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A Social Metrics Based Process Model on Complex Social System

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***Abstract*—In previous work, we investigated the performance of Cultural Algorithms (CA) over the complete range of system complexities in a benchmarked environment. In this paper the goal is to discover whether there is a similar internal process going on in CA problem solving, regardless of the complexity of the problem. We are to monitor the "vital signs" of a cultural system during the problem solving process to determine whether it was on track or not and infer the complexity class of a social system based on its "vital signs". We first demonstrate how the learning curve for a Cultural System is supported by the interaction of the knowledge sources. Next a circulatory system metaphor is used to describe how the exploratory knowledge sources generate new information that is distributed to the agents via the Social Fabric network. We then conclude that the Social Metrics are able to indicate the progress of the problem solving in terms of its ability to periodically lower the innovation cost for the performance of a knowledge source which allows the influenced population to expand and explore new solution possibilities as seen in the dispersion metric. Hence we present the possibility to assess the complexity of a system's environment by looking at the Social Metrics.**

Keywords—Cultural Algorithm, Complex Systems, Optimization, problem solving process.

I. Introduction

Complex systems are an important research topic in all of the sciences [1, 2]. Cultural Algorithms[3, 4] can provide a flexible framework in which to study the emergence of organizational complexity of any social systems using Multi-Agent System (MAS) approaches. As shown in Fig. 1, a Cultural Algorithm is a dual inheritance system that characterizes evolution in human culture at both the macro-evolutionary level that takes place within the Belief Space; and, at the micro-evolutionary level, that occurs in the Population Space. Knowledge produced in the Population Space is selectively accepted or passed to the Belief Space and used to adjust the knowledge structures there. This knowledge can then be used to influence the changes made by the population in the next generation. In this way, the population component and the Belief Space interact with, and support each other, in a manner analogous to the evolution of human culture [5, 6]. Previously, Peng[7] found that the similarities in social structures that emerge in similar cultures, are produced as a result of the integration of knowledge sources in the problem solving process. Ali[8] expanded on the ability of a knowledge source to influence a population through a "Social Fabric" which represents the extent to which the influence of a knowledge source can spread throughout a population via a temporary interconnecting "social network" between agents in the population.

In order to study the relationship between system performance, social structure, and problem complexity in a complex social system, Che [9] adopted a generalized complex system environment: a cones world framework taken from the work of De Jong [11], and based upon theoretical models of complex systems by Langton and others [12]. The cones world is composed of resource cones juxtaposed on a landscape and the combination of the surfaces of these cones will produce a functional landscape that can be explored by a population. Che also extended the "Social Fabric" approach by allowing the networks to have a memory with which the network organization can sustain over the whole problem solving process. Che found the organization of the social structure for a culture does reflect the nature and number of problems presented to it by its environment [9], which has been suggested by other researchers such as Barabási [13] that the prevalence of small world networks in biological systems may reflect an evolutionary advantage of such an architecture. One possibility is that small world networks are more robust to perturbations than other network architectures. If this were the case, it would provide an advantage to biological systems that are subject to damage by mutation or viral infection.

The fact that networks of a similar structure appear in different environments suggest that there are underlying similarities within which cultural formation processes take place. If one uses certain metrics to monitor the status of a living organism and if cultural systems can be viewed as living organisms, we can develop certain metrics that can be used to track the well-being of such a cultural evolution process. It is suggested that these metrics can be applied to all cultural systems, since the underlying computational processes are the same across the board in our Cultural Algorithms model.

In this paper we introduce four metrics that we use to monitor the culture system's vital signs in a given problem environment and use them to assess the expression of the social networks on the functional landscape produced by the Cultural Algorithms. The metrics assess the extent to which the influence function is able to generate diversity at each of its several stages. These metrics are used as a vehicle to estimate the entropy, pre-determined by the generator function, of the problem solving system used to find the optimal solution. These metrics relate to the dispersion of individuals in problem space and the dispersal of knowledge within a social network composed of these individuals.

