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The Auto-Revising Method for Fuzzy Rule-Base

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***Abstract*—To refine the fuzzy rule base, it is important to find out and remove the redundant rules in the large scale rule base automatically. On the topic, a synthetical process is described. The abstraction relationship between the clauses is presented firstly, and then the redundant rule is defined in strictly formal criteria on the base of three laws: Transitive Law, Precondition Specific Law and Conclusion Abstraction Law. Based on the theorems, the algorithm to revise the knowledge base is presented.**

Keywords—intelligent system; rule base; redundant rule; fