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Adaptive T-S Fuzzy Sliding Mode Control of MEMS Gyroscope

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Abstract—**In this paper, a MIMO Takagi–Sugeno (T-S) fuzzy model is built on the basis of the nonlinear model of micro -electro mechanical system (MEMS) gyroscope. A robust adaptive sliding mode control with on-line identification for the upper bounds of external disturbance and estimator for the model uncertainty parameters is proposed. Based on Lyapunov methods, these adaptive laws can guarantee that the system is asymptotically stable, and force the proof mass of the MEMS gyroscope to oscillate in the x and y direction at given frequency and amplitude. The controller is implemented on the nonlinear model of MEMS gyroscope at the same time. Numerical simulations are investigated to verify the effectiveness of the proposed control scheme on the T-S model and the nonlinear model.**

I. INTRODUCTION

MEMS gyroscopes have become the most growing micro-sensors for measuring angular velocity in recent years due to its compact size, low cost and high sensitivity. The constant vibration of the proof mass in the MEMS gyroscope is essential in the process of measuring the angular velocity. Cross stiffness and damping resulting from fabrication imperfections, time-varying parameters, quadrature errors and external disturbances have negative influence on the performance of the MEMS gyroscope. In the last few years, many control approaches have been proposed to control the MEMS gyroscope to oscillate in the given direction at given frequency and amplitude, and improve its performance and stability. Park et al. [1] presented an adaptive controller for a MEMS gyroscope which drive both axes of vibration and controls the entire operation of the gyroscope. Leland [2] proposed an adaptive control of a MEMS gyroscope using force-to-rebalance operation. John and Vinay [3] proposed a novel concept for an adaptively controlled triaxial angular velocity sensor device. Sliding mode control is a robust control technique which has many attractive features such as robustness to parameter variations and insensitivity to external disturbance, having some limitation such as chattering or high frequency oscillation in practical applications. Utkin [4][5] showed that variable structure control is insensitive to parameters perturbations and external disturbances. Batur et al. [6] developed a sliding mode control for a MEMS gyroscope system. Adaptive sliding mode control has the advantages of combining the robustness of variable structure methods with the tracking capability of adaptive control strategies. Adaptive sliding mode control approaches have been developed to control the MEMS gyroscope in [7]. Fei et al. [8] derived an adaptive sliding mode control for a MEMS gyroscope. [9] presented a robust adaptive sliding mode controller for triaxial gyroscope. An adaptive sliding mode controller with upper bound estimation has been developed to control the vibration of MEMS gyroscope by [10]. Intelligent control approaches such as fuzzy control do not depend on mathematical models and have ability to approximate nonlinear systems. Adaptive fuzzy sliding mode controller can be utilized to compensate the model uncertainties and disturbances since it combines the merits of the sliding mode control, the fuzzy inference mechanism and the adaptive algorithm. In [11], an adaptive fuzzy sliding mode control (AFSMC) for micro-electro-mechanical system (MEMS) triaxial gyroscope is proposed. A fuzzy logic adaptive sliding mode control using feedback linearization approach is proposed for the micro-electro mechanical system triaxial gyroscope with unknown system nonlinearities in [12].

The system nonlinearities in MEMS gyroscope model have been described in [13][14]. Using the fuzzy implications and the fuzzy reasoning methods suggested by Takagi and Sugeno [15], a real nonlinear plant model could be constructed by local linear models referring to [16]. Chien et al. [17] developed robust adaptive controller design for a class of uncertain nonlinear systems using online T-S fuzzy-neural modeling approach. Park et al. [18] designed T-S model based indirect adaptive fuzzy controller using online parameter estimation. Systematic stability analysis and controller design of the robust adaptive fuzzy controller using T-S fuzzy model for MEMS gyroscope have not been investigated before. Therefore, adaptive sliding mode control using T-S model is utilized to approximate the nonlinear system and compensate model uncertainties and external disturbances in the control of MEMS gyroscope, thus improving the tracking and compensation performance.

In this paper, the Lyapunov-based robust adaptive sliding model control strategy is applied to the MEMS gyroscope using T-S fuzzy model. The T-S fuzzy model can represent the system nonlinearity by using IF-THEN rules. The proposed adaptive sliding mode controller can guarantee the asymptotical stability of the closed loop system and improve the robustness of control system in the presence of model uncertainties and external disturbances. The control strategy proposed here has the following characteristics and contributions:

1) T-S modeling method provides a possibility for developing a systematic analysis and design method for complex nonlinear control systems. Since there exists nonlinearities in MEMS gyroscope system, it is necessary to utilize T-S fuzzy

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model to represent the nonlinear system approximately. The MIMO T-S fuzzy model with both parametric uncertainties and input disturbance of the MEMS gyroscope, is established based on the non-dimensional vector equation of motion of MEMS gyroscope.

