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# Visual Examination of the Behavior of EMO Algorithms for Many-Objective Optimization with Many Decision Variables

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*Abstract*—**Various evolutionary multiobjective optimization (EMO) algorithms have been proposed in the literature. They have different search mechanisms for increasing the diversity of solutions and improving the convergence to the Pareto front. As a result, each algorithm has different characteristics in its search behavior. Multiobjective search behavior can be visually shown in an objective space for a test problem with two or three objectives. However, such a visual examination is difficult in a high-dimensional objective space for many-objective problems. The use of distance minimization problems has been proposed to examine many-objective search behavior in a two-dimensional decision space. This idea has an inherent limitation: the number of decision variables should be two. In our former study, we formulated a four-objective distance minimization problem with 10, 100, and 1000 decision variables. In this paper, we generalize our former study to many-objective problems with an arbitrary number of objectives and decision variables by proposing an idea of specifying reference points on a plane in a high-dimensional decision space. As test problems for computational experiments, we generate six-objective and eight-objective problems with 10, 100, and 1000 decision variables. Our experimental results on those test problems show that the number of decision variables has large effects on multiobjective search in comparison with the choice of an EMO algorithm and the number of objectives.**

## I. Introduction

A number of evolutionary algorithms have been proposed for solving multiobjective optimization problems [1]-[4]. Those algorithms are often called evolutionary multiobjective optimization (EMO) algorithms. In the design of an efficient EMO algorithm, it is important to strike a balance between convergence and diversity. Especially for many-objective problems with four or more objectives, the realization of a good convergence-diversity balance is very important because Pareto dominance-based selection pressure is very weak [5]-[8]. A variety of convergence improvement methods have been proposed for many-objective optimization [9]. However, convergence improvement often decreases the diversity of solutions due to convergence-diversity tradeoff in the search for many-objective optimization [10]. Thus it is important to understand the search behavior of each EMO algorithm when we try to improve its search ability.

For the visual examination of the search behavior of EMO algorithms, the use of distance minimization problems in a two-dimensional decision space has been proposed [11]-[15]. Distance minimization in a high-dimensional space was also discussed for performance evaluation of EMO algorithms in [16]-[18]. In a distance minimization problem, each objective is defined by the distance between a solution and a reference point in the decision space. Thus the number of objectives is the same as the number of reference points. This means that we can easily specify an arbitrary number of objectives.

By specifying reference points on a two-dimensional plane, we can visually observe the search behavior of each EMO algorithm in the decision space. Fig. 1 shows an example of a four-objective problem with four reference points (red circles) and 100 solutions at the 100th generation in a single run of NSGA-II [19]. We can visually examine the distribution of solutions in the decision space. Fig. 2 shows 100 solutions at the 10th, 20th, and 100th generations in a single run of MOEA/D [20] on the same test problem. In this manner, we can visually monitor the search behavior of EMO algorithms in the two-dimensional decision space.

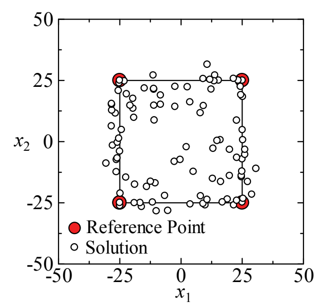


Fig. 1. A four-objective distance minimization problem and solutions at the 100th generation of NSGA-II.

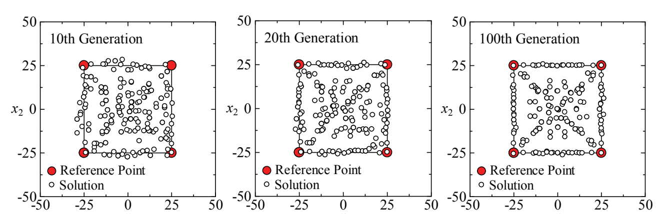


Fig. 2. Solutions at the 10th, 20th, and 100th generations of MOEA/D.

The visual examination in the decision space in Fig. 1 and Fig. 2 has an inherent limitation: the number of decision variables is two. It is easy to generate a many-objective distance minimization problem in a high-dimensional decision space. However, the use of a high-dimensional decision space makes the visual examination very difficult. In our former study [18], we generated four-objective distance minimization problems with 10, 100, and 1000 variables.

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Four reference points A, B, C, and D were specified so that the relation $\boldsymbol{v}_{AB} + \boldsymbol{v}_{AD} = \boldsymbol{v}_{AC}$ held among the three vectors $\boldsymbol{v}_{AB}$, $\boldsymbol{v}_{AC}$, and $\boldsymbol{v}_{AD}$. Under this relation, the four reference points are on the same plane. All Pareto-optimal points P can be represented by the following formulation: $\boldsymbol{v}_{AP} = w_1\boldsymbol{v}_{AB} + w_2\boldsymbol{v}_{AD}$ in the decision space where $w_1$ and $w_2$ are weights in the closed interval [0, 1].

In this paper, we generalize our former study [18] by proposing a simple specification method of an arbitrary number of reference points on a plane in a high-dimensional decision space. Our idea in this paper is to transform a two-dimensional decision space with a number of reference points to a plane in a high-dimensional decision space. In this manner, we can easily generate a distance minimization problem with an arbitrary number of objectives and decision variables. Using the generated test problem, we can visually examine the behavior of EMO algorithms.

This paper is organized as follows. In Section II, we explain many-objective distance minimization problems with two decision variables. In Section III, we propose an idea of generating many-objective distance minimization problems with an arbitrary number of decision variables. We show experimental results on six-objective and eight-objective test problems with 10, 100, and 1000 decision variables in Section IV. This paper is concluded in Section V.

## II. Two-Variable Distance Minimization Problems

An $m$-objective distance minimization problem with two decision variables can be generated by specifying $m$ points in a two-dimensional decision space. Let us denote those $m$ points by $A_i$, $i = 1, 2, ..., m$. The $i$th objective is the Euclidean distance from the $i$th reference point $A_i$ to a solution $\boldsymbol{x}$, which is a point in the decision space. For example, a six-objective problem with a regular hexagonal Pareto-optimal region is shown in Fig. 3, which is defined by the following reference points for $m = 6$:

$$A_i : \begin{pmatrix} \cos(2\pi(i-1)/m) & -\sin(2\pi(i-1)/m) \\ \sin(2\pi(i-1)/m) & \cos(2\pi(i-1)/m) \end{pmatrix} \begin{pmatrix} 0 \\ r \end{pmatrix} + \boldsymbol{c}, \quad i = 1, 2, \ldots, m, \tag{1}$$

where $A_i$ is a reference point corresponding to a vertex of the hexagon, $m$ is the number of vertices ($m = 6$ in Fig. 3), $r$ is the radius of the hexagon ($r = 25$ in Fig. 3), and $\boldsymbol{c}$ is the position vector of the center of the hexagon ($\boldsymbol{c} \in \Re^2$; $\boldsymbol{c} = \boldsymbol{0}$ in Fig. 3).

