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T-S Fuzzy Models Based Approximation for General Fractional Order Nonlinear Dynamic Systems

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***Abstract*—In this paper, a novel approach is presented to approximate the general fractional order nonlinear dynamic systems. Firstly, a generalized T-S fuzzy method is used to approximate the original models. Then a new method to approximate the fractional order T-S models is utilized, and obtain a series of integer order linear model. It is revealed for the first time that a general fractional order nonlinear system (FONS) can be approximated by a series of integer order linear models to any degree of accuracy on any compact set. Finally, numerical simulation results are provided to illustrate the effectiveness of the proposed approach.**

I. INTRODUCTION

In recent years, fractional order systems have attracted increasing attention from control community, since many engineering plants and processes, such as electronic circuit, heat conduction and abnormal diffusion [1]–[5], can be more concisely described by fractional order differential equations. Due to the great efforts devoted by researchers, a number of valuable results on system modeling [6] and identification [7][8], controllability and observability [10], stability analysis [11] and controller synthesis [12][13] of fractional order systems have been reported in papers.

As we all know, the most difficult problem of analysis and synthesis of fractional order dynamic system is how to calculate the related fractional order equation, especially for nonlinear system [3]. Fortunately, some available means were presented, such as the approaches mentioned in literature [14]–[18]. The author Wei proposed a new approach based on the piecewise approximation and proved that the original system and the approximation system have the same controllability and observability [19].

Since T-S fuzzy control [20] appeared, a large number of successful applications of nonlinear system control based on it have been developed [21]–[23]. It's worth noting that a generalized T-S fuzzy model is proposed in [24], whose ability to approximate non-affine nonlinear systems has been proved. The corresponding research on FONS has received little attention despite its practical significance. [25] modifies the T-S fuzzy model and proposes a new technique to stabilize a class of fractional order chaotic systems. Some control techniques incorporated with fuzzy control have been implemented in the control FONS, for example, H_∞ control [26], sliding mode control [27] and PID control [28].

Despite of the previous achievements, there is room for further research. The related research focuses on the control of a small special class FONS only. However, we have not yet found any literature which introduce the application of fuzzy control in FONS systematically, so far. What's more, we also haven't found the related research on the non-affine nonlinear systems or the non-commensurate order systems, although these are very important. All of these motivate us to finish this work.

The rest of this article is organized as follows: In section II, we will briefly review a number of concepts of fractional calculus and fractional order systems and give the problem discussed later. Section III is devoted to use our method to approximate the general FONS. In section IV, some numerical simulations are executed to illustrate the effectiveness and superiority of the proposed approaches. Conclusions are given in section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Fractional order calculus

The fractional $\alpha\,\mathrm{th}$ order $(\alpha > 0)$ Riemann-Liouville integral of a function $f(t)$ is defined by [3]

$$\mathcal{I}_\alpha\left(f(t)\right) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau, \tag{1}$$

where $\Gamma(\alpha)$ is the gamma function $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\mathrm{d}x$.

Obviously, $\mathcal{I}_\alpha\left(f(t)\right)$ is the convolution of the function $f(t)$ with the impulse function $h_\alpha(t) = t^{\alpha-1}/\Gamma(\alpha)$ is the Laplace transform of $h_\alpha(t)$

$$\mathcal{I}_\alpha(s) = \mathcal{L}\left(h_\alpha(t)\right) = s^{-\alpha}. \tag{2}$$

The so-called fractional order differentiation is just the dual operation of the fractional order integration. Define $v(t)$

and $y(t)$ as the input and output of the fractional integration operator $\mathcal{I}_\alpha(s)$, respectively, then

$$\left\{\begin{array}{l} y(t)=\mathcal{I}_\alpha(v(t)) \\ v(t)=\mathcal{D}^\alpha(y(t)) \end{array}\right. . \tag{3}$$