2) The on-line adaptive algorithm to estimate the upper bound of the uncertainties and external disturbance is proposed referring to [19]. A robust adaptive sliding mode control with on-line identification of the upper bounds of external disturbance and estimator of the uncertainty parameters is proposed in the control of MEMS gyroscope using T-S model. The upper bound of external disturbance on-line adaptation could lower sliding mode gain and as a result, reduce the chattering that could damage the actuator in the practical application. The adaptive algorithm of model uncertainty parameters could compensate the error between optimal T-S model and the designed T-S model and alleviate the fluctuation of the sliding surface.

II. DYNAMICS OF MEMS GYROSCOPE

This section describes the dynamic of Z-axis MEMS gyroscope through non-dimensional transformation. Assume that the gyroscope is moving with a constant linear speed; the gyroscope is rotating at a constant angular velocity; the gyroscope undergoes rotations along z axis. A z-axis MEMS gyroscope is depicted in Fig.1. The nonlinear dynamic equations of such a gyroscope system can be derived as:

$$\begin{aligned} m\ddot{x}+d_{xx}\dot{x}+\left(d_{xy}-2m\Omega_z^*\right)\dot{y}+\left(k_{xx}-m\Omega_z^{*2}\right)x+k_{xy}y+k_{x^3}x^3&=u_x^* \\ m\ddot{y}+d_{yy}\dot{y}+\left(d_{xy}+2m\Omega_z^*\right)\dot{x}+\left(k_{yy}-m\Omega_z^{*2}\right)y+k_{xy}x+k_{y^3}y^3&=u_y^* \end{aligned} \quad (1)$$

where x, y represent the system generalized coordinates, m is the mass of proof mass. Fabrication imperfections contribute mainly to the asymmetric spring term $\mathrm{d_{xy}}$, and asymmetric damping terms $\mathrm{k_{xy}}$; d_{xx}, d_{yy} are damping terms; $\mathrm{k_{xx}}, \mathrm{k_{yy}}$ are linear spring terms; k_{x^3}, k_{y^3} are nonlinear spring terms; Ω_z^* is the input angular velocity; u_x^*, u_y^* are the control forces.

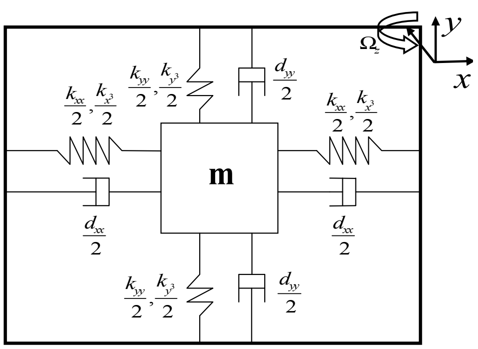

Fig.1. Z-axis vibratory MEMS gyroscope with nonlinear effective spring.

Dividing the equation by the proof mass, and because of the non-dimensional time $t^* = \omega_0 t$, dividing both sides of equation by reference frequency ω_0^2 and reference length q_0 and rewriting the dynamics in vector forms result in

$$\frac{\ddot{q}_1^*}{q_0}+\frac{D^*}{m\omega_0}\frac{\dot{q}_1^*}{q_0}+2\frac{S^*}{\omega_0}\frac{\dot{q}_1^*}{q_0}-\frac{\Omega_z^{*2}}{m\omega_0}\frac{q_1^*}{q_0}+\frac{K_1^*}{m\omega_0^2}\frac{q_1^*}{q_0}+\frac{K_3^*}{m\omega_0^2}\frac{q_1^{*3}}{q_0}=\frac{u^*}{m\omega_0^2 q_0} \quad (2)$$

where

$$q_1^* = \begin{bmatrix} x \\ y \end{bmatrix}, u^* = \begin{bmatrix} u_x^* \\ u_y^* \end{bmatrix}, D^* = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}, S^* = \begin{bmatrix} 0 & -\Omega_z^* \\ \Omega_z^* & 0 \end{bmatrix},$$

$$K_1^* = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}, K_3^* = \begin{bmatrix} k_{x^3} & 0 \\ 0 & k_{y^3} \end{bmatrix}.$$