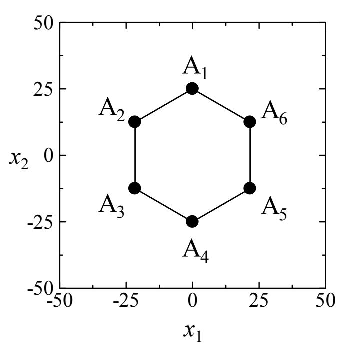


Fig. 3. A hexagon obtained by (1) with $m = 6$, $r = 25$, and $\boldsymbol{c} = \boldsymbol{0}$.

## III. Many-Variable Distance Minimization Problems

An $m$-objective distance minimization problem with $n$ decision variables can be generated by specifying $m$ reference points in an $n$-dimensional decision space [18]. For the sake of the visual examination of many-objective search behavior, we propose an idea of specifying reference points on a plane in an $n$-dimensional decision space. In this section, we explain how to specify those reference points in detail.

### A. Transformation of the Plane

Our idea is to transform a two-dimensional decision space to a plane in an $n$-dimensional decision space. We specify the plane by a vector space with a basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ where $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are independent $n$-dimensional vectors. That is, the plane is the vector space spanned by $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

A point $\boldsymbol{a} = (a_1, a_2)^T$ in a two-dimensional decision space is represented by

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_2\begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{2}$$

We map this point to a point $\boldsymbol{b} = (b_1, b_2, ..., b_n)^T$ on the plane (i.e., the vector space with the basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$) in the $n$-dimensional decision space as follows:

$$\boldsymbol{b} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 \,. \tag{3}$$

It is clear from (3) that the point $\boldsymbol{b}$ is on the plane spanned by $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. When multiple points in the two-dimensional decision space are mapped by (3), all points are on the same plane after the mapping. Geometric relations among those points in the two-dimensional decision space are preserved on the plane in the $n$-dimensional decision space when $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are orthogonal and their length is the same.

It is also clear from (3) that the origin $(0, 0, ..., 0)^T$ of the $n$-dimensional decision space is always on the plane. That is, the plane spanned by $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ always passes through the origin of the $n$-dimensional decision space. To move the plane away from the origin, we modify the mapping in (3) as

$$\boldsymbol{b} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \boldsymbol{t}\,, \tag{4}$$

where $\boldsymbol{t}$ is an $n$-dimensional vector to move the plane away from the origin. The vector $\boldsymbol{t}$ can be also viewed as a point in the $n$-dimensional decision space to which the origin of the two-dimensional decision space is mapped by (4). By specifying the three vectors $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, and $\boldsymbol{t}$ in (4), we can generate an $m$-objective distance minimization problem with $n$ decision variables from a two-dimensional $m$-objective problem.

For example, let us assume that $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, and $\boldsymbol{t}$ are specified as $\boldsymbol{v}_1 = (1, 0, 1, 0)^T$, $\boldsymbol{v}_2 = (0, 1, 0, 1)^T$, and $\boldsymbol{t} = (0, 0, 0, 0)^T$. In this case, a four-dimensional problem is generated by mapping reference points in the two-dimensional decision space. A reference point $\boldsymbol{a} = (a_1, a_2)^T$ is mapped to $\boldsymbol{b} = (a_1, a_2, a_1, a_2)^T$.

### B. Visual Examination in the Decision Space

One very simple method for visualizing the $n$-dimensional decision space is to project it into a two-dimensional plane using two decision variables. However, it is difficult to examine the distribution of solutions in the entire decision space since such a simple visualization uses only two decision variables. Moreover, the Pareto-optimal region is not always shown nicely for all combinations of two decision variables. For example, let us assume that reference points are in the form of $\boldsymbol{b} = (a_1, a_2, a_1, a_2)^{\mathrm{T}}$ as in the above-mentioned example. In this case, they are projected to $(a_1, a_1)^{\mathrm{T}}$ if we use the first and third decision variables. That is, all the projected reference points are on the same line. This means that the Pareto-optimal region is projected to a line even when it has some area on a plane in the four-dimensional decision space.

Our idea is to project all solutions to the plane spanned by the basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$, which is used to generate an $n$-dimensional $m$-objective distance minimization problem. Let us denote a solution in the $n$-dimensional decision space by $\boldsymbol{x}$, which is an $n$-dimensional vector. We project $\boldsymbol{x}$ to the plane spanned by $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. The coordinates of the projected $\boldsymbol{x}$ on the plane are calculated with respect to the basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ as $\boldsymbol{x} \cdot \boldsymbol{v}_1 / \|\boldsymbol{v}_1\|^2$ and $\boldsymbol{x} \cdot \boldsymbol{v}_2 / \|\boldsymbol{v}_2\|^2$ using the inner product operation. By using the calculated coordinates as the coordinates with respect to the standard two-dimensional basis $\{(1\ \ 0)^{\mathrm{T}}, (0\ \ 1)^{\mathrm{T}}\}$, the $n$-dimensional solution $\boldsymbol{x}$ is mapped to the following two-dimensional vector $\boldsymbol{y}$ for visualization:

$$\boldsymbol{y} = \frac{\boldsymbol{x} \cdot \boldsymbol{v}_1}{\|\boldsymbol{v}_1\|^2}\begin{pmatrix}1\\0\end{pmatrix} + \frac{\boldsymbol{x} \cdot \boldsymbol{v}_2}{\|\boldsymbol{v}_2\|^2}\begin{pmatrix}0\\1\end{pmatrix}. \tag{5}$$