B. Finite dimension approximation

In fact, $\mathcal{I}_\alpha(s)$ $(\alpha>0)$ is a linear frequency distributed system, with input $v(t)\in\mathcal{R}$ and output $y(t)\in\mathcal{R}$. Its equivalent frequency distributed model with distributed state $z(\omega,t)\in\mathcal{R}$ satisfies

$$\left\{\begin{array}{l} \frac{\partial z(\omega,t)}{\partial t}=-\omega z(\omega,t)+v(t) \\ y(t)=\int_0^\infty \eta_\alpha(\omega)\, z(\omega,t)\,\mathrm{d}\omega \end{array}\right. , \tag{4}$$

where $\eta_\alpha(\omega)=\omega^{-\alpha}\sin(\alpha\pi)/\pi$.

In view of the frequency distributed model of $\mathcal{I}_\alpha(s)$ usually cannot be used directly, now we choose finite distributed frequency points $\omega_0,\ \omega_1,\ \cdots,\ \omega_k$ instead of continuous frequency point, then the finite dimension approximate model can be described as

$$\left\{\begin{array}{l} \frac{\mathrm{d}z(\omega_0,t)}{\mathrm{d}t}=-\omega_0 z(\omega_0,t)+v(t) \\ \qquad\qquad\vdots \\ \frac{\mathrm{d}z(\omega_k,t)}{\mathrm{d}t}=-\omega_k z(\omega_k,t)+v(t) \\ \hat{y}(t)=\sum_{i=0}^{k} c_i z(\omega_i,t) \end{array}\right. , \tag{5}$$

where $y(t)=\lim\limits_{k\to\infty}\hat{y}(t)$, c_i is the ith element $z(\omega_i,t)$.

Supposing $\hat{\mathcal{I}}_\alpha(s)$ is the transfer function of system Eq.(5), then we have

$$\mathcal{I}_\alpha(s)=\lim_{k\to\infty}\hat{\mathcal{I}}_\alpha(s), \tag{6}$$

where the zero-pole model of $\hat{\mathcal{I}}_\alpha(s)$ can be denoted as

$$\hat{\mathcal{I}}_\alpha(s)=\frac{G_\alpha}{s+\omega_0}\prod\nolimits_{i=1}^{k}\frac{s+\bar{\omega}_i}{s+\omega_i}.$$

We have proved in [19] that $\hat{\mathcal{I}}_\alpha(s)$ can approximate $\mathcal{I}_\alpha(s)$ very well, when we select the zeros, poles and static gain as

$$\left\{\begin{array}{l} \bar{\omega}_i=\lambda^{i-1}\gamma^{-0.5}\omega_l, i\in\Omega \\ \omega_0=\left\{\begin{array}{ll} \lambda^{-\alpha}\gamma^{-0.5}\omega_l & ,0<\alpha\le 0.6 \\ 0 & ,0.6<\alpha<1 \end{array}\right. \\ \omega_i=\lambda^{i-\alpha}\gamma^{-0.5}\omega_l, i\in\Omega \\ G_\alpha=\arg\min\limits_{G_\alpha}\left|\mathcal{I}_\alpha\left(j\sqrt{\omega_l\omega_h}\right)-\hat{\mathcal{I}}_\alpha\left(j\sqrt{\omega_l\omega_h}\right)\right| \end{array}\right. , \tag{7}$$

where $\Omega\triangleq\{1,2,\cdots,k\}, \lambda=(\gamma\omega_h/\omega_l)^{\frac{1}{k-\alpha}}$.