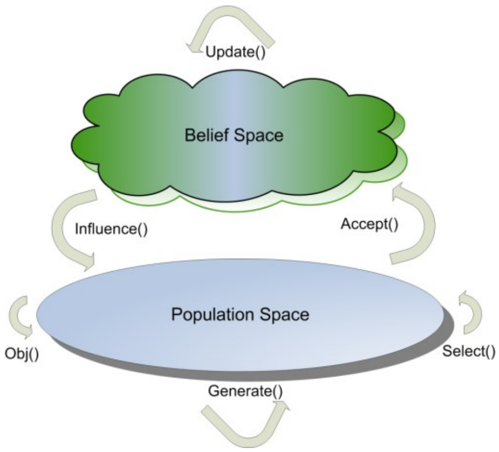

Fig. 1 Cultural Algorithm Frameworks [9]

In section II we introduce social metrics to assess the impact that the various components of the Social Fabric influence function have on the problem solving process. Next in section III the experimental framework is presented. In that framework, the Cones World, problem landscapes are constructed by the overlaying of cones on a multi-dimensional landscape. The generator processor can vary in terms of entropy or predictability from static to chaotic. Sample outoput including social metrics summary table and knowledge sourlce learning curve are presented. In Section IV we provide an example of the vital signs produced in a successful run for a complexity of A=1.01. In Section V we propose a metric based process model based on the experiment results and analysis. Section VI provides the conclusion and future work.

II. SOCIAL METRICS

In this section we describe the four metrics that we will use to monitor the Culture's vital signs in a given environment. We expect that each Knowledge Source is learning during the search process. Our assumption is that the Social Fabric is able to distribute performance information about each knowledge source to its directly connected set of neighbors.

The metrics assess the extent to which the influence function is able to generate diversity at each of its several stages. The influence function affects the decision making of each individual in the population space at each generation with the following steps:

1) The update function which adjusts the Knowledge Sources based upon agent experiences. Adjustments were made here to increase the diversity of the certain Knowledge Source (e.g., Situational Knowledge Source whose data also influenced other Knowledge Sources)
2) The MVT (Marginal Value Theorem) [7, 24] as implemented via a roulette wheel mechanism assigns each population agent a knowledge source. That Knowledge Source is said to be the agent's direct influence. The MVT is also a co-evolutionary device that associated Knowledge Sources with predators and the population of agents as prey spread out over a functional landscape.
3) The direct influence for each agent is distributed to its neighbors.
4) With multiple Knowledge Sources providing conflicting decision choices, each individual agent adopts a vector voting scheme based a weighted voting (bidding) mechanism in which the Knowledge Source with the highest weighted-total is the winner. The winning Knowledge Source then is able to control the behavior of the individual at that time step. Depending on the weighs for each knowledge source, there might be two different winning scenarios: "majority win" in which the majority Knowledge Source wins the bidding and "minority win" in which the minority of Knowledge Source wins the bidding when the drop in performance associated with the need to experiment with new solutions.

Here we employ metrics that assess the vital signs of the system in terms of steps 2 and 4 above. This is because step 2 reflects the impact of step 1, and step 4 reflects the impact of step 3 above.

A. *The Dispersion Coefficient*

The metric associated with step 2 is called the **Dispersion Coefficient**. It measures the distance on the functional landscape over which directly connected individuals are spread. A real world analog for this metric would be the following: the current economy may force related individuals to search for work in places distant from each other. That is viewed as producing a social tension or disconnect between related individuals so that their experiences are now potentially much different.