### C. Some Extensions

Our $m$-objective distance minimization problem with $n$ decision variables can be further generalized in several ways. One idea is to formulate $n$-dimensional test problems with multiple equivalent Pareto-optimal regions in the decision space in the same manner as in the case of two-dimensional test problems [13]-[15]. An example of those test problems is shown in Fig. 4 where four sets of six reference points are specified from (1) with $m = 6$, $r = 10$, and different settings of $\boldsymbol{c}$: $\mathrm{A}_i$ from $\boldsymbol{c} = (30, 30)^{\mathrm{T}}$, $\mathrm{B}_i$ from $\boldsymbol{c} = (30, -30)^{\mathrm{T}}$, $\mathrm{C}_i$ from $\boldsymbol{c} = (-30, -30)^{\mathrm{T}}$, and $\mathrm{D}_i$ from $\boldsymbol{c} = (-30, 30)^{\mathrm{T}}$. The $i$th objective is the distance from $\boldsymbol{x}$ to the nearest point among $\mathrm{A}_i$, $\mathrm{B}_i$, $\mathrm{C}_i$, and $\mathrm{D}_i$. All points in each hexagon are Pareto-optimal. That is, this problem has the four equivalent Pareto-optimal regions. By transforming the two-dimensional decision space into a plane in an $n$-dimensional decision space, we can generate an $n$-dimensional six-objective test problem with the four hexagonal equivalent Pareto-optimal regions.

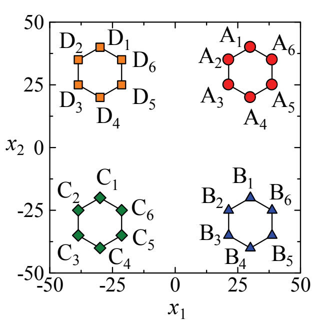


Fig. 4. Four equivalent Pareto-optimal regions on a single plane.

It is also possible to generate equivalent Pareto-optimal regions on different planes in an $n$-dimensional decision space. This can be done by using a different basis for the mapping of a different group of reference points. Let us explain this idea for the test problem with the four sets of six reference points in Fig. 4. We use a different basis for a different group of six reference points. For example, those bases can be as follows for a 100-dimensional decision space (all of them are constructed by repeating a sequence of four values):

$\mathrm{A}_i$: $\{(1, 0, 1, 0, ..., 1, 0, 1, 0)^{\mathrm{T}}, (0, 1, 0, 1, ..., 0, 1, 0, 1)^{\mathrm{T}}\}$
$\mathrm{B}_i$: $\{(1, 0, 0, 1, ..., 1, 0, 0, 1)^{\mathrm{T}}, (0, 1, 1, 0, ..., 0, 1, 1, 0)^{\mathrm{T}}\}$
$\mathrm{C}_i$: $\{(1, 0, -1, 0, ..., 1, 0, -1, 0)^{\mathrm{T}}, (0, 1, 0, -1, ..., 0, 1, 0, -1)^{\mathrm{T}}\}$
$\mathrm{D}_i$: $\{(1, 0, 0, -1, ..., 1, 0, 0, -1)^{\mathrm{T}}, (0, 1, -1, 0, ..., 0, 1, -1, 0)^{\mathrm{T}}\}$

Each group of six reference points in Fig. 4 is mapped to a different plane in the 100-dimensional decision space. Those points are shown in Fig. 5 by projecting them to each plane.

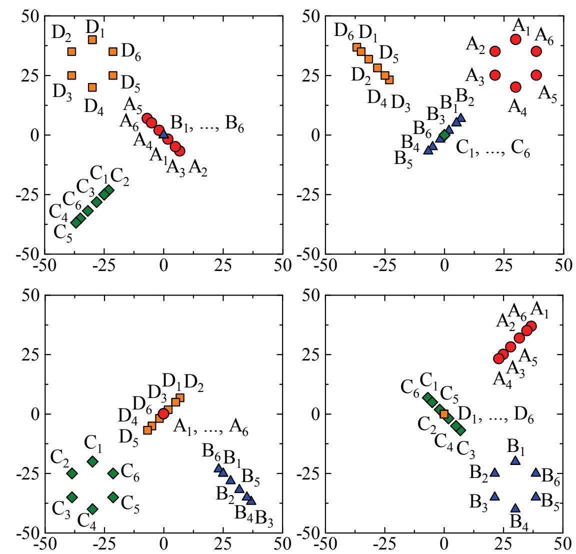


Fig. 5. Four equivalent Pareto-optimal regions on the four different planes.

When we formulate a test problem with multiple equivalent Pareto-optimal regions, the distance between reference points in the same group should be small in comparison with the distance between different groups. For example, if the four groups of reference points in Fig. 4 are very close to each other, Pareto-optimal regions can be not only inside each hexagon but also between different hexagons.

## IV. Computational Experiments

In this section, we show experimental results of NSGA-II [19] and MOEA/D [20] on six-objective and eight-objective distance minimization problems. Each test problem has 10, 100, or 1000 decision variables. First we generated those test problems using the proposed method in Section III. More specifically, we generated three six-objective test problems with 10, 100, and 1000 decision variables from the test problem in Fig. 3. The six reference points were mapped to high-dimensional decision spaces using the basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\} = \{(1, 0, 1, 0, ..., 1, 0)^{\mathrm{T}}, (0, 1, 0, 1, ..., 0, 1)^{\mathrm{T}}\}$. For generating eight-objective test problems, we first generated a test problem in a two-dimensional decision space by specifying the eight reference points by (1) with $m = 8$, $r = 25$, and $\boldsymbol{c} = \mathbf{0}$. Then those reference points were mapped to high-dimensional decision spaces using the same basis as in the case of the six-objective test problems.

Our computational experiments were performed under the following settings:

The range of the decision space: $[-50, 50]^n$,
The number of variables: $n = 10$, 100, and 1000,
Initial solutions: Random real vectors in $[-50, 50]^n$,
Crossover probability: 1.0 (SBX with $\eta_c = 15$) ,
Mutation probability: 0.5 (Polynomial mutation $\eta_m = 20$).

The population size in NSGA-II was specified as 100 for the six-objective and eight-objective test problems. Due to the combinatorial nature of the number of uniform weight vectors, the population size in MOEA/D was specified as 126 for the six-objective test problems and 120 for the eight-objective test problems. MOEA/D with the weighted Tchebycheff function was implemented with no archive population. A reference point for the Tchebycheff function calculation was updated after the evaluation of each solution using the minimum value of each objective. The neighborhood size was specified as 10% of the population size (i.e., 13 for the six-objective problems and 12 for the eight-objective problems).