Define a set of new parameters as follows:

$$q_1 = \frac{q_1^*}{q_0}, u = \frac{u_1^*}{m\omega_0^2 q_0}, \Omega_z = \frac{\Omega_z^*}{\omega_0}, \mathrm{S} = \frac{S}{\omega_0}, D = \frac{D^*}{m\omega_0},$$
$$K_1 = \frac{K_1^*}{m\omega_0^2}, K_3 = \frac{K_3^* q_0^2}{m\omega_0^2}.$$

The final form of the non-dimensional vector equation of motion for the z-axis gyroscope is

$$\ddot{q}_1 = \left(2S - D\right)\dot{q}_1 + \left(\Omega_z^2 - K_1\right)q_1 - K_3 q_1^3 + u \quad (3)$$

III. T-S FUZZY MODEL

T-S model is based on a set of fuzzy rules to describe a global nonlinear system in terms of a set of local linear models which are smoothly connected by fuzzy membership functions. T-S fuzzy models include two kinds of knowledge: one is qualitative knowledge represented by fuzzy IF-THEN rules, and the other is quantitative knowledge represented by local linear models [20]. The MIMO T-S fuzzy model with both parametric uncertainties and input disturbance of the MEMS gyroscope, is established based on the vector equation of MEMS gyroscope (3). The T-S fuzzy model of the MEMS gyroscope could be composed by 9 IF–THEN rules, which include both fuzzy inference rules and local analytic linear models. The ith rule has the form

Rule i : *IF* x *is about* M_{i1} *and* y *is about* M_{i2}

and $\dot{x}$ *is about* M_{i3} $\dot{y}$ *is about* M_{i4}

THEN $\dot{q} = \left(A_i + \Delta A_i\right)q + B_i u + H_i\delta, i = 1, 2, \cdots, 9$

By using the strategy of singleton fuzzification, product inference and center-average defuzzification, defuzzification fuzzy dynamic T-S model is

$$\begin{aligned} \dot{q}(t) &= \sum_{i=1}^{9}\mu_i\left[\left(A_i+\Delta A_i\right)q + B_i u + H_i\delta\right] \\ &= \sum_{i=1}^{9}\mu_i A_i q + \sum_{i=1}^{9}\mu_i\Delta A_i q + \sum_{i=1}^{9}\mu_i B_i u + \sum_{i=1}^{9}\mu_i H_i\delta \\ &= Aq + \Delta A q + Bu + H\delta \end{aligned} \quad (4)$$

where

$$A = \sum_{i=1}^{9}\mu_i A_i, A_i = \begin{bmatrix} a_{11}^i & a_{12}^i & a_{13}^i & a_{14}^i \\ a_{21}^i & a_{22}^i & a_{23}^i & a_{24}^i \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \Delta A = \sum_{i=1}^{9}\mu_i\Delta A_i,$$

$$\Delta A_i = \begin{bmatrix} \Delta a_{11}^i & \Delta a_{12}^i & \Delta a_{13}^i & \Delta a_{14}^i \\ \Delta a_{21}^i & \Delta a_{22}^i & \Delta a_{23}^i & \Delta a_{24}^i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \sum_{i=1}^{9} \mu_i B_i, B_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$H = \sum_{i=1}^{9} \mu_i H_i, H_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, q = \begin{bmatrix} \dot{x} \\ \dot{y} \\ x \\ y \end{bmatrix}, u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \delta = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix},$$

$\mu_i = \frac{\eta_i}{\sum_{i=1}^{4} \eta_i}$ is the normalized membership function, where $\eta_i = \eta_{M_{i1}} \eta_{M_{i2}} \eta_{M_{i3}} \eta_{M_{i4}}, \eta_{M_{i1}}, \eta_{M_{i2}}, \eta_{M_{i3}}, \eta_{M_{i4}}$ are membership function values of the fuzzy variable $x, y, \dot{x}, \dot{y}$ and with respect to fuzzy set $M_{i1}, M_{i2}, M_{i3}, M_{i4}$ respectively, ΔA_i is the parameter uncertainties and δ is the input disturbance.

Assumption 1(Matching condition): There exists matrix of appropriate dimensions G such that $H = BG$, where BG is the matched disturbance.

According to (4), there exists $G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Under the assumption of matching condition, (4) can be rewritten as:

$$\begin{aligned} \dot{q}(t) &= Aq + \Delta A q + Bu + H\delta \\ &= Aq + \Delta A q + Bu + BG\delta \\ &= Aq + \Delta A q + B(u + f) \end{aligned} \tag{5}$$

where Bf represents the lumped external disturbances, which is given by $f = G\delta$.

IV. ADAPTIVE FUZZY SLIDING MODE CONTROL

In this section, a robust adaptive sliding mode control strategy using T-S model for MEMS gyroscopes is proposed as shown in Figure 2. A detailed study of the robust sliding mode control algorithm with proportional sliding surface is presented in the presence of matched parameter uncertainties and external disturbances. The on-line identification of the upper bounds of external disturbance and the adaptive algorithm for parameter uncertainties is proposed with the sliding mode controller to alleviate the chattering in the control force and the fluctuation of the sliding surface. The controller contains three parts U_{eq}, U_n, U_s. The controller is also implemented on the nonlinear model to verify the correctness of the T-S model and the feasibility of the proposed control scheme on the nonlinear model of MEMS gyroscope.

The control target for MEMS gyroscope is to force the proof mass to oscillate in the x and y direction at given frequency and amplitude such as $\begin{matrix} x = A_x \sin(\omega_x t) \\ y = A_y \sin(\omega_y t) \end{matrix}$. The state-space equation of reference model can be defined as

$$\dot{q}_r = A_r q_r \tag{6}$$

where

$$A_r = \begin{bmatrix} 0 & 0 & -\omega_x^2 & 0 \\ 0 & 0 & 0 & -\omega_y^2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \dot{q}_r = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A_x \omega_x^2 \sin(\omega_x t) \\ A_y \omega_y^2 \sin(\omega_y t) \\ A_x \omega_x \cos(\omega_x t) \\ A_y \omega_y \cos(\omega_y t) \end{bmatrix}, q_r = \begin{bmatrix} A_x \omega_x \cos(\omega_x t) \\ A_y \omega_y \cos(\omega_y t) \\ A_x \sin(\omega_x t) \\ A_y \sin(\omega_y t) \end{bmatrix}.$$

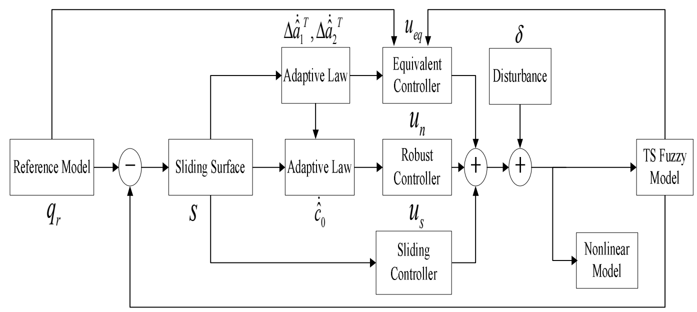

Fig.2. Block diagram of adaptive fuzzy sliding mode control

Define the tracking error as

$$e = q - q_r \tag{7}$$

Then the derivative of tracking error is

$$\dot{e} = \dot{q} - \dot{q}_r = Aq + \Delta A q + B(u + f) - A_r q_r \tag{8}$$

Define the sliding surface as

$$s = Ce \tag{9}$$

where c is a constant matrix as

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \end{bmatrix}$$

And the derivative of sliding surface is

$$\dot{s} = C\dot{e} = CAq + C\Delta A q + CB(u + f) - CA_r q_r \tag{10}$$

As we know, the sliding mode control requires the upper bound of uncertainties and disturbances to specify the sliding mode gain to satisfy the requirement of stability and robustness. In conventional sliding mode control, the upper bound of uncertainties, which includes parameter variations and external disturbances, must be available. However, the bound of the uncertainties is difficult to measure in advance for practical applications. If the upper bound is chosen to be too small, it may not compensate the model uncertainties and external disturbances to guarantee reaching condition of sliding mode, so the control system may be unstable. High sliding mode gain will cause large chattering. Therefore, an on-line identification algorithm for the upper bound of external disturbance is proposed to estimate the optimal upper bound to alleviate the chattering.

Assumption 2(Bounded condition): The matched lumped disturbance f is bounded such as $\|f\| = \|G\delta\| \le c_0$, where c_0 is unknown positive constant.

Define the estimation error of the optimal upper bound as $\tilde{c}_0 = \hat{c}_0 - c_0$, where $\hat{c}_0$ is the estimate of unknown positive constant c_0. Define the estimation error of parameter variations as $\Delta\tilde{A} = \Delta\hat{A} - \Delta A$, where $\Delta\hat{A}$ is estimate of unknown constant ΔA, referring to (4), the third and fourth column of ΔA is always zero, so define the $\Delta\tilde{A}$ as $\Delta\tilde{A} = [\Delta\tilde{a}_1 \quad \Delta\tilde{a}_2 \quad 0 \quad 0]$

where

$\Delta\tilde{a}_1 = \Delta\hat{a}_1 - \Delta a_1$,

$\Delta\hat{a}_1 = \left[\Delta\hat{a}_{11} \quad \Delta\hat{a}_{12} \quad \Delta\hat{a}_{13} \quad \Delta\hat{a}_{14}\right], \Delta a_1 = \left[\Delta a_{11} \quad \Delta a_{12} \quad \Delta a_{13} \quad \Delta a_{14}\right]$,

$\Delta\tilde{a}_2 = \Delta\hat{a}_2 - \Delta a_2$,

$\Delta\hat{a}_2 = \left[\Delta\hat{a}_{21} \quad \Delta\hat{a}_{22} \quad \Delta\hat{a}_{23} \quad \Delta\hat{a}_{24}\right], \Delta a_2 = \left[\Delta a_{21} \quad \Delta a_{22} \quad \Delta a_{23} \quad \Delta a_{24}\right]$

The adaptive controller is proposed as

$$u = u_{eq} + u_s + u_n \tag{11}$$

where $u_{eq} = (CB)^{-1}\left(CA_r q_r(t) - CAq(t) - C\Delta\hat{A}q(t)\right)$ is equivalent control, which describes the behavior of the system when the trajectories stay over the sliding manifold and a variable structure control part that enforces the trajectories to reach the sliding manifold and prevent them leaving the sliding manifold, $u_s = -(CB)^{-1}Ks$ is robust item, which can guarantee that the control system is asymptotically stable, $u_n = -\frac{B^T C^T s}{\|B^T C^T s\|}\hat{c}_0$ is sliding mode term which represents the nonlinear feedback control for suppressing the effect of the uncertainty.

Substituting (11) into (10)

$$\begin{aligned}
\dot{s} &= CAq + C\Delta Aq + CB[(CB)^{-1}\left(CA_r q_r - CAq - C\Delta\hat{A}q\right) \\
&\quad - (CB)^{-1}Ks - \frac{B^T C^T s}{\|B^T C^T s\|}\hat{c}_0 + f] - CA_r q_r \\
&= CAq + C\Delta Aq + CA_r q_r - CAq - C\Delta\hat{A}q \\
&\quad - Ks - CB\frac{B^T C^T s}{\|B^T C^T s\|}\hat{c}_0 + CBf - CA_r q_r \\
&= C\Delta Aq - C\Delta\hat{A}q - Ks - CB\frac{B^T C^T s}{\|B^T C^T s\|}\hat{c}_0 + CBf \\
&= -C\Delta\tilde{A}q - Ks - CB\frac{B^T C^T s}{\|B^T C^T s\|}\hat{c}_0 + CBf
\end{aligned} \tag{12}$$

Define a Lyapunov function candidate as

$$V = \frac{s^T s}{2} + \frac{\tilde{c}_0^{\,2}}{2r_1} + \frac{\Delta\tilde{a}_1\Delta\tilde{a}_1^T}{2r_2} + \frac{\Delta\tilde{a}_2\Delta\tilde{a}_2^T}{2r_3} \tag{13}$$

Differentiating V with respect to time yields

$$\dot{V} = s^T\dot{s} + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0 + \frac{\Delta\tilde{a}_1\Delta\dot{\tilde{a}}_1^T}{r_2} + \frac{\Delta\tilde{a}_2\Delta\dot{\tilde{a}}_2^T}{r_3} \tag{14}$$

where r_1, r_2, r_3 are positive constants.

Substituting (12) into (14) yields

$$\begin{aligned}
\dot{V} &= -s^T C\Delta\tilde{A}q - s^T Ks - \|B^T C^T s\|\hat{c}_0 + s^T CBf \\
&\quad + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0 + \frac{\Delta\tilde{a}_1\Delta\dot{\tilde{a}}_1^T}{r_2} + \frac{\Delta\tilde{a}_2\Delta\dot{\tilde{a}}_2^T}{r_3} \\
&= -s^T C\left[\Delta\tilde{a}_1 \quad \Delta\tilde{a}_2 \quad 0 \quad 0\right]q - s^T Ks - \|B^T C^T s\|\hat{c}_0 \\
&\quad + s^T CBf + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0 + \frac{\Delta\tilde{a}_1\Delta\dot{\tilde{a}}_1^T}{r_2} + \frac{\Delta\tilde{a}_2\Delta\dot{\tilde{a}}_2^T}{r_3} \\
&= -s^T CP_1\Delta\tilde{a}_1 q - s^T CP_2\Delta\tilde{a}_2 q - s^T Ks - \|B^T C^T s\|\hat{c}_0 \\
&\quad + s^T CBf + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0 + \frac{\Delta\tilde{a}_1\Delta\dot{\tilde{a}}_1^T}{r_2} + \frac{\Delta\tilde{a}_2\Delta\dot{\tilde{a}}_2^T}{r_3}
\end{aligned} \tag{15}$$