The definition of the Dispersion Coefficient for one generation in a certain social environment, ST, is defined as the sum of the Euclidean Distances between each individual (X_1, $X_{2,}$ ... X_{Dim}) and its immediate neighbors in the Social Fabric(a_1, $a_{2,}$ ... a_M). It is described as follows:

$$ST = \sum_{i=1}^{N} \sum_{j=1}^{M} \sqrt{\sum_{k=1}^{Dim} (X_{i,k} - a_{i,j,k})^2}$$

Here, *N* is total number of individuals,
Dim is total number of dimensions of this environment,
M is the number of neighbors directly adjacent to each individual,
$X_{i,k}$ represents the coordinate on dimension k for individual i, and
$a_{i,j,k}$ is the coordinate of j[th] neighbor of individual i on dimension k.

B. *Minority/Majority Win Scores and Innovation Cost*

There are three metrics associated with step 4. They are:

Majority Win Score – the average value of the score when the majority Knowledge Source wins the bidding in a time step.

Minority Win Score – the average score for the time period when a minority Knowledge Source wins the bidding.The Innovation Cost – The difference between 2 and 1 above assuming that the majority Win Score will be greater than the minority win score. This represents the drop in performance associated with the need to experiment with new solutions. For maximization problems, the score is an index that reflects the cost of innovation in terms of the reduction in performance that results when the majority does not win in a given situation

The Innovation Cost index is computed to represent the loss of performance revenue that results from initially allowing a minority Knowledge Source to win. This will happen when a Knowledge Source with few individuals finds a new promising region and as a result is average performance for that time period that is high enough to beat the sum of the majority influences.

Like the Dispersion Coefficient these three metrics are calculated in each time step. However, they are only used to distribute information in designated time steps in all of our experiments when share information every three time steps. This gives us time to evaluate the changes to each knowledge source, so that new weights based upon performance are less noise and a better index of relative performance.

The dispersion coefficient and the innovation cost coefficient can reflect the entropy of the Population Space and the Belief Space respectively. This will allow us to assess the changes in system behavior in entropic-like terms in response to changes in the complexity of the environment. Hence we suggest that the values and cycles associated with the "information pumping" action of the extended influence function can be a useful technique for understanding and predicting system behavior in a variety of environments.

III. EXPERIMENTAL FRAMEWORK

The experimental framework uses an implementation of a Cultural Algorithm Tool Kit [3,8,9] in a Repast Agent Based simulation Integrate Development Environment

A. Cultural Algorithm Implementation in Repast

The Cultural Algorithm as described earlier has four major components: the Population Space, the Belief Space, the problem landscape, and the communication protocol. Each of the components will now be described.

1. *The Population Space:*

The population space can support any population-based computational model, such as Genetic Algorithms, Evolutionary Programming, Genetic programming, PSO, ACO, and agent-based systems. Here we employ the Genetic Algorithm population framework.

2. *The Belief Space:*

In the belief space, we have five basic types of knowledge sources: **Normative, Situational, Domain, Topographic, and History** knowledge. Although for specific problems, we might need more specific domain knowledge, we are most interested in studying system behavior relative to changing problem complexity. These knowledge sources drive the problem solving process.

3. *The Problem Landscape:*

The performance of an individual agent in the population space for a given problem is evaluated within a real-valued performance environment obj(). In this paper this performance environment is a problem landscape generator that produces a problem landscape by the placement of cones of varying within a multi-dimensional landscape. This environment is called the Cone's World. It is adapted from the work of DeJong and Morrison [11]. Although problems can be expressed in terms of many dimensions, we focus on the optimization problems defined over a two-dimensional real-valued landscape which is easy to visualize. Peng [7, 23] coined the term **Cones World** when she employed it to test various Cultural Algorithm configurations. When Ali[9] extended Peng's Cultural Algorithm framework in his CAT system, he employed the Cones World as one of the problems available to system users along with the traditional benchmark problems.

