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Effects of Ensemble Action Selection on the Evolution of Iterated Prisoner's Dilemma Game Strategies

Takahiko Sudo, Yusuke Nojima, and Hisao Ishibuchi

Abstract—**Iterated prisoner's dilemma (IPD) games have been frequently used for examining the evolution of cooperative game strategies. It has been pointed out in some studies that the choice of a representation scheme (i.e., coding mechanism) has a large effect on the evolution. A choice of a different representation scheme often leads to totally different results. In those studies on IPD games, a single representation scheme is assigned to all players. That is, all players have the same representation scheme. In our former studies, we reported experimental results in an inhomogeneous setting where a different representation scheme was assigned to each player. The evolution of cooperation among different types of game strategies was examined. In this paper, we report experimental results in another interesting setting where each player is assumed to have multiple strategies with different representation schemes. The next action of each player is determined by a majority vote by its strategies. That is, each player is assumed to have an ensemble decision making system. Experimental results in such an ensemble IPD model are compared with those in the standard IPD model where each player has a single strategy.**

I. INTRODUCTION

THE prisoner's dilemma (PD) is a well-known non-zero sum game. Its iterated version (IPD: iterated prisoner's dilemma) has been frequently used to examine the evolution of cooperation among independent players since the 1980s [1]-[3]. It has been demonstrated that various factors are related to the evolution of cooperative IPD game strategies.

One important factor is the choice of a representation scheme (i.e., coding mechanism). Ashlock et al. [4] examined a variety of representation schemes such as binary strings, neural networks, decision trees and finite state machines. It was demonstrated that totally different results were obtained from each representation scheme. In their experiments [4], a single representation scheme was selected and assigned to all players in each run as in many studies on IPD games [1]-[3]. That is, all players had the same representation scheme. This homogeneous setting was extended to an inhomogeneous setting in our former studies [5]-[7] where a different representation scheme was assigned to each player.

In this paper, we examine another interesting setting where each player is assumed to have multiple IPD game strategies with different representation schemes. We further assume that each player decides its next action by a majority vote by its strategies. In our computational experiments, we use three types of binary strings with different string length and memory length as representation schemes: 3-bit binary strings with memory length 1, 7-bit binary strings with memory length 2, and 15-bit binary strings with memory length 3. Each player can have a 3-bit string, a 7-bit string, and a 15-bit string. The next action is decided by a majority vote by the strings. In a noise-free version of our simulations, the result of such an ensemble decision making is always used as the actual action. In a noisy version, the result of the majority vote is changed with a pre-specified probability. The aim of this paper is to examine the potential benefit of having multiple strategies for the evolution of cooperative IPD game strategies.

Another important factor is a spatial structure of players [8]-[11]. The use of a different structure leads to different results with respect to the evolution of cooperative IPD game strategies. Recently, network structures have been actively studied as spatial models in evolutionary computation [12]-[15]. In this paper, we use a ring graph with additional edges as a spatial structure. Each player is assigned to a node of the graph. The neighbors of each player are defined by edges from its node. In this spatial model, the number of additional edges is an important parameter since it defines the neighborhood size. These edges are randomly added to a ring graph in this paper.

This paper is organized as follows. We first explain our ensemble IPD in Section II. Next we explain our spatial genetic algorithm for the evolution of IPD game strategies in Section III. Then we show experimental results in Section IV. Finally, we conclude this paper in Section V.

II. IPD GAME WITH OUR ENSEMBLE MODEL

A. Ensemble Action Selection in the IPD Game

The prisoner's dilemma (PD) is a two-player non-zero sum game. The player and the opponent choose either cooperation ("C") or defection ("D"), simultaneously. We use a standard payoff matrix in Table I. The player receives the largest payoff 5 when the player defects and the opponent cooperates. This payoff is larger than the payoff 3 from the mutual cooperation (i.e., the payoff 3 when both the player and the opponent cooperate). The player receives the smallest payoff 0 when the player cooperates and the opponent defects. This payoff is smaller than the payoff 1 from the mutual defection.

Takahiko Sudo, Yusuke Nojima, and Hisao Ishibuchi are with Osaka Prefecture University, Japan (phone: +81-72-254-9198, fax: +81-72-254-9915, e-mail: {takahiko.sudo@ci., nojima@, hisaoi@}cs.osakafu-u.ac.jp).

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From Table I, it is clear that the defection is a better action for the player independent of the action of the opponent. That is, the player always obtains a higher payoff by choosing D than C in Table I. This is also the case for the opponent. However, their rational actions lead to the payoff 1 from the mutual defection, which is smaller than the payoff 3 from the mutual cooperation. This is the dilemma in the PD game.

In an iterated version of the PD game (i.e., IPD: iterated prisoner's dilemma), the PD game is iterated between the same pair of the player and the opponent. Each of them continues to play the IPD game to maximize its own total payoff over a number of rounds of the PD game.

Each player usually has a single strategy in many studies on the IPD game. However, in our ensemble IPD game model, each player has three strategies. This setting can be easily generalized to the case of an ensemble decision making based on more strategies. Table II illustrates how the final action is decided by a majority vote by three strategies. For example, when the suggested actions by the three strategies are "C", "D" and "C", the result of their majority vote is "C".

TABLE I
PAYOFF MATRIX OF OUR IPD GAME

Player's Action	Opponent's Action	
	C: Cooperation	D: Defection
C: Cooperation	Player Payoff: 3 Opponent Payoff: 3	Player Payoff: 0 Opponent Payoff: 5
D: Defection	Player Payoff: 5 Opponent Payoff: 0	Player Payoff: 1 Opponent Payoff: 1

TABLE II
ILLUSTRATION OF ENSEMBLE ACTION SELECTION

Player's action	D	D	D	C	D	C	C	C
Strategy 1	D	D	D	D	C	C	C	C
Strategy 2	D	D	C	C	D	D	C	C
Strategy 3	D	C	D	C	D	C	D	C

B. *Binary String Strategies*

We use 3-bit, 7-bit and 15-bit binary strings to represent player's strategies. In those binary strings, "0" and "1" mean "D" and "C", respectively. Table III shows how a player with a 3-bit binary string "$x_1x_2x_3$" plays the IPD game. The first bit x_1 determines the player's action in the first round. The other bits x_2 and x_3 determine the player's actions in the subsequent rounds. When the opponent's action is "D" ("C") in the (t−1)th round, x_2 (x_3) is used to determine the player's action in the t-th round. For example, the 3-bit binary string "101" chooses "C" in the first round, and chooses the same action as the opponent's action in the previous round. As shown in Table III, 3-bit strategies have a memory of length 1, which contains the opponent's action in the previous round.

Table IV explains the action selection by a 7-bit binary string "$x_1x_2x_3x_4x_5x_6x_7$". The first three bits "$x_1x_2x_3$" are used in the same manner as in Table III in the first two rounds. The other four bits "$x_4x_5x_6x_7$" are used to determine the player's actions in the subsequent rounds. As shown in Table IV, 7-bit strategies have a memory of length 2, which contains the opponent's actions in the previous two rounds. Table V shows the action selection by a 15-bit string. As shown in Table V, 15-bit strategies have a memory of length 3, which contains the opponent's actions in the previous three rounds.

In this paper, the PD game is iterated for 100 rounds between the same pair of players. In a noise-free setting, each player chooses the suggested action by its strategy. In a noisy setting, each player chooses a different action from the suggested one with a pre-specified error probability. In this paper, we examined the following five specifications of the error probability: 0.00, 0.01, 0.03, 0.05, and 0.10. The error probability 0.00 means a noise-free setting. Each value shows an error probability with respect to each action selection.

TABLE III
BINARY STRATEGY OF LENGTH 3

Opponent's action at the (t−1)th round	-	D	C
Player's action at the t-th round	x_1	x_2	x_3

TABLE IV
BINARY STRATEGY OF LENGTH 7

Opponent's action at the (t−2)th round	-	-	-	D	D	C	C
Opponent's action at the (t−1)th round	-	D	C	D	C	D	C
Player's action at the t-th round	x_1	x_2	x_3	x_4	x_5	x_6	x_7

TABLE V
BINARY STRATEGY OF LENGTH 15

(t−3)th round	-	-	-	-	-	-	-	D	D	D	D	C	C	C	C
(t−2)th round	-	-	-	D	D	C	C	D	D	C	C	D	D	C	C
(t−1)th round	-	D	C	D	C	D	C	D	C	D	C	D	C	D	C
Player's action	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}

III. STRATEGY EVOLUTION

In our standard IPD model with a single strategy, one of the three representation schemes (i.e., 3-bit, 7-bit and 15-bit binary strings) is chosen and assigned to all players. In our ensemble IPD model with three strategies, a combination of three representation schemes is chosen and assigned to all players. We examine ten different combinations of three representation schemes in Table VI where the length of each binary string is shown for each combination. For example, Combination B has three 7-bit strings while Combination J has a single 3-bit, a single 7-bit, and a single 15-bit string. One of those combinations is selected and assigned to all players. Initial strategies are generated by randomly assigning 0 and 1 with the same probability to each value in binary strings of the specified length.