Choose the adaptive laws for parameter variations as

$$\begin{aligned}
\Delta\dot{\tilde{a}}_1^T &= r_2 q s^T CP_1 \\
\Delta\dot{\tilde{a}}_2^T &= r_3 q s^T CP_2
\end{aligned} \tag{16}$$

where $P_1 = [1 \quad 0 \quad 0 \quad 0]^T$, $P_2 = [0 \quad 1 \quad 0 \quad 0]^T$.

Substituting (16) into (15) yields

$$\begin{aligned}
\dot{V} &= -s^T Ks - \|B^T C^T s\|\hat{c}_0 + s^T CBf + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0 \\
&\le -s^T Ks - \|B^T C^T s\|\hat{c}_0 + \|B^T C^T s\|\|f\| + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0 \\
&\le -s^T Ks - \|B^T C^T s\|\left(\hat{c}_0 - c_0\right) + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0 \\
&= -s^T Ks - \|B^T C^T s\|\tilde{c}_0 + \frac{1}{r_1}\tilde{c}_0\dot{\tilde{c}}_0
\end{aligned} \tag{17}$$

Choose the adaptive laws for upper bound as

$$\dot{\tilde{c}}_0 = r_1\|B^T C^T s\| \tag{18}$$

Substituting (18) into (17) yields

$$\dot{V} = -s^T Ks \le \lambda_{\min}(K)\|s\| < 0 \tag{19}$$

where $\lambda_{\min}(K)$ is the minimum eigenvalues of K.

Therefore it has been proved that $\dot{V}$ is a negative definite, implying that $s, \tilde{c}_0, \Delta\tilde{a}_1, \Delta\tilde{a}_2$ are all bounded. From (12) it can be known that $\dot{s}$ is also bounded. $\dot{V} \le -\lambda_{\min}(K)\|s\|$ implies that s is integrable as $\int_0^t \|s\|dt \le \frac{1}{\lambda_{\min}(K)}[V(0) - V(t)]$. Since $V(0)$ is bounded and $V(t)$ is nonincreasing and bounded, $\lim_{t\to\infty}\int_0^t \|s\|dt$ is bounded. Since $\lim_{t\to\infty}\int_0^t \|s\|dt$ is bounded and $\dot{s}$ is also bounded, according to Barbalat's lemma, $s(t)$ will asymptotically converge to zero, $\lim_{t\to\infty} s(t) = 0$.

Remark : To eliminate the problem of integral wind-up in the adaptation of the upper bound of the unknown disturbance, the adaptive law is modified as

$$\dot{\tilde{c}}_0 = r_1\left(-r_4\tilde{c}_0 + \|B^T C^T s\|\right) \tag{20}$$

where r_4 is a positive constant.

V. SIMULATION STUDY

In this section, we will evaluate the proposed adaptive fuzzy control using T-S model approach on the lumped MEMS gyroscope sensor model. The control objective is design an adaptive fuzzy sliding mode controller so that the position q can track the reference model q_r. Parameters of the MEMS gyroscope sensor are as follows:

$m = 0.57e-8\text{kg}, \omega_0 = 1kHz, q_0 = 10e-6m, \Omega_z = 5.0rad/s$,

$k_{xx} = 80.98N/m, k_{yy} = 71.62N/m, k_{xy} = 5N/m$,

$k_{x^3} = 3.56e6N/m, k_{y^3} = 3.56e6N/m, d_{xx} = 0.429e-6Ns/m$,

$d_{yy} = 0.0429e-6Ns/m, d_{xy} = 0.0429e-6Ns/m$.