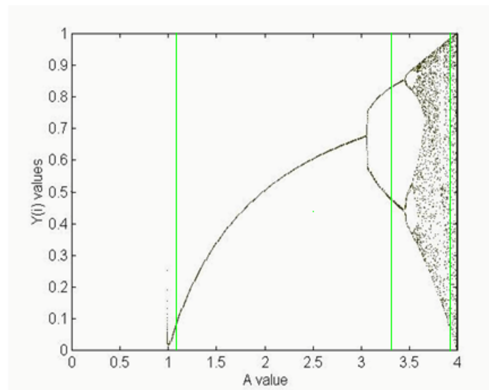

Fig. 2 Logistic Function with characteristic A values

The Cones World Generator generates a problem landscape, in which a field of resource cones of different heights and different slopes that are randomly scattered across a multi-dimensional landscape. The Cones-World algorithm generates a dynamic cones-world in two steps:

First step is to specify a baseline static landscape of the desired morphological complexity, and then add the desired dynamics. The base landscape is given by:

$$f(< x_1, x_2, \ldots, x_n >) = \max_{j=1,k} (H_j - R_j * \sqrt{\sum_{i=1}^{n} (x_i - C_{j,i})^2})$$

Where:

k : the number of cones,

n : the dimensionality,

H_j: height of cone j,

R_j : slope of cone j, and

$C_{j,i}$: coordinate of cone j in dimension i.

The values for each cone (H_j, R_j, and $C_{j,i}$) are randomly assigned based on the following user specified ranges:

$H_j \in$ (Hbase, Hbase + Hrange)

$R_j \in$ (Rbase, Rbase + Rrange)

$C_{j,i} \in$ (-1, 1)

Each of these independently specified cones are "blended" together using the max function, i.e., if two cones overlap, the height at a point is the height of the cone with the largest value at that point.

After the initial random landscape generation, the second step is to specify the dynamics. For each cone j, its every single parameter (every dimension $C_{j,i}$, height H_j, and, and slope R_j) can be changed individually and independently. In order to control the complexity of a landscape, we use the logistics function given as:

$$Y_i = A * Y_{i-1} * (1 - Y_{i-1})$$

where A is a constant and Y_i is the value at iteration i.

A bifurcation map of this function is provided in Fig. 2 which shows the values of Y that can be generated in each iteration of the logistic function given values of A between 1.0 and 4.0. The particular value of A chosen for each of the dynamic features specifies whether the movement will be small same-sized steps, large same-sized steps, steps of few different sizes, or chaotically changing step sizes. We are particularly interested in characteristic points in terms of the complexity in the cones world as shown in Fig. 2. Here we pick A = 1.01, 3.35 and 3.99 for our test environment complexity as marked with vertical mark line in green. From left to right we have A = 1.01, 3.35, 3.99 corresponding to one step change, two steps change and totally chaotic step size change. Each of these represents one of the computational classes proposed by Langton as discussed below. Based upon that he established several basic computational classes as follows:

Fixed - For problems of low entropy a fixed set of rules can be given to each cell in order to allow them to exchange the information needed to solve the problem. In our case this is equivalent to having a fixed topology over which information is exchanged, around 1.
Periodic – For problems of this nature the cells need to switch from one set of rules to another depending on the number of bifurcations.
Chaotic – Problems for which the number of bifurcations is so large the system is inherently chaotic. Thus, there are no specific rule sets that apply.

Fig. 3[9] shows example landscapes with k = 15, Hbase = 1, Hrange = 9, Rbase = 8, and Rrange = 12 and A = 1.01.

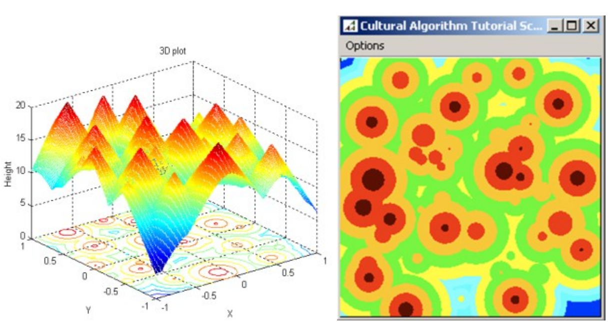

Fig. 3 [9] Example Landscapes displayed in 3D and 2D
x ∈ (-1.0, 1.0), y ∈ (-1.0, 1.0), H∈(1, 10), and R∈ (8, 20)

4. *The Communication Protocol:*

The communication protocol of a Cultural Algorithm System is composed of two functions: the **acceptance function** which determines which individuals are used to impact the Belief Space; and **the influence function** which determines how the Belief Space influences the population space in generating new solutions.