Rewrite $\hat{\mathcal{I}}_\alpha(s)$ as follows

$$\hat{\mathcal{I}}_\alpha(s)=\sum\nolimits_{i=0}^{k}\frac{c_i}{s+\omega_i}, \tag{8}$$

considering the equivalence of the two forms, yields

$$\left\{\begin{array}{l} c_0=G_\alpha\prod_{j=1}^{k}\frac{\omega_0-\bar{\omega}_j}{\omega_0-\omega_j} \\ c_i=G_\alpha\frac{(\omega_i-\bar{\omega}_i)}{(\omega_i-\omega_0)}\prod_{j=1,j\ne i}^{k}\frac{\omega_i-\bar{\omega}_j}{\omega_i-\omega_j}, i\in\Omega \end{array}\right. . \tag{9}$$

C. Problem statement

Consider the pseudo state space model of the general fractional order nonlinear model (FONM)

$$\left\{\begin{array}{l} \mathcal{D}^{\alpha_1}y_1=G_1(y_1,y_2,\cdots,y_n,u_1,u_2,\cdots,u_m) \\ \mathcal{D}^{\alpha_2}y_2=G_2(y_1,y_2,\cdots,y_n,u_1,u_2,\cdots,u_m) \\ \qquad\qquad\vdots \\ \mathcal{D}^{\alpha_n}y_n=G_n(y_1,y_2,\cdots,y_n,u_1,u_2,\cdots,u_m) \end{array}\right. , \tag{10}$$

where the order $\alpha=[\alpha_1\ \alpha_2\ \cdots\ \alpha_n]^{\mathrm{T}}$ with $\alpha_i\in(0,1]$ for any $i\in N\triangleq\{1,2,\cdots,n\}$, $u=[u_1\ u_2\ \cdots\ u_m]^{\mathrm{T}}\in\mathcal{R}^m$ and $y=[y_1\ y_2\ \cdots\ y_n]^{\mathrm{T}}\in\mathcal{R}^n$ are the input and output of the system, respectively, with $y\in Y=\prod_{i=1}^{n}[\underline{y}_i,\bar{y}_i]$, $u\in U=\prod_{i=1}^{m}[\underline{u}_i,\bar{u}_i]$, $Y\times U\subset\mathcal{R}^n\times\mathcal{R}^m$ is a compact set, without loss of generality, we assume $y(0)\in Y$, $u(0)\in U$.

In order to describe briefly, we rewrite Eq.(10) as

$$\mathcal{D}^\alpha y=G(y,u), \tag{11}$$

where $G(y,u)\in\mathcal{R}^n$ is a linear or nonlinear function on $Y\times U$, origin for the equilibrium point $G(0,0)=0$, for any $(y_a,u_a),(y_b,u_b)\in Y\times U$, there will always be a positive constant L which keeps

$$\|G(y_a,u_a)-G(y_b,u_b)\|\le L(\|y_a-y_b\|+\|u_a-u_b\|). \tag{12}$$

Our goal is to approximate the model Eq.(11) by integer order linear model. There are two methods which can realize the approximation.

The method 1 consists of two steps:

- S1 First use integer order nonlinear models (IONM) to approximate the original model in frequency domain.
- S2 Based on T-S fuzzy method, integer order linear model (IOLM) are used to approximate the middle model.

There are two steps in method 2 too.

- S1 We use the generalized T-S fuzzy model to approximate the original model, obtaining a series of fractional order linear models (FOLM).
- S2 Secondly, using the method in II.B, we get the final integer order linear approximation model.

Supposing the number of approximated models zeros is k, the number of the fuzzy set for each state is $\kappa\ge 2$, then we can get the model type, the number of models and the number of models dimension in every step of approximation with the two methods. The results are shown in TABLE I–TABLE II. It can be clearly seen that method 2 need less dimension models than method 1, with less computational complexity. As a result we adopt the second method in Section III.

III. Main Results

A. Approximation in time domain

Our objective is to develop an approach to approximate such general nonlinear system as Eq.(11) by the following class of generalized T-S fuzzy models:

TABLE I. THE CHARACTERISTICS OF MODELS OBTAINED BY TWO METHODS

step	method 1		method 2	
	FONM		FONM	
	number	1	number	1
	dimension	n	dimension	n
	IONM		FOLM	
S1	number	1	number	κ^{n+m}
	dimension	$n\left(k+1\right)$	dimension	n
	IOLM		IOLM	
S2	number	$\kappa^{n(k+1)+m}$	number	κ^{n+m}
	dimension	$n\left(k+1\right)$	dimension	$n\left(k+1\right)$