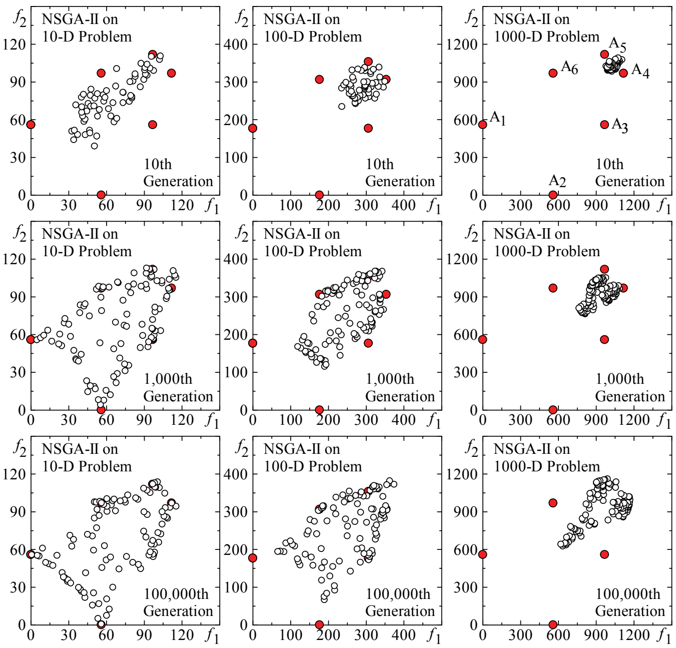


Fig. 6. NSGA-II on the six-objective problems ($f_1$-$f_2$ Space).

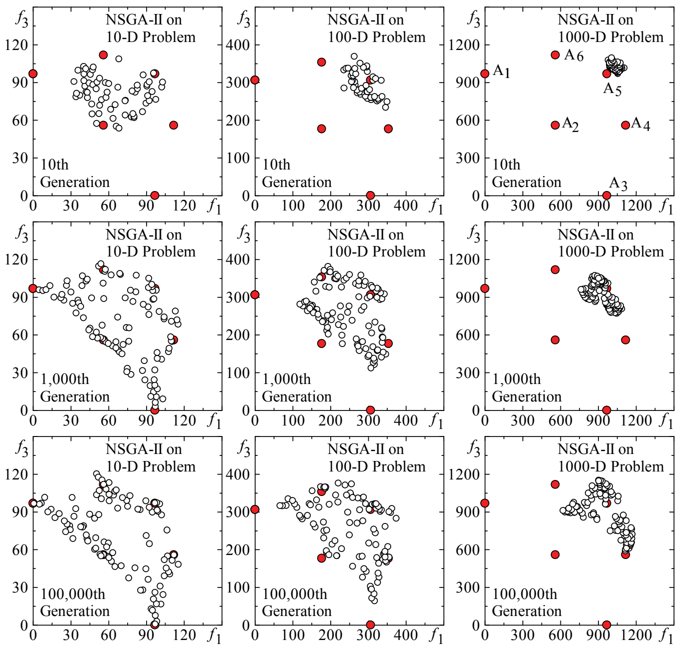


Fig. 8. NSGA-II on the six-objective problems ($f_1$-$f_3$ Space).

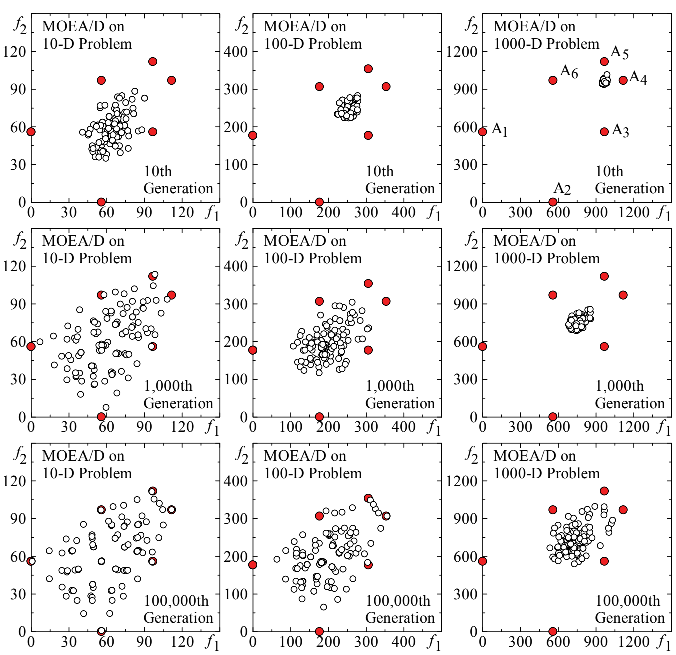


Fig. 7. MOEA/D on the six-objective problems ($f_1$-$f_2$ Space).

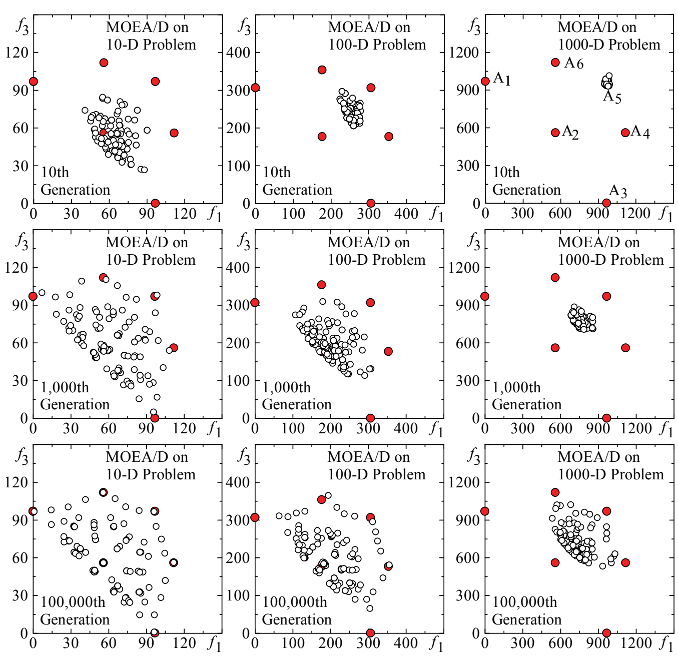


Fig. 9. MOEA/D on the six-objective problems ($f_1$-$f_3$ Space).