Acceptance Function:

The acceptance function determines which individuals and their behaviors can update the Belief Space knowledge. It is often expressed as a percentage of the number of current individuals ranging between 1% and 100% of the population size, based upon selected parameters such as performance. In our case, we employ 50 individuals and a landscape constructed of 500 cones. Given the small size of the population relative to the potential complexity of the environment, we use 100% of the information gathered by the agents in our version here.

Influence Function:

The key activity of the influence function is to integrate the multiple knowledge sources together in the early system. The influence function affects the decision making of each individual in the population space at each generation with the 4 steps described in Section II.

The network topologies supported are: lbest (degree of two for each agent), square (degree of 4 for each agent), hexagon (degree of 6 for each agent), octagon (degree of 8 for each agent), hexadecagon (degree of 16 for each agent) and gbest (degree of n-1 nodes for each agent).

B. Social Metrics Summary Tables

These tables give the statistics for the social metrics that are used to generate the vital signs for a given run. For a given complexity class, basic statistics for the metrics produced by each topology for the 50 runs are given. Table I presents the social metrics results for runs with A=1.01, in which:

Dispersion_Run_Ave: The average Dispersion metric for each run
MajorityWinScore: the average winning score when everyone conforms, i.e. the average fitness value of the winning KS when everybody agree with each other.
MinorityWinScore: the average fitness of the winning KS when there is a conflict between an individual and its neighbors. The conflict will be solved by bidding .i.e. incentive-based majority win.
Innovation Cost Index: the difference of Conformity mean and conflict mean reflects the opportunity of innovation

C. Knowledge Source Learning Curve Graphs

Based on the raw data we recorded for each generation, we are able to reproduce the real time learning curve graph over generations relative to each knowledge source. Fig.4 is an example graph for one single run showing how best-overall fitness and best Knowledge source individuals change over generations.

TABLE I SOCIAL METRIC SUMMARY TABLE

Topology		Minimum	Maximum	Mean	Std.Dev
lBest	Dispersion_Run_Ave	.42	1.00	.6642	.12334
	MinorityWinScore	.10	.10	.1028	.00047
	MajorityWinScore	.18	.19	.1823	.00266
	InnovationCost	.08	.08	.0794	.00235
square	Dispersion_Run_Ave	.44	.92	.6375	.12219
	MinorityWinScore	.19	.20	.1954	.00204
	MajorityWinScore	.25	.26	.2553	.00260
	InnovationCost	.05	.07	.0598	.00350
Hexagon	Dispersion_Run_Ave	.43	.91	.6302	.10194
	MinorityWinScore	.23	.24	.2322	.00242
	MajorityWinScore	.33	.35	.3389	.00420
	InnovationCost	.10	.11	.1061	.00345
Octagon	Dispersion_Run_Ave	.40	.96	.6370	.13252
	MinorityWinScore	.30	.31	.3047	.00328
	MajorityWinScore	.40	.43	.4141	.00624
	InnovationCost	.10	.12	.1070	.00677
16-gon	Dispersion_Run_Ave	.41	.85	.6115	.10013
	MinorityWinScore	.50	.54	.5164	.00751
	MajorityWinScore	.67	.73	.7045	.01377
	InnovationCost	.14	.18	.1595	.01093
Global	Dispersion_Run_Ave	.39	.92	.6013	.12445
	MinorityWinScore	.92	1.34	1.2088	.10424
	MajorityWinScore	1.50	2.09	1.8033	.13089
	InnovationCost	.03	.08	.0529	.00892

IV. OBSERVATION OF AN EXEMPLARY RUN

In Fig.4, at the top of the figure is the change in best value found over the course of the run. Notice the incremental character of the process. There is an initial increase, around time step 6 in performance, and then a plateau is reached at a false peak. Then, additional exploration causes a second wave of the learning curve around time step 41. Additional small increments take place after that as process focuses in on the optimal peak.