TABLE VI
COMBINATIONS OF THREE BINARY STRINGS OF DIFFERENT LENGTH

Ensemble Model	A	B	C	D	E	F	G	H	I	J
Strategy 1	3	7	15	3	3	3	7	3	7	3
Strategy 2	3	7	15	3	3	7	7	15	15	7
Strategy 3	3	7	15	7	15	7	15	15	15	15

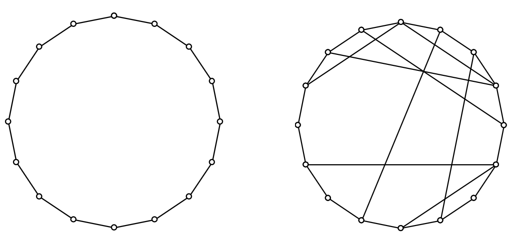
Each player has its neighbors which are specified by a graph structure. If the player i is connected to the player j by an edge, the player j is a neighbor of the player i (and vice versa). The IPD game is played by each player against all its neighbors. After the execution of the IPD game is completed for all players, the fitness of each player is calculated as the average payoff per round over all executions of the IPD game.

For each player, two neighbors (including the player itself) are selected by binary tournament selection with replacement. In our standard IPD model with a single strategy, one-point crossover is applied to the strategies of the selected two neighbors with a pre-specified crossover probability (which is 1.0 in this paper). One of the generated offspring is randomly selected, to which bit-flip mutation is applied to generate a new strategy. In our ensemble IPD model with three strategies, each neighbor has three strategies. One-point crossover is applied to the first strategies of the selected neighbors, their second strategies, and their third strategies, separately (e.g., to the first strategy of one neighbor and the first strategy of the other neighbor). One of the generated strategies is randomly selected, to which bit-flip mutation is applied to generate the first strategy in a new ensemble strategy. The same procedure is applied to the second and the third strategies, separately.

Bit-flip mutation is applied to each bit of the generated offspring with a pre-specified mutation probability. In our standard IPD model, we specify the mutation probability as $1/NL$ where N is the number of players and L is the string length. This means that only a single bit is mutated in a population of N binary strings of length L on average. In our ensemble IPD model, we specify the mutation probability separately for each of the three strategies in the same manner. After new strategies are generated for all players, their current strategies are replaced with the newly generated ones in a synchronized manner. The generation update is iterated 1000 times in this manner (i.e., for 1000 generations).

IV. Experimental Results

We use a ring graph as the basic graph structure. A number of edges are randomly added to the basic ring graph. The basic ring graph structure and the addition of random edges are illustrated for the case of 16 nodes in Fig. 1. In our computational experiments, we use a ring graph with 1024 nodes and its five modifications with 8, 64, 512, 1024, and 2048 random edges.



(a) Ring graph with 16 nodes. (b) Addition of eight edges.

Fig. 1. Illustration of a ring graph and the addition of random edges.

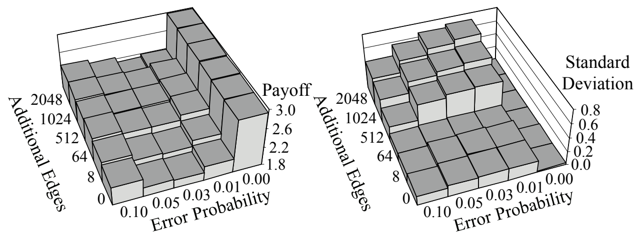
The random addition of edges is performed as follows. First, we randomly choose a node from the nodes with the smallest number of edges in the graph. Next we connect the selected node to another node, which is randomly chosen from the unconnected nodes to the selected node. This procedure is simply iterated to add a pre-specified number of random edges to the basic 1024-node ring graph.

We examine the following 13 settings in this paper:

Standard model: 3 choices of the string length (3, 7, 15),
Ensemble model: 10 combinations in Table VI.

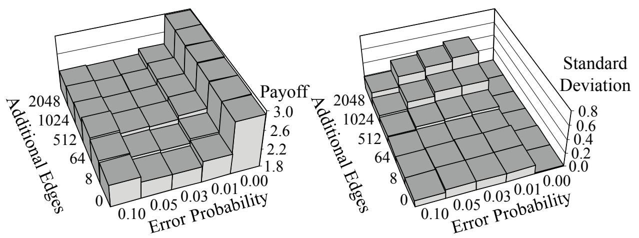
In each setting, five specifications of the error probability (0.00, 0.01, 0.03, 0.05, and 0.10) are examined for each of the six graph structures (1024-node ring graphs with 0, 8, 64, 512, 1024, and 2048 random edges). That is, we examine 5×6 cases for each of the 13 settings of the representation schemes. This means that we examine 390 different cases in total. For each case, the evolution of cooperative IPD game strategies is examined by 100 runs of our genetic algorithm. Since we use 1024-node graphs, the population size is 1024. The average payoff for each case is calculated over those 100 runs.

From Table I in Section II, we can see that the highest average payoff for the player and the opponent is 3, which is obtained from the mutual cooperation. The lowest average payoff in Table I is 1, which is obtained from the mutual defection. In the other situations where one cooperates and the other defects, the average payoff is 2.5. These average payoff values are used for interpreting experimental results. Our experimental results are summarized in Figs. 2-17.



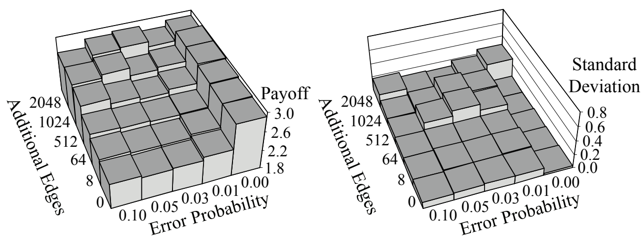
(a) Average Payoff. (b) Standard Deviation.

Fig. 2. Standard IPD model with 3-bit binary strings.



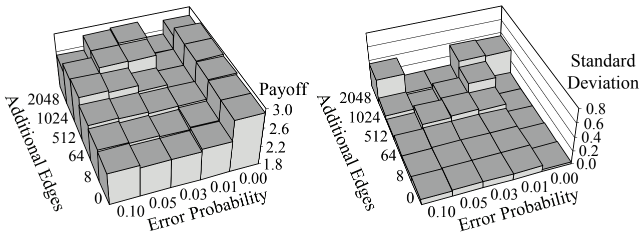
(a) Average Payoff. (b) Standard Deviation.

Fig. 3. Ensemble IPD model with (3, 3, 3)-bit binary strings.



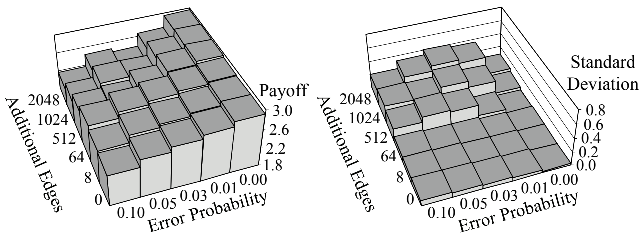
(a) Average Payoff. (b) Standard Deviation.

Fig. 4. Standard IPD model with 7-bit binary strings.



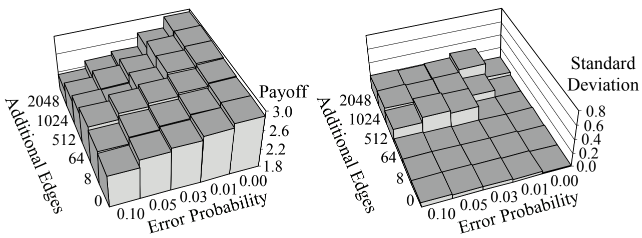
(a) Average Payoff. (b) Standard Deviation.

Fig. 5. Ensemble IPD model with (7, 7, 7)-bit binary strings.



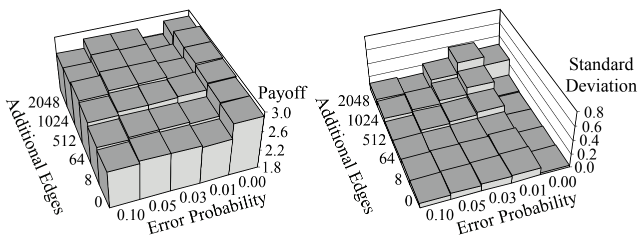
(a) Average Payoff. (b) Standard Deviation.

Fig. 6. Standard IPD model with 15-bit binary strings.



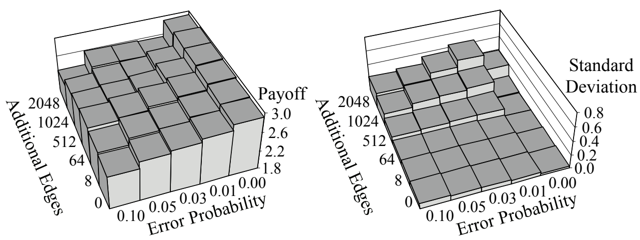
(a) Average Payoff. (b) Standard Deviation.

Fig. 7. Ensemble IPD model with (15, 15, 15)-bit binary strings.



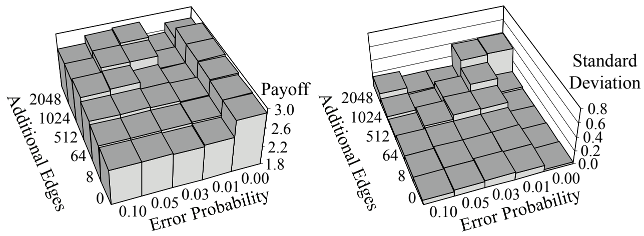
(a) Average Payoff. (b) Standard Deviation.

Fig. 8. Ensemble IPD model with (3, 3, 7)-bit binary strings.



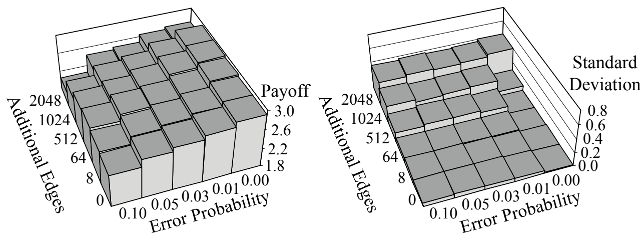
(a) Average Payoff. (b) Standard Deviation.

Fig. 9. Ensemble IPD model with (3, 3, 15)-bit binary strings.



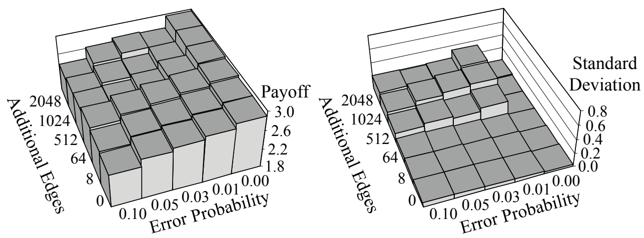
(a) Average Payoff. (b) Standard Deviation.

Fig. 10. Ensemble IPD model with (3, 7, 7)-bit binary strings.



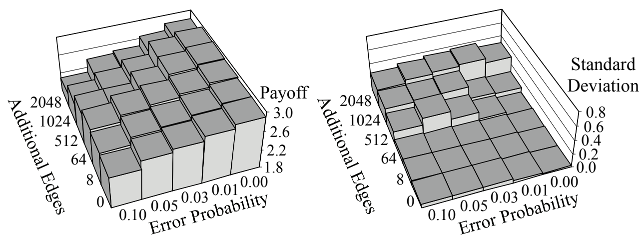
(a) Average Payoff. (b) Standard Deviation.

Fig. 11. Ensemble IPD model with (7, 7, 15)-bit binary strings.



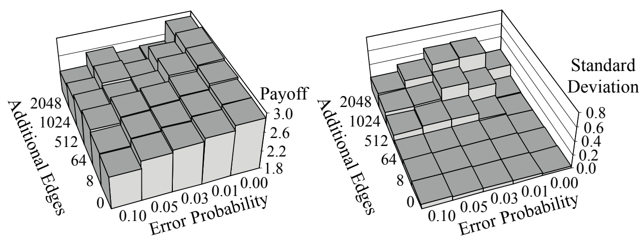
(a) Average Payoff. (b) Standard Deviation.

Fig. 12. Ensemble IPD model with (3, 15, 15)-bit binary strings.



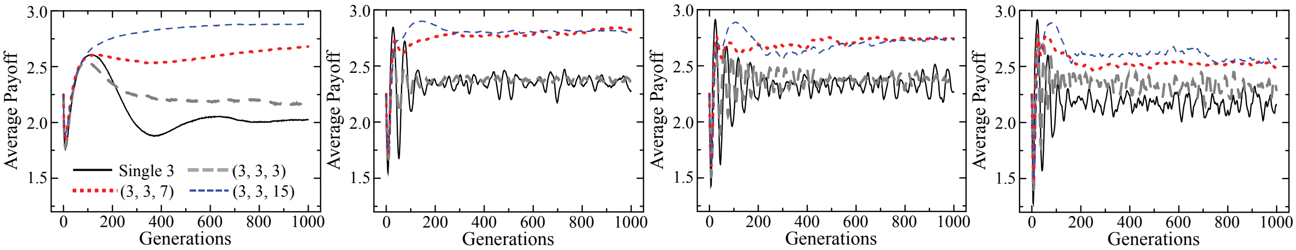
(a) Average Payoff. (b) Standard Deviation.

Fig. 13. Ensemble IPD model with (7, 15, 15)-bit binary strings.



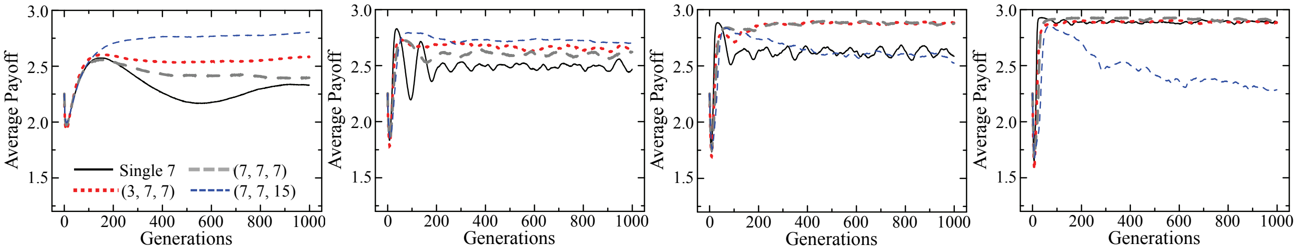
(a) Average Payoff. (b) Standard Deviation.

Fig. 14. Ensemble IPD model with (3, 7, 15)-bit binary strings.



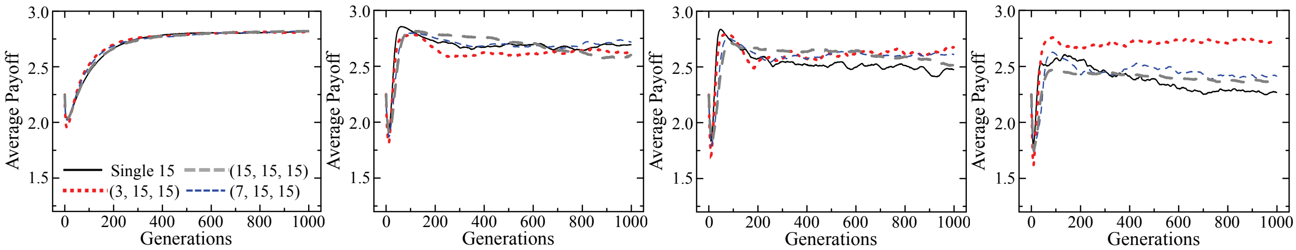
(a) With no random edge. (b) With 512 random edges. (c) With 1024 random edges. (d) With 2048 random edges.

Fig. 15. Effects of random edges on the standard IPD model with a single 3-bit binary strategy and the three ensemble IPD models (3, 3, 3), (3, 3, 7) and (3, 3, 15). Average payoff is calculated at each generation for each IPD model under the error probability 0.01. The black line shows the results by the standard IPD model with a single 3-bit binary strategy. The dashed bold gray line shows the results by the ensemble IPD model with three 3-bit binary strategies: (3, 3, 3). The dotted red line shows the results by the ensemble IPD model with two 3-bit and a single 7-bit binary strategies: (3, 3, 7). The dashed blue line shows the results by the ensemble IPD model with two 3-bit and a single 15-bit binary strategies: (3, 3, 15).



(a) With no random edge. (b) With 512 random edges. (c) With 1024 random edges. (d) With 2048 random edges.

Fig. 16. Effects of random edges on the standard IPD model with a single 7-bit binary strategy and the three ensemble IPD models (7, 7, 7), (3, 7, 7) and (7, 7, 15). Average payoff is calculated at each generation for each IPD model under the error probability 0.03. The black line shows the results by the standard IPD model with a single 7-bit binary strategy. The dashed bold gray line shows the results by the ensemble IPD model with three 7-bit binary strategies: (7, 7, 7). The dotted red line shows the results by the ensemble IPD model with a single 3-bit and two 7-bit binary strategies: (3, 7, 7). The dashed blue line shows the results by the ensemble IPD model with two 7-bit and a single 15-bit binary strategies: (7, 7, 15).



(a) With no random edge. (b) With 512 random edges. (c) With 1024 random edges. (d) With 2048 random edges.