Figs. 6, 8, and 10 show all solutions in the objective space at the 10th, 1000th, and 100000th generations of NSGA-II on the three six-objective test problems. Experimental results by MOEA/D are shown in Figs. 7, 9, and 11. The six points $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, and $A_6$ are also projected to each plot as the red points. Some of the projections (e.g., $f_2$-$f_3$ space) are omitted because similar results are visually observed from different projections with the same shape of the projected six points. From Figs. 6-11, we can see that the increase in the number of decision variables has a severe negative effect on the diversity of solutions in NSGA-II and MOEA/D. This observation is consistent with our previous study [18].

The same experimental results are shown in the decision space in Fig. 12 and Fig. 13 using our visualization method in Subsection III.B. From the results in the objective space and the decision space, we can obtain similar observations for the diversity of solutions. For the distribution of solutions, we may be able to obtain more detailed understanding from the results in the decision space (e.g., see the bottom-left plot in Fig. 13). However, with respect to the convergence of solutions to the Pareto front, the results in the decision space can be misleading. For example, the top-right plot in Fig. 12 seems to show that all solutions are in the Pareto-optimal region. This is not the case in Fig. 10.

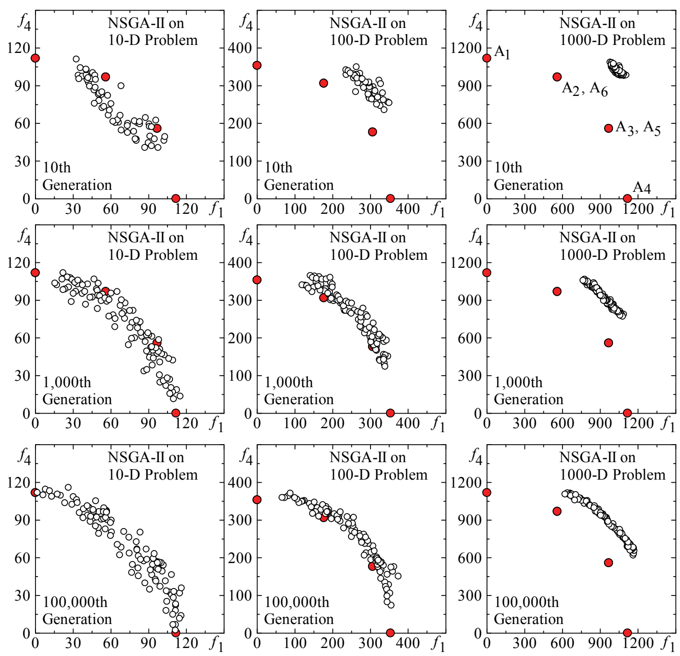

Fig. 10. NSGA-II on the six-objective problems ($f_1$-$f_4$ Space).

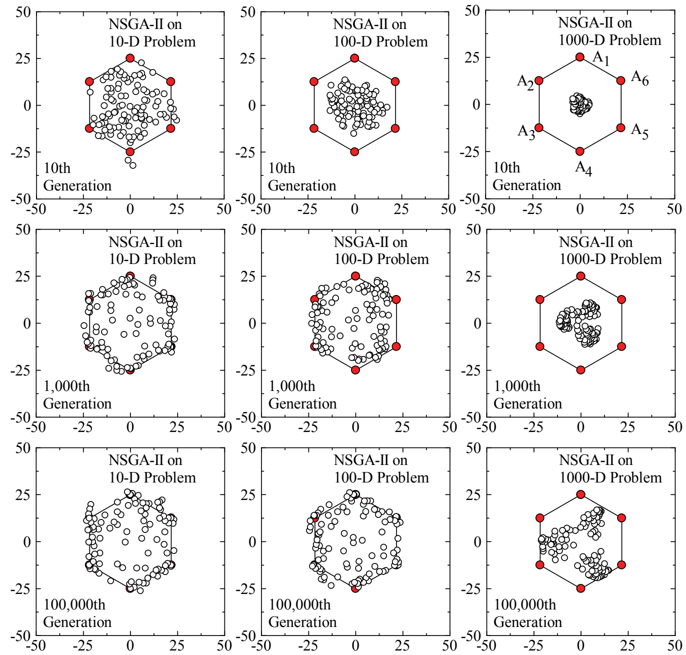

Fig. 12. NSGA-II on the six-objective problems (Decision Space).

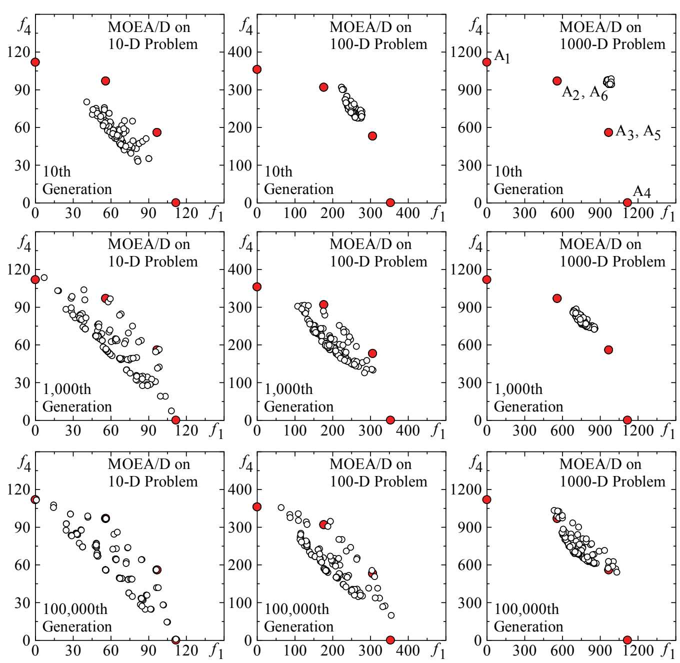

Fig. 11. MOEA/D on the six-objective problems ($f_1$-$f_4$ Space).