This learning process is precipitated by the distribution of knowledge through the Social Fabric initially by the exploratory knowledge sources. The new knowledge is produced by the explorer knowledge sources, normative and topographic. We view this as the **'heartbeat".** This is the process that injects innovation and new knowledge into the system through the Social Fabric. The best way to read these two graphs is to look for a downward spike. That reflects the movement of the bounding box to another region based upon the Marginal Value Theorem. Generally the movement is into the most promising area at that time. Notice that the sharp drops or downward spikes for both knowledge sources are not in synchrony. They complement each other. For example, in the first learning episode we see an initial downward spike in topographic knowledge around timestep 2 which triggers a series of lesser ones in normative knowledge in time steps 8 and 9. Subsequently in the middle of the first learning step topographic knowledge has another spike around 18 followed by a major spike in normative knowledge around time step 20.

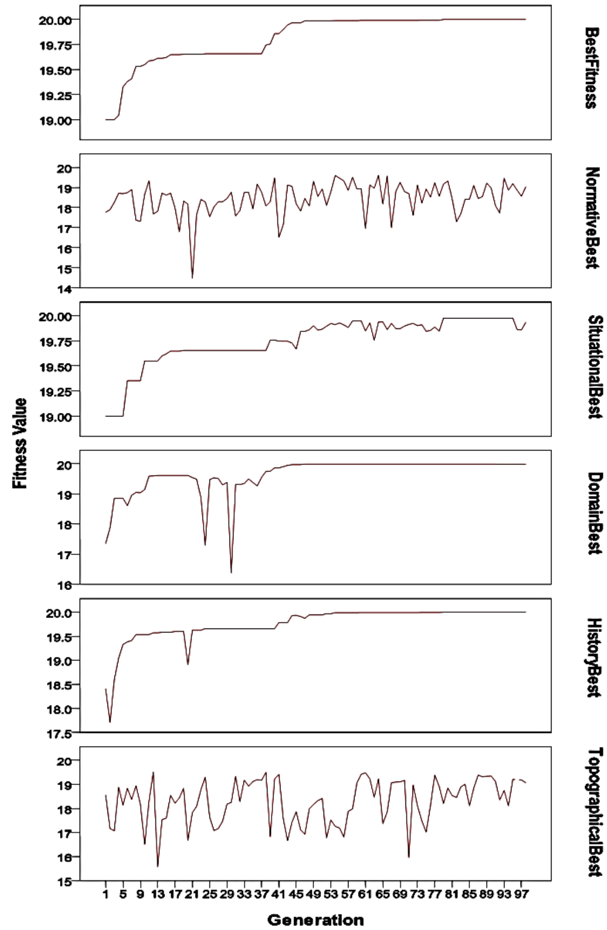

Fig.4 Knowledge Source Learning Curve Graphs (A=1.01)

There is a second learning activity that starts around generation 38. First with a spike in topographic knowledge which corresponds to a shift in its bounding box. It results in an improvement that then reduces the share of normative knowledge, causing it to move its bounding box. As a result, we see a large spike in topographic knowledge around generation 73 followed by spikes in normative knowledge. This suggests another but much smaller learning adjustment.