TABLE II. THE COMPUTATION COMPLEXITY OF TWO METHODS

	The computation complexity in **S1**
method 1	$O\left(n\left(k+1\right)\right)$
method 2	$O\left(\kappa^{n+m}n\left(m+n\right)\right)$
	The computation complexity in **S2**
method 1	$O\left(\kappa^{n(k+1)+m}n\left(k+1\right)\left[m+n\left(k+1\right)\right]\right)$
method 2	$O\left(\kappa^{n+m}n\left(k+2\right)\left[m+n\left(k+2\right)\right]\right)$

Plant rule R^l: **IF** $y_1(t)$ is μ_1^l AND$\cdots$ AND $y_n(t)$ is μ_n^l; $u_1(t)$ is ν_1^l AND$\cdots$ AND $u_m(t)$ is ν_m^l, **THEN**

$$\mathcal{D}^{\alpha}y=\bar{A}_l y+\bar{B}_l u, l\in S\triangleq\left\{1,2,\cdots,s\right\}, \tag{13}$$

where R^l denotes the lth fuzzy inference rule, s is the rules' total number, μ_i^l and ν_j^l are the fuzzy sets for any $i\in N\triangleq\{1,2,\cdots,n\}$ and $j\in M\triangleq\{1,2,\cdots,m\}$, $y\in\mathcal{R}^n$ is the output vector, $u\in\mathcal{R}^m$ is the input vector, and $\left[\bar{A}_l,\bar{B}_l\right]$ is the system matrix of the lth local model.

According to the standard fuzzy inference method, that is, using a singleton fuzzifier, product fuzzy inference, and center-average defuzzifier, the T-S fuzzy model in Eq.(13) can be rewritten as

$$\mathcal{D}^{\alpha}y=\sum\nolimits_{l=1}^{s}\mu_l\left(y,u\right)\left(\bar{A}_l y+\bar{B}_l u\right), \tag{14}$$

with

$$\mu_l\left(y,u\right)=\frac{\prod_{i=1}^{n}\mu_i^l\left(y_i\right)\prod_{j=1}^{m}\nu_j^l\left(u_j\right)}{\sum_{l=1}^{s}\prod_{i=1}^{n}\mu_i^l\left(y_i\right)\prod_{j=1}^{m}\nu_j^l\left(u_j\right)},$$

where $\mu_l\left(y,u\right)$ is the so-called normalized membership functions, satisfying $\mu_l\left(y,u\right)\geq 0$, for each $\mu_l\left(y,u\right)\geq 0$, and $\sum_{l=1}^{s}\mu_l\left(y,u\right)$=1.

Lemma 1 [24] Let a general nonlinear system be given in Eq.(11), where $G\left(y,u\right)$ is continuously differentiable on the compact set $X\times U$ with $G\left(0,0\right)=0$. Then, for any $\varepsilon_1>0$ and $\varepsilon_2>0$, there exists a T-S fuzzy model given in Eq.(13) such that

$$G\left(y,u\right)=G_{TS}\left(y,u\right)+\Delta G\left(y,u\right), \tag{15}$$

$$\left\|\Delta G\left(y,u\right)\right\|\leq\varepsilon_1\left\|y\right\|+\varepsilon_2\left\|u\right\|, \tag{16}$$

where

$$\left\{\begin{array}{l} G_{TS}\left(y,u\right)=\sum_{l=1}^{s}\mu_l\left(y,u\right)\left(\bar{A}_l y+\bar{B}_l u\right)\\ \Delta G\left(y,u\right)=\Delta A\left(y,u\right)y+\Delta B\left(y,u\right)u\end{array}\right. .$$

Proof: See [27, pp. 1145, Theorem 2.1]. Based on Lemma 1 the general fractional order nonlinear system can be approximated by T-S fuzzy models.