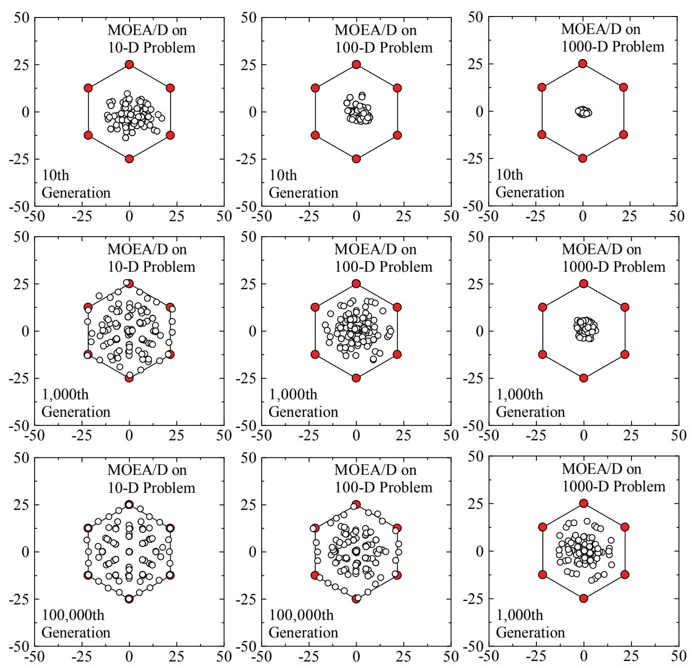

Fig. 13. MOEA/D on the six-objective problems (Decision space).

In the same manner as in Figs. 6-11 for the six-objective test problems, experimental results for the eight-objective test problems are shown in the objective space in Figs. 14-21 for NSGA-II and MOEA/D. The eight reference points in each test problem are shown by red circles in each plot. Whereas the number of objectives is increased from six in Figs. 6-11 to eight in Figs. 14-21, similar experimental results are obtained. For example, Fig. 6 and Fig. 14 are similar to each other. The diversity of solutions is severely degraded by the increase in the number of decision variables from 10 to 1000 in both Fig. 6 and Fig. 14.

In our former studies, we obtained similar observations from two-objective and four-objective distance minimization problems with 10, 100, and 1000 variable. These observations suggest that the number of decision variables has a dominant effect on the search behavior of NSGA-II and MOEA/D in our computation experiments.

We can also see that similar results were also obtained from NSGA-II and MOEA/D (e.g., in Fig. 14 and Fig. 15). This observation suggests that the number of decision variables has a larger effect than the choice of the EMO algorithm in our computational experiments.

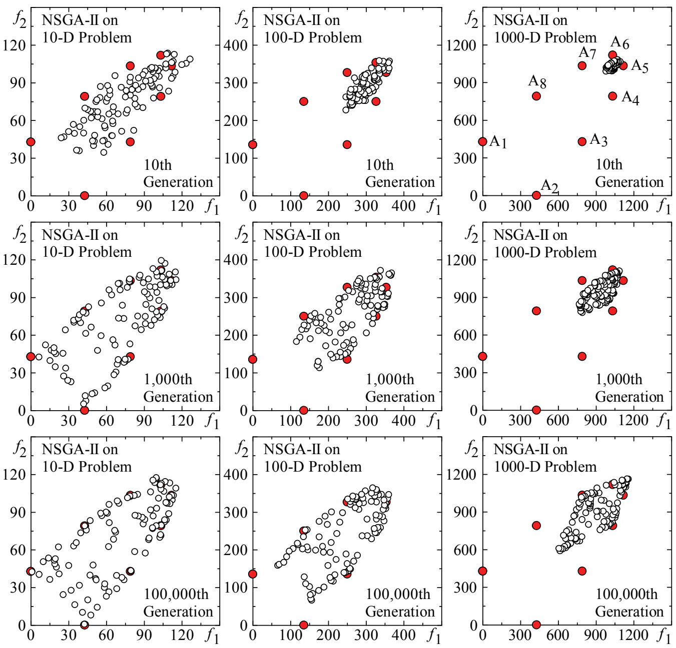


Fig. 14. NSGA-II on the eight-objective problems ($f_1$-$f_2$ Space).

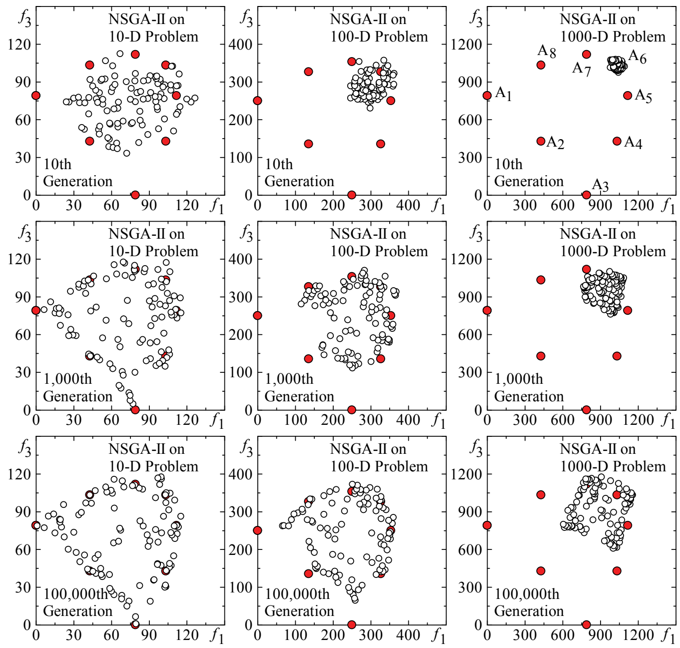


Fig. 16. NSGA-II on the eight-objective problems ($f_1$-$f_3$ Space).

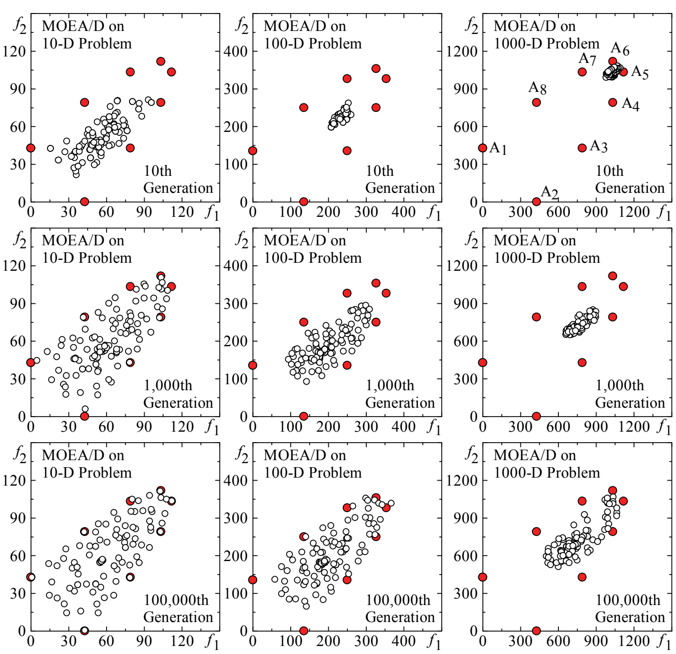


Fig. 15. MOEA/D on the eight-objective problems ($f_1$-$f_2$ Space).