The other three knowledge sources focus on the exploitation of promising regions: historic, domain, and situational. Each of them focusses on a promising area and therefore has a more stable curve. Still, in each of the two major learning steps here we notice a downward spike which precipitates the movement of their corresponding bounding boxes. The spikes take place in general during the rise of the curve instead of anticipating the rise. Once the explorative knowledge sources have found a promising area, these knowledge source tend to move their own bounding boxes in order to track the new findings. This process has been called "**knowledge swarming" [**7] and it is clearly taking place in the successful runs here.

V. A METRICS BASED PROCESS MODEL

In order to tie these learning "events" together, we adopt a metaphor from the human circulatory system in order to model how the exploratory processes "pumped" information into the system. In terms of this metaphor, the influence function pumps new information into the population. The Social Fabric serves as the circulatory network over which the information is pumped. We suggested that certain changes in the search results can be anticipated in terms of changes in the underlying knowledge sources that produced them. In this section we look at how changes in the Social Metrics can be used to track these internal knowledge source changes as well. That is, while the Population Space and Knowledge Sources comprise the internal organ components of the system, the social metrics provide the vital signs for Cultural Systems' search effectiveness.

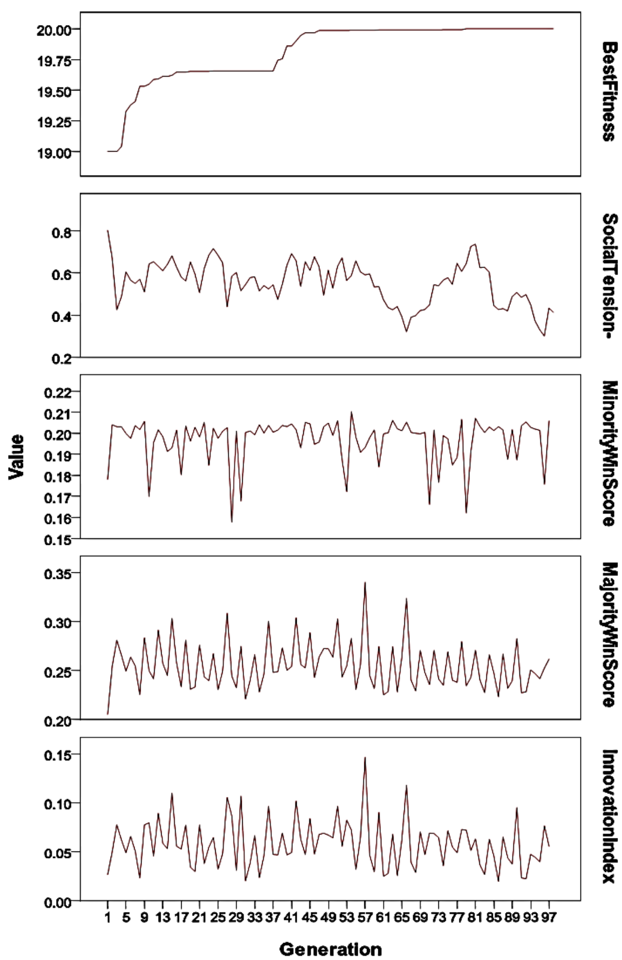

Fig.5 Social metric changes in each generation (A=1.01)

A successful Culture should provide sufficient diversity and to allow on occasion low cost opportunities for social innovation. Fig.5 shows an example graph for social metric changes over each generation when A=1.01. We have the same performance reference curve as in Fig.4. We see the Social Metric graphs for the same run described in the previous section, a square topology. The social metrics just tell us how well the search process is going from a social perspective. Recall that most of the changes in our system were related to improving its potential to support co-evolution. That means that a successful Culture should provide sufficient diversity and to allow on occasion low cost opportunities for social innovation.