Remark1 Supposing all the following assumptions hold,

- For any $i\in N, j\in M$, there always exist $y_i\in[\underline{y}_i,\bar{y}_i]$, $u_j\in[\underline{u}_j,\bar{u}_j]$ satisfying $\mu_i^l\left(y_i\right)=1$, $\nu_j^l\left(u_j\right)=1$.
- For any $i\in N, j\in M$, $[\underline{y}_i,\bar{y}_i]$ and $[\underline{u}_j,\bar{u}_j]$ have the same fuzzy partition number κ, and they are uniform fuzzy partition.
- For any $y_i\in[\underline{y}_i,\bar{y}_i]\,(i\in N)$, there is always an $l\in S$ which keeps $\mu_i^l\left(y_i\right)+\mu_i^{l+1}\left(y_i\right)=1$ and $\mu_i^l\left(y_i\right)>0$, and for any $u_j\in[\underline{u}_j,\bar{u}_j]\,(j\in M)$, there is always an $l\in S$ which keeps $\nu_j^l\left(u_j\right)+\nu_j^{l+1}\left(u_j\right)=1$ and $\nu_j^l\left(u_j\right)>0$.

one can obtain that

- For any $(y,u)\in Y\times U$, the number of rules is $2^k\,(0\leq k\leq n+m)$ whose $\prod_{i=1}^{n}\mu_i^l\left(y_i\right)\prod_{j=1}^{m}\nu_j^l\left(u_j\right)$ is positive for all $l\in S$.
- The approximation error of our method satisfies
$$\left\|\Delta G\left(y,u\right)\right\|\leq\frac{d^2}{2(\kappa-1)^2}\sup_{(y,u)}f\left(y,u\right),$$
in which $d=\max\{\bar{y}_i-\underline{y}_i,\bar{u}_j-\underline{u}_j,i\in N,j\in M\}$, $f\left(y,u\right)=\left\|\frac{\partial^2 G(y,u)}{\partial y^2}\right\|+2\left\|\frac{\partial^2 G(y,u)}{\partial y\partial u}\right\|+\left\|\frac{\partial^2 G(y,u)}{\partial u^2}\right\|$.

B. Approximation in frequency domain

According to the preceding discussion in the Section II, the local model in Eq.(13) can be equivalently described as a frequency distributed model as follows

$$\left\{\begin{array}{l}\frac{\partial z(\omega,t)}{\partial t}=-\omega z\left(\omega,t\right)+\bar{A}_l y+\bar{B}_l u\\ y=\int_0^{\infty}\eta_\alpha\left(\omega\right)z\left(\omega,t\right)\mathrm{d}\omega\end{array}\right. , \tag{17}$$

where $z\left(\omega,t\right)\in\mathcal{R}^n$ denotes the frequency distributed state, $\eta_\alpha\left(\omega\right)=\mathrm{diag}\left(\eta_{\alpha_1}\left(\omega\right),\eta_{\alpha_2}\left(\omega\right),\cdots,\eta_{\alpha_n}\left(\omega\right)\right)$, $\eta_{\alpha_i}\left(\omega\right)=\omega^{-\alpha_i}\sin\left(\alpha_i\pi\right)/\pi$, for any $i\in N$.

As we know, the model in Eq.(17) is the exact model of the original system. Utilizing our approximation method, we obtain the corresponding finite dimension approximate model

$$\left\{\begin{array}{l}\frac{\mathrm{d}z(\omega_i,t)}{\mathrm{d}t}=-\omega_i z\left(\omega_i,t\right)+\bar{A}_l y+\bar{B}_l u, i\in K\\ y=\sum_{i=0}^{k}C_i z\left(\omega_i,t\right)\end{array}\right. , \tag{18}$$

where $C_i=\mathrm{diag}\left(c_{\alpha_1,i},c_{\alpha_2,i},\cdots,c_{\alpha_n,i}\right)$, $c_{\alpha_j,i}$ is the weight coefficient c_i of the discrete frequency distributed model of fractional order integrator $\mathcal{I}_{\alpha_j}\left(s\right)$ at ω_i for any $i\in K$.