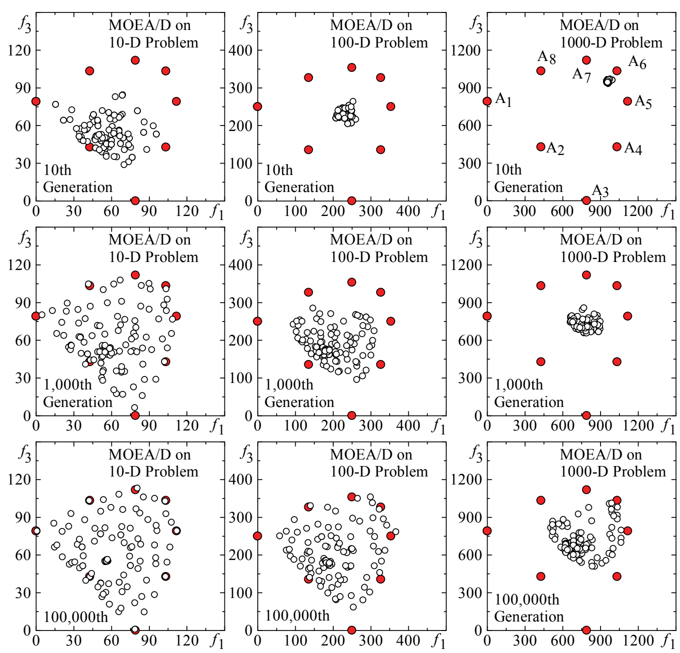


Fig. 17. MOEA/D on the eight-objective problems ($f_1$-$f_3$ Space).

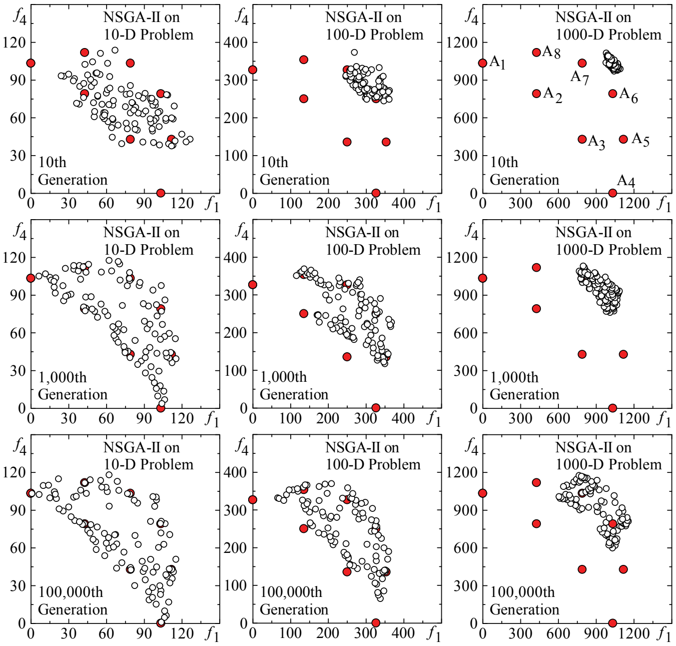


Fig. 18. NSGA-II on the eight-objective problems ($f_1$-$f_4$ Space).

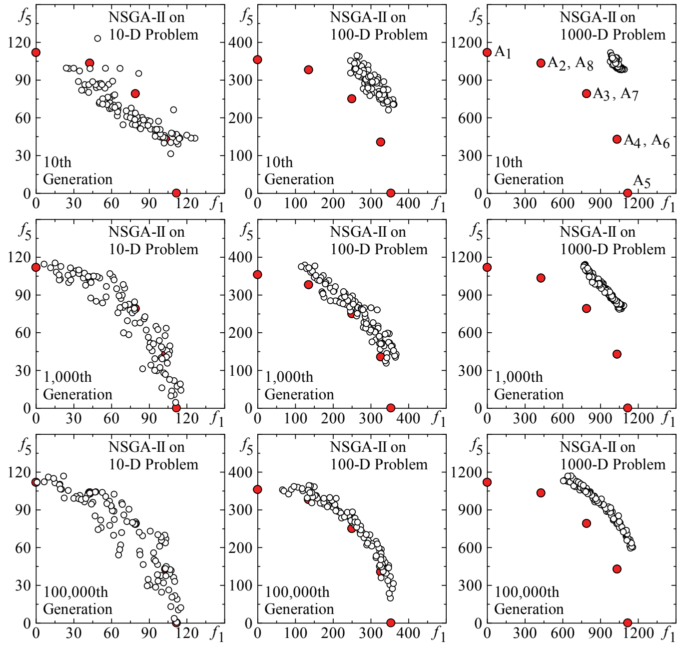


Fig. 20. NSGA-II on the eight-objective problems ($f_1$-$f_5$ Space).

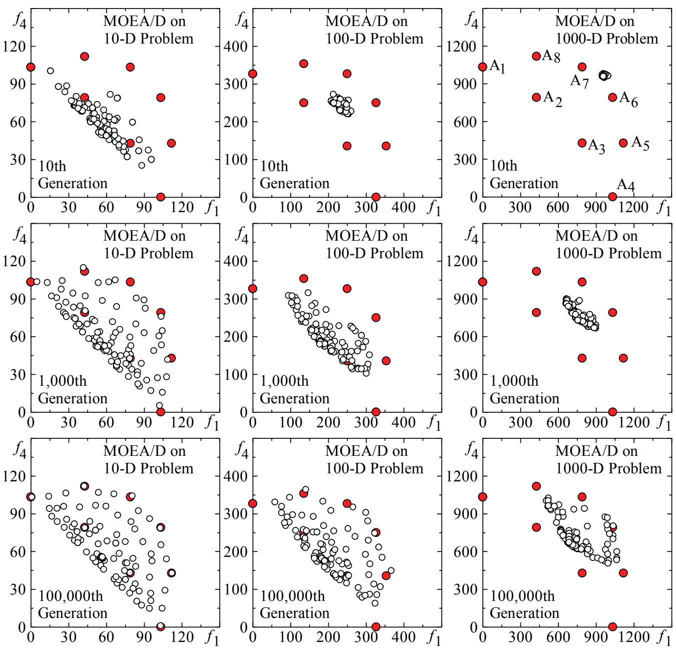


Fig. 19. MOEA/D on the four-objective problems ($f_1$-$f_4$ Space).