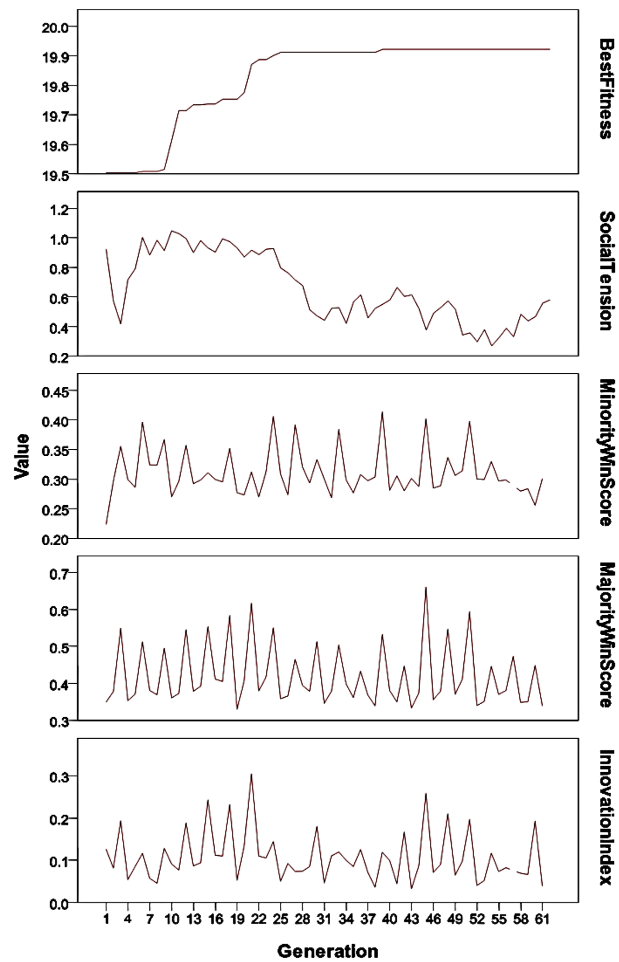

Fig.6 social metric changing over generation (A=3.35)

Here our key "vital signs" are the dispersion and innovation cost metrics. Rather than reflecting what is learned they reflect how the learning process is performing. If we return to our circulatory system analogy, we expect to see a sequence of agent dispersions and contractions that result from the pumping process that moves the bounding boxes for the knowledge sources around. Likewise, we expect to see that the innovation cost also cycles from high to low in order to allow the infusion of new information that can be used to disperse the knowledge sources and subsequently their populations. Notice that once the system has found the solution, it begins to produce a more clustered population as indicated by the social stress metric.

Fig.6 shows a graph for social metric changing over generation using the same setting as of Fig.5 except with A=3.35. The key question here is whether we still see a similar pattern for the vital signs even though the configuration and complexity class are completely different. In other words, do the vital signs that we propose characterize a successful search process regardless of topology or complexity?

Notice a quick rise in the dispersion metric that coincides with the first learning increment. Once the optimum is achieved the system tries to increasingly distribute the agents in an attempt to improve on the score. Likewise the innovation cost is periodically kept low which allows more opportunities to generate innovations. The average innovation cost is quite stable over all the generations, even after a solution is found. Analysis on same output for more chaotic complexity classes reveals more stress on the system. Overall, the search process has similarities to the others, but also one can see the additional stress on the system that the environment is producing.

VI. Conclusion and Future Work

In this paper we demonstrated how the learning curve for the Cultural System is supported by the interaction of the knowledge sources. We used a circulatory system metaphor to describe how the exploratory knowledge sources generated new information that was distributed to the agents via the Social Fabric network. We then observed that the Social Metrics were able to indicate the progress of the search in terms of its ability to periodically lower the innovation cost for a knowledge source to drop which allows the influenced population to expand as seen in the dispersion metric.

As a result it is clear that the same process underlies a successful search regardless of the topology and the complexity of the environment. However, as stress increase one can see that the spikes of innovation cost index, although still there, get more erratic. It is then possible to assess the complexity of a system's environment by just looking at the Social Metrics.

For future work, it would be interesting to measure the social metrics on different social system framework such as PSO and ACO and look at how these metrics monitor complex social systems in general.

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