Since the general displacement range of the MEMS gyroscope sensor in each axis is sub-micrometer level, it is reasonable to choose $1\mu m$ as the reference length q_0. Given that the usual natural frequency of each axle of a vibratory MEMS gyroscope sensor is in the kHz range, ω_0 is chosen as 1 kHz. The unknown angular velocity is assumed $\Omega_z = 5.0rad/s$. The desired motion trajectories are $x = A_x \sin(\omega_x t), y = A_y \sin(\omega_y t)$, where $A_x = 1, A_y = 1.2$, $\omega_x = 6.71KHz$, $\omega_y = 5.11KHz$. The sliding mode parameter of (9) is chosen as $C = \begin{bmatrix} 0.01 & 0 & 1 & 0 \\ 0 & 0.01 & 0 & 1 \end{bmatrix}$, the controller parameters in $u_s = -(CB)^{-1} Ks$ is chosen as $K = \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix}$. The plant parameters are adjusted online by adaptive law (16) where the adaptive gain $r_2 = 0.01, r_3 = 0.01$, while the initial value $c_0 = 0$. The adaptive parameter of upper bound in (19) is $r_1 = 10, r_4 = 10$ and the initial value $\Delta\tilde{\alpha}_1 = [0 \quad 0 \quad 0 \quad 0]$, initial value $\Delta\tilde{\alpha}_2 = [0 \quad 0 \quad 0 \quad 0]$. The physical parameters in the T-S fuzzy model are not known exactly, hence, there exists error between optimal T-S model and the designed T-S model. In this simulation, we choose the control matrix of the T-S model in (4) as $A = \begin{bmatrix} -0.075 & 0.002 & -14207 & -877 \\ -0.017 & -0.12 & -877 & -12564 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, the initial state values of T-S model are $[0 \quad 0 \quad 0.5 \quad 0.6]$, initial state values of the nonlinear model are $[0 \quad 0 \quad 0.5 \quad 0.6]$, the external disturbance is $\begin{bmatrix} 10\sin(2\pi t) \\ 10\sin(2\pi t) \end{bmatrix}$. The fuzzy rules for T-S fuzzy model for the system can obtained from linearizing the nonlinear model (3) at the points $x \in \{-A_x \quad 0 \quad A_x\}, \dot{x} \in \{-A_x\omega_x \quad 0 \quad A_x\omega_x\}$, $y \in \{-A_y \quad 0 \quad A_y\}, \dot{y} \in \{-A_y\omega_y \quad 0 \quad A_y\omega_y\}$.

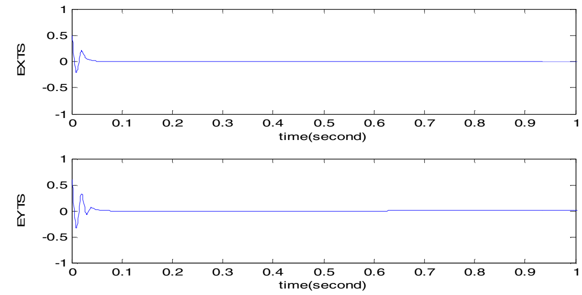

Fig.3. The tracking error e_{T-S}

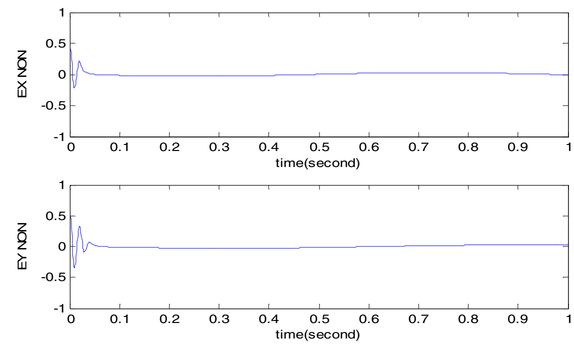

Fig.4. The tracking error e_{NON}

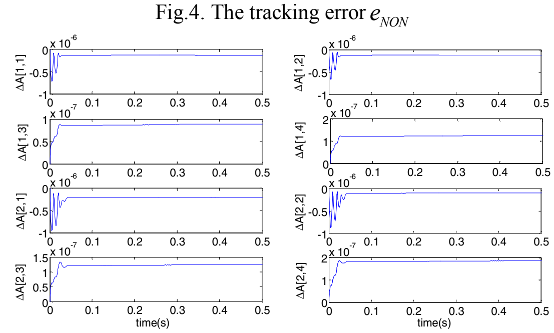

Fig.5. Convergence of the ΔA.

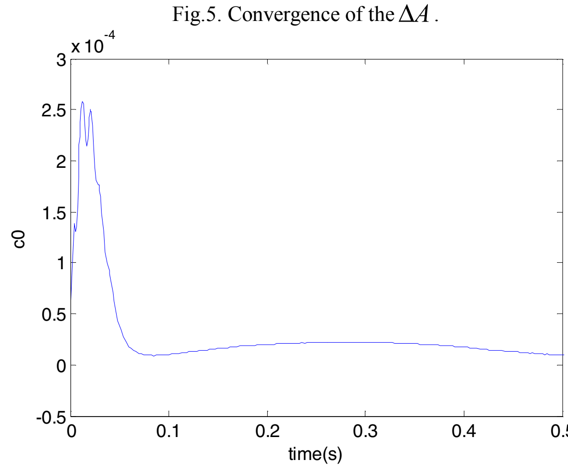

Fig.6. Convergence of the c_0.

The tracking error e_{T-S} between the reference model and T-S model is shown in Fig.3. and Fig.4 depicts the tracking error e_{NON} between the reference model and nonlinear model. From Fig.3. and Fig.4, we can see the tracking errors e_{T-S} and e_{NON} converge to zero asymptotically. It can be concluded that the MEMS gyroscope can maintain the proof mass to oscillate in the x and y direction at given frequency and amplitude in the presence of the external disturbances and uncertainty parameters. Fig.4 confirms the effectiveness of the proposed control on the nonlinear model. Fig.5 plots the estimation of model uncertainty parameters. It can be found that the model uncertainty parameters ΔA converge to constant values. Fig.6 demonstrates the upper bound estimation of external disturbance converge to a constant value in very short time.

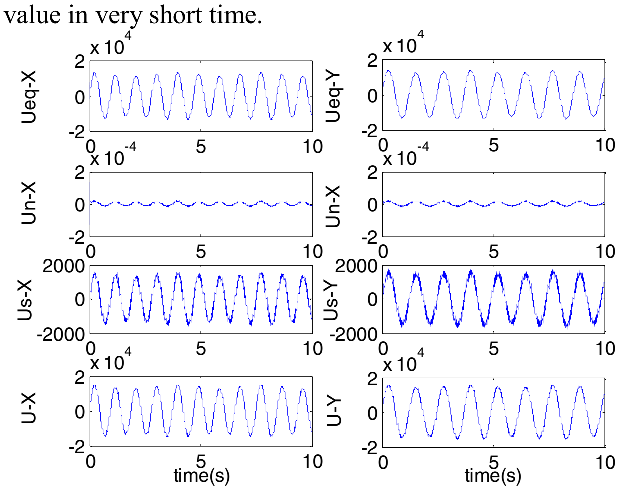

Fig.7. Control efforts with adaptive upper bound.

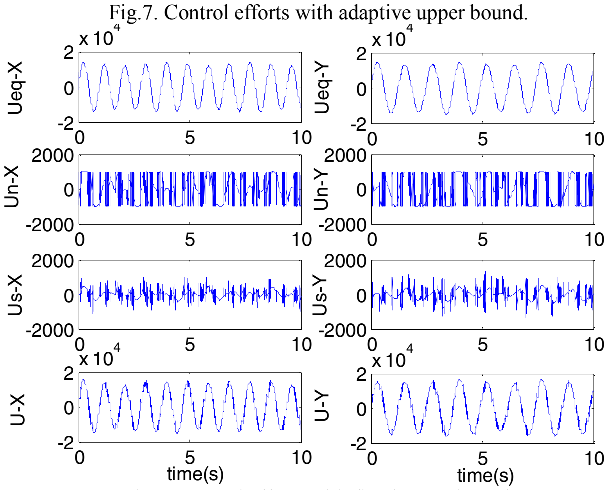

Fig.8. Control efforts with fixed upper bound.

Fig.7 and Fig.8 compare the control efforts between the control strategy with adaptive upper bound and the conventional control strategy with fix upper bound. It is obvious that the control input in Fig. 7 is better than that of Fig. 8 and the chattering is reduced greatly when using the adaptive estimate of the upper bound of system disturbances.

VI. CONCLUSION

A robust adaptive sliding mode control for angular velocity sensor of MEMS gyroscope using T-S fuzzy model is presented in this paper. The asymptotical stability of the closed-loop system can be guaranteed with the proposed control strategy. The upper bound of external disturbance on-line adaptation could lower sliding mode gain and as a result, reduce the chattering that could damage the actuator in the practical application. The adaptive algorithm for uncertainty parameters could compensate the error between optimal T-S model and the T-S model we choose, as well as improved the sliding surface convergence performance. The proposed control scheme can achieve a good robust and favorable tracking performance against parameter uncertainties and external disturbances. Simulation studies are implemented to verify the effectiveness of the proposed adaptive fuzzy control for angular velocity sensor in the presence of unknown upper bound of external disturbance and model uncertainties.

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