For any $i\in K\cup\{0\}$ and $j\in N$, if we define $x\left(t\right)=\left[z^{\mathrm{T}}(\omega_0,t)\;z^{\mathrm{T}}(\omega_1,t)\;\cdots\;z^{\mathrm{T}}(\omega_k,t)\right]^{\mathrm{T}}$ as the system state variable, $A=\mathrm{diag}\left(A_0,A_1,\cdots,A_k\right)$, where $A_i=\mathrm{diag}\left(-\omega_{\alpha_1,i},-\omega_{\alpha_2,i},\cdots,-\omega_{\alpha_n,i}\right)$, $\omega_{\alpha_j,i}$ is the ith pole of the discrete frequency distributed model of fractional order integrator $\mathcal{I}_{\alpha_j}\left(s\right)$, $B=\left[B_0\;B_1\;\cdots\;B_k\right]^{\mathrm{T}}$, $B_i=I_n$, I_n is the $n\times n$ identity matrix, $C=\left[C_0\;C_1\;\cdots\;C_k\right]$, then the local T-S model in Eq.(13) can be written as

$$\left\{\begin{array}{l}\dot{x}=\left(A+B\bar{A}_l C\right)x+B\bar{B}_l u, l\in S\\ y=Cx\end{array}\right. . \tag{19}$$

The global T-S model can be described as

$$\left\{ \begin{array}{l} \dot{x} = \sum_{l=1}^{s} \mu_l\left(Cx, u\right)\left[\left(A + B\bar{A}_l C\right)x + B\bar{B}_l u\right] \\ y = Cx \end{array} \right. . \quad (20)$$

Remark 2 The advantage of the approximation is that we can avoid much complex computing about fractional order and nonlinear problem. The cost of the approximation is that the degree and the number of the final system in Eq.(20) become larger than the original system in Eq.(11).

IV. Numerical Examples

A. *Affine nonlinear system*

Consider a fractional order affine nonlinear system

$$\left\{ \begin{array}{l} \mathcal{D}^{0.055} y_1 = y_2 \\ \mathcal{D}^{0.200} y_2 = y_3 \\ \mathcal{D}^{0.745} y_3 = -y_1^3 - y_2^2 - y_3 + u \end{array} \right. .$$

Select the approximation parameters as $\gamma = 100$, $[\omega_l, \omega_h] = [0.001, 1000]$ Hz, $k = 20$, $\kappa = 3$, then there are 9 fuzzy rules. The uniform distribution of triangular membership functions curve of y_1 and y_2 are shown as Fig. 1.

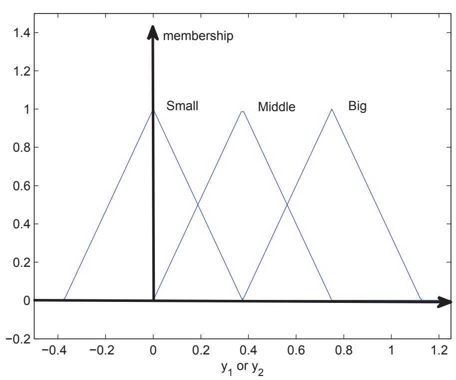

Fig. 1. The membership functions curve of y_1 and y_2

The system matrices $\left[\bar{A}_l, \bar{B}_l\right]$ of the lth local model $(l \in S)$ are shown as

$$\bar{A}_{3(i-1)+j} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a\left(i\right) & b\left(j\right) & -1 \end{bmatrix}, \bar{B}_{3(i-1)+j} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

where $a = [0.029 \ -0.113 \ -0.544], b = [0 \ -0.375 \ -0.75]$.

Based on the initial conditions, we can get the step responses of the original model, the middle approximation model shown in Eq.(14) and the final approximation model shown in Eq.(20) as Fig. 2.

Obviously, the three step response curves are very close, which just indicate that the proposed approximation method is very effective for the fractional order affine nonlinear system.

