8	0.045	-25.4	-26.0	0.011	-25.5	0.043	-24.8	-21.1	0.009	-18.0	0.016	-26.0
	100	-86.4	-86.1	0.004	-85.4	0.012	-13.8	-55.2	0.008	-19.9	0.030	-63.8	-86.0	0.005	-84.5	0.022	255.5	-56.3	0.017	-20.1	0.045	-80.2
	300	-135.4	-191.7	0.005	-168.8	0.011	1,016.9	-113.8	0.020	59.8	0.060	-73.2	-197.1	0.006	-162.4	0.020	1,016.9	-82.5	0.035	90.3	0.068	-208.7



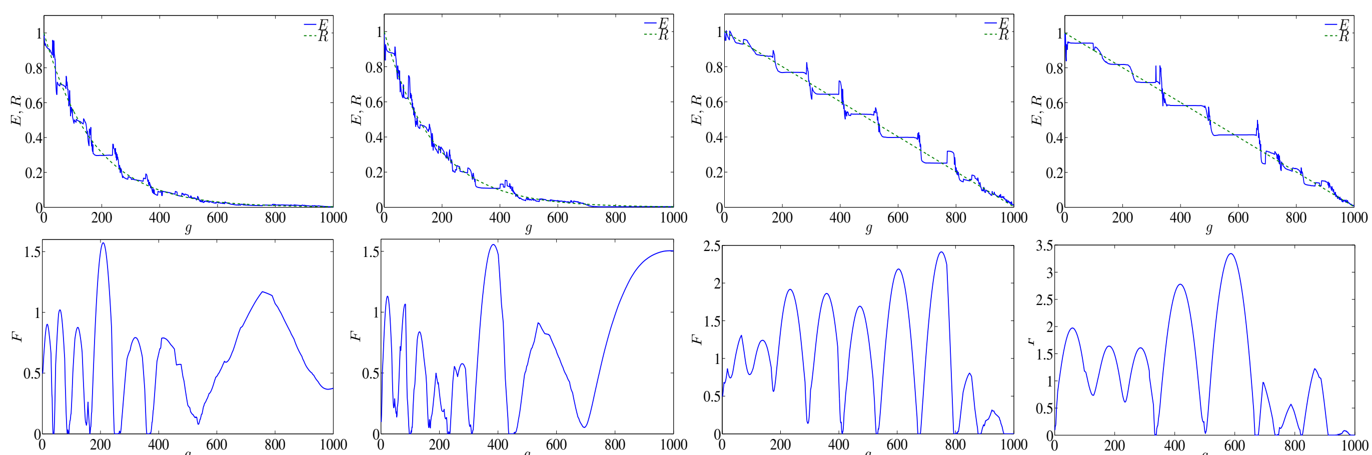
(a) $(F(0), C) = (0.5, 0.1)$, error = 0.012 (b) $(F(0), C) = (0.5, 0.1)$, error = 0.025 (c) $(F(0), C) = (0.5, 0.1)$, error = 0.036 (d) $(F(0), C) = (0.5, 0.1)$, error = 0.033

Fig. 8. Trajectories of $E(g), R(g)$ and $F(g)$ (Sphere, $n = 100$)



(a) $(F(0), C) = (0.5, 0.1)$, error = 0.012 (b) $(F(0), C) = (0.5, 0.9)$, error = 0.029 (c) $(F(0), C) = (0.5, 0.1)$, error = 0.034 (d) $(F(0), C) = (0.5, 0.9)$ error = 0.057

Fig. 9. Trajectories of $E(g), R(g)$ and $F(g)$ (2^n minima, $n = 100$)



(a) $(F(0), C = (0.5, 0.1))$, error = 0.015 (b) $(F(0), C = (0.1, 0.1))$, error = 0.015 (c) $(F(0), C = (0.5, 0.1))$, error = 0.028 (d) $(F(0), C = (0.1, 0.1))$, error = 0.028

Fig. 10. Trajectories of $E(g), R(g)$ and $F(g)$ (Rastrigin, $n = 100$)