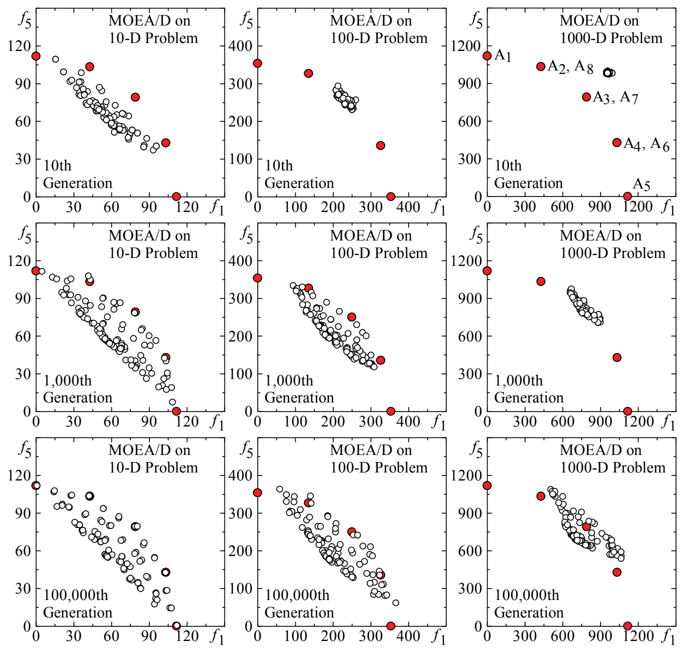


Fig. 21. MOEA/D on eight-objective problems ($f_1$-$f_5$ Space).

Our experimental results on the eight-objective problems are also shown in Fig. 22 and Fig. 23 in the decision space using the proposed method. The distribution of solutions can be further examined in Fig. 22 and Fig. 23. Comparison between Fig. 12 and Fig. 22 (Fig. 13 and Fig. 23) shows that similar distributions of solutions are obtained for our test problems with six and eight objectives. Fig. 22 and Fig. 23 also show that the number of decision variables has a dominant effect on the behavior of NSGA-II and MOEA/D. Of course, if we use test problems with a huge number of objectives (e.g., 100 and 1000 objectives), we may observe that the number of objectives has a dominant effect.

## V. CONCLUSIONS

We proposed an idea of generating many-objective and many-variable distance minimization problems. We also proposed a visual examination method of the search behavior of EMO algorithms on the generated distance minimization problems. The point of the proposed two methods was the use of a plane in a high-dimensional decision space. Such a plane was defined by two independent vectors (i.e., a basis of a vector space). Reference points in a two-dimensional distance minimization problem were mapped to those on a plane in a high-dimensional decision space. Solutions at each generation were observed by projecting them to the plane.

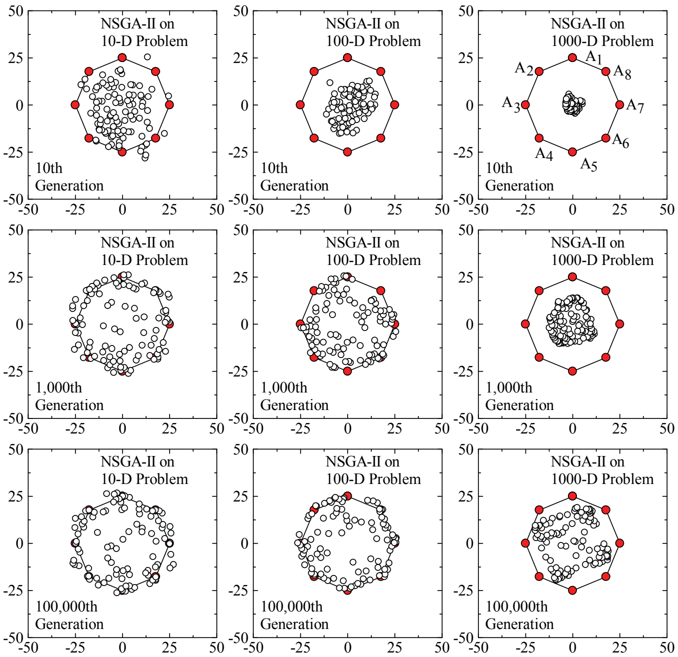


Fig. 22. NSGA-II on the eight-objective problems (Decision Space).

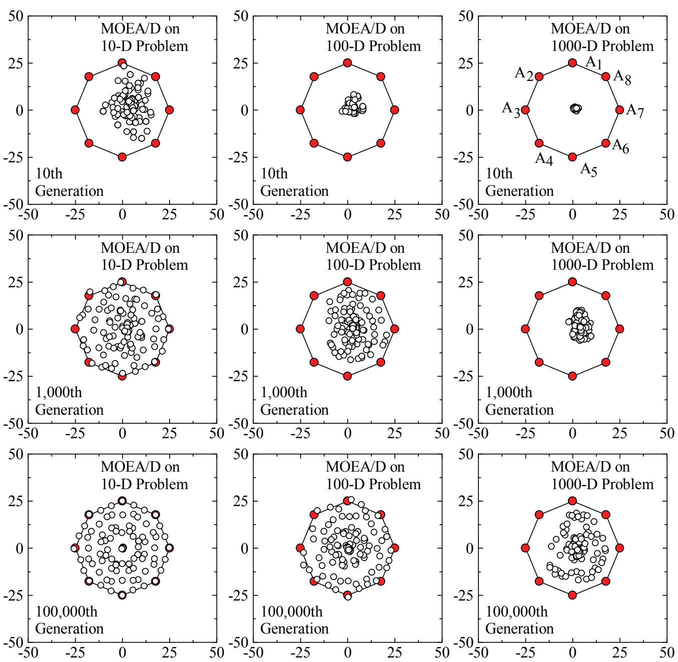


Fig. 23. MOEA/D on the eight-objective problems (Decision Space).

Through computational experiments on the six-objective and eight-objective distance minimization problems, we showed that the number of decision variables had a dominant effect on the search behavior of EMO algorithms. It was also shown that the distribution of solutions can be examined in more detail in the decision space than the objective space.

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