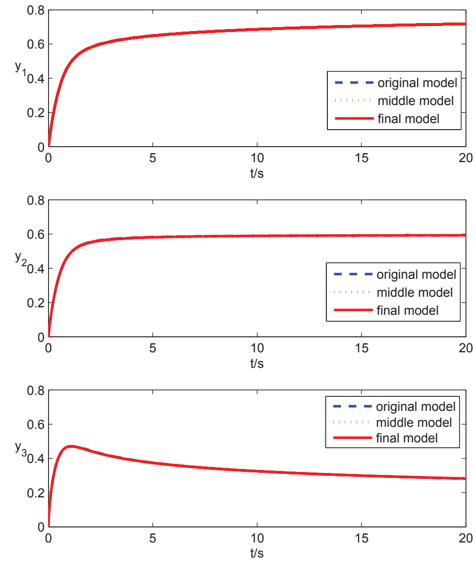

Fig. 2. The step response of y_1, y_2 and y_3

B. *Non-affine nonlinear system*

Consider a fractional order non-affine nonlinear system

$$\left\{ \begin{array}{l} \mathcal{D}^{0.6} y_1 = y_2 \\ \mathcal{D}^{0.3} y_2 = 0.6\cos\left(y_1\right)\left[u + \sin\left(u\right)\right] + y_2 \end{array} \right.$$

We select the same approximation parameters as the affine nonlinear example, then there are 9 fuzzy rules. and have the same uniform distribution of triangular membership functions. The fuzzy sets center of y_1 and u are $\{0.01,\ 1.255,\ 2.5\}$ and $\{0.01, 0.505, 1\}$, respectively.

The system matrices $\left[\bar{A}_l, \bar{B}_l\right]$ of the lth local model $(l \in S)$ are shown as

$$\bar{A}_{3(i-1)+j} = \begin{bmatrix} 0 & 1 \\ a\left(i,j\right) & 1 \end{bmatrix}, \bar{B}_{3(i-1)+j} = \begin{bmatrix} 0 \\ b\left(i,j\right) \end{bmatrix}.$$

where the parameters $a = \begin{bmatrix} 0.0024 & 0.006 & 0.23 \\ -0.0028 & -0.024 & -0.27 \\ -0.0034 & -0.025 & -0.18 \end{bmatrix}$ and $b = \begin{bmatrix} 1.23 & 1.15 & 1.11 \\ 0.706 & 0.70 & 0.69 \\ -0.196 & -0.91 & -0.48 \end{bmatrix}$.

Based on the assumptions, we can get the simulation results which are illustrated in Fig. 3.

We can easily see that the approximation performance is excellent. Meanwhile, it is found that the approximation error in **S2** is much smaller than that in **S1**. The approximation performance is a bit poor when the original curve changes severely.

V. Conclusion

The fractional order nonlinear dynamic system is a kind of special, important and difficult system. The specific contribution of the article is that we propose a novel approach

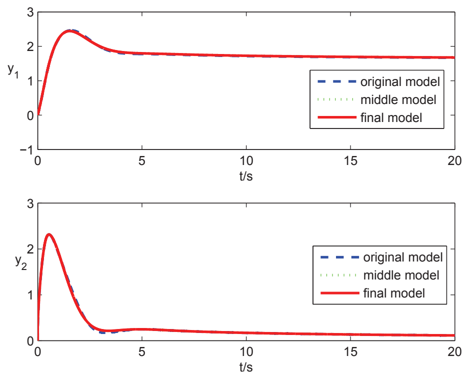

Fig. 3. The step response of y_1 and y_2

to approximate fractional order dynamic system based on T-S fuzzy dynamic models. Firstly, a generalized T-S fuzzy method is used to approximate the general FONS. Secondly, a new approximation method for fractional order integrator is extended to approximate the fractional order T-S models. Theoretical analysis and numerical simulation show that a general FONS can be approximated by a series of IOLM to any degree of accuracy on any compact set. It is believed that the approach provide a new avenue to solve the related problem of FONS.

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