Fig. 17. Effects of random edges on the standard IPD model with a single 15-bit binary strategy and the three ensemble IPD models (15, 15, 15), (3, 15, 15) and (7, 15, 15). Average payoff is calculated at each generation for each IPD model under the error probability 0.03. The black line shows the results by the standard IPD model with a single 15-bit binary strategy. The dashed bold gray line shows the results by the ensemble IPD model with three 15-bit binary strategies: (15, 15, 15). The dotted red line shows the results by the ensemble IPD model with a single 3-bit and two 15-bit binary strategies: (3, 15, 15). The dashed blue line shows the results by the ensemble IPD model with a single 7-bit and two 15-bit binary strategies: (7, 15, 15).

In Figs. 2-14, the standard deviation shows the variation of the average payoff among the 100 trials. First, let us compare Fig. 2 with Fig. 3. Fig. 2 uses a single 3-bit binary strategy while Fig. 3 uses three 3-bit binary strategies. In the noise-free case (i.e., when the error probability is specified as 0.00), the average payoff is close to 3.0 in all settings in Fig. 2 and Fig. 3. This means that the mutual cooperation is almost always achieved in early generations. However, high average payoff is not obtained in any settings in Fig. 2 and Fig. 3 in the noisy cases. This is because 3-bit binary strategies with memory length 1 cannot distinguish the two types of defection: One is the intentional defection based on the suggestion by the strategy of the opponent, and the other is the unintentional defection by accident due to the error probability.

From the comparison between Fig. 2 and Fig. 3, we can observe that the use of three 3-bit binary strategies in the ensemble IPD model in Fig. 3 slightly increases the average payoff and slightly decreases the standard deviation from the case of the standard IPD model in Fig. 2 with a single 3-bit binary strategy.

However, the difference between Fig. 4 with a single 7-bit strategy and Fig. 5 with three 7-bit strategies is unclear. That is, we cannot observe any clear effects of the use of three 7-bit strategies in comparison with a single 7-bit strategy. The difference between Fig. 6 with a single 15-bit strategy and Fig. 7 with three 15-bit strategies is unclear, too.

Figs. 2-7 show that there is no clear difference between the standard IPD model with a single strategy and the ensemble IPD model with three strategies of the same type. This may be consistent with well-known results of ensemble classifiers: The use of similar classifiers does not significantly improve the classification performance of each individual classifier.

In ensemble classifier design, it is often observed that an ensemble classifier of totally different classifiers has higher classification ability than each individual classifier. We have some corresponding observations in Figs. 2-14. For example, the average payoff from the ensemble IPD model (3, 3, 3) in Fig. 3 is clearly improved by replacing one of three 3-bit binary strategies with a 7-bit binary strategy in Fig. 8 with the ensemble IPD model (3, 3, 7) and a 15-bit binary strategy in Fig. 9 with the ensemble IPD model (3, 3, 15). It is interesting to observe that high average payoff is obtained even in the noisy setting from those ensemble models. A majority of strategies in those ensemble IPD models are still 3-bit binary strategies that cannot handle the noisy setting as shown in Fig. 2 and Fig. 3.

In order to further examine the effect of including a different type of a strategy (i.e., a different representation scheme) in our ensemble IPD models, we calculate the average payoff at each generation of our genetic algorithm. Experimental results on the noisy setting with the error probability 0.01 are shown in Fig. 15 for the standard IPD model with a 3-bit binary strategy and the three ensemble IPD models (3, 3, 3), (3, 3, 7), and (3, 3, 15). It is clearly shown in Fig. 15 that higher average payoff is obtained from the ensemble IPD models (3, 3, 7) and (3, 3, 15) than (3, 3, 3) and the standard model with a single 3-bit strategy.

Similar observations are obtained from Fig. 16 (a) and Fig. 17 (d). In those plots, higher average payoff is obtained from the ensemble IPD models with two different representation schemes than the models with a single representation scheme. For example, higher average payoff is obtained from the ensemble IPD model (7, 7, 15) than (7, 7, 7) in Fig. 16 (a).

We can also observe clear effects of adding random edges in Figs. 15-17. For example, we can see from the experimental results by the standard IPD model with 3-bit binary strategies in Fig. 15 (i.e., continuous black line in each plot in Fig. 15) that the increase in the number of randomly added edges makes the evolution of cooperative IPD game strategies fast and unstable. The same observation is obtained from Fig. 16 and Fig. 17 for the standard IPD models with 7-bit and 15-bit binary strategies (i.e., continuous black line in Fig. 16 (a)-(c) and Fig. 17 (a)-(c)). This is because randomly added edges make the propagation of good strategies throughout the entire network easier, which leads to faster evolution of cooperative IPD game strategies. At the same time, they disturb the locality in the spatial IPD game, which leads to unstable evolution of game strategies.

Among 12 plots in Figs. 15-17, totally different results are obtained in Fig. 16 (d) from the other results. Except for Fig. 16 (d), the average payoff obtained from the ensemble IPD models is clearly higher than or almost equal to that from the standard IPD models. However, the average payoff from the ensemble IPD model (7, 7, 15) is clearly lower than that of the standard IPD model with a single 7-bit strategy. This is a strange observation, which cannot be easily explained. Moreover, the experimental results by the standard IPD model with a single 7-bit strategy in Fig. 16 are different from those by a single 3-bit strategy in Fig. 15 and a single 15-bit strategy in Fig. 17. The increase in the number of randomly added edges from 512 to 1024 and 2048 decreases the average payoff of the standard IPD models in Fig. 15 and Fig. 17 (see the continuous line in each plot). However, the same increase has a positive effect on the average payoff of the standard IPD model with a single 7-bit binary strategy in Fig. 16. This is also a strange observation, which cannot be easily explained.

In order to further examine the evolution of cooperative IPD game strategies, we count the number of rounds where each player chooses the cooperation in various situations with respect to the previous actions of the opponent. Table VII shows the average percentage of choosing the cooperation after each of the eight cases of the previous three actions of the opponent. For example, "91.1" in the last cell in the bottom row shows that the ensemble IPD model (3, 7, 15) chooses "C" in 91.1% rounds (and "D" in 8.9% rounds) after the three actions "CCC" of the opponent in the previous three rounds. This model chooses "C" in 20.4% rounds after "DDD". All of those percentages are calculated from our computational experiments on the 1024-node ring graph with 512 randomly added edges under the error probability 0.05. This error probability "0.05" can be viewed as being high in the IPD model since each player makes an error every 20 rounds on average. The evolution of cooperative IPD game strategies is very difficult under such a high error probability. This is because occasional defection of each player by accident based on such a high error probability disturbs the evolution of cooperative IPD game strategies.

In Table VII, cooperation percentages larger than 50% are highlighted in boldface. In general, IPD game strategies tend to cooperate when the opponent cooperates in the previous round. This is shown by large cooperation percentages in the four columns of Table VII with "C" as the opponent's previous action in the $(t-1)$th round. However, the largest cooperation percentage "97.8" is obtained in Table VII by the single 7-bit model when "D" is the opponent's previous action in the $(t-1)$th round (and "C" in the $(t-2)$th round). This large cooperation probability after the opponent's previous action "D" may be related to the strange behavior of the single 7-bit model in Fig. 16.

TABLE VII
PERCENTAGE OF CHOOSING THE COOPERATION OF EACH IPD MODEL IN EACH OF THE EIGHT CASES OF THE PREVIOUS THREE ACTIONS OF THE OPPONENT

		Opponent's previous actions							
$(t-3)$th round		D	D	D	D	C	C	C	C
$(t-2)$th round		D	D	C	C	D	D	C	C
$(t-1)$th round		D	C	D	C	D	C	D	C
Standard	Single 3-bit	39.0	**68.5**	39.0	**68.5**	39.0	**68.5**	39.0	**68.5**
	Single 7-bit	0.5	**77.5**	**97.8**	**67.4**	0.5	**77.5**	**97.8**	**67.4**
	Single 15-bit	8.6	**60.2**	**59.3**	**65.9**	**88.8**	**59.5**	16.8	**80.3**
Ensemble	(3, 7, 15)	20.4	**74.5**	**65.5**	**71.4**	**71.8**	**68.7**	26.6	**91.1**

V. CONCLUSION

In this paper, we first proposed the use of an ensemble decision making scheme based on multiple strategies in the IPD game. Then we examined the evolution of multiple game strategies of each player in the ensemble IPD game model and that of a single game strategy of each player in the standard IPD game model in ring graphs. Our computational experiments were performed under various settings with respect to the choice of representation schemes of strategies, the error probability in the action selection, and the number of randomly added edges to a ring graph.

Some expected observations were obtained in this paper. For example, the average payoff was increased by the use of different representation schemes in the ensemble IPD model from the case with the same representation scheme. The increase in the error probability decreased the average payoff. The increase in the number of random edges made the evolution of cooperative IPD game strategies fast and unstable. However, some observations were counter-intuitive. For example, when we used 7-bit binary strategies, the increase in the number of random edges made the evolution of cooperative IPD game strategies fast and stable. Further studies are needed to discuss these counter-intuitive observations from our computational experiments in this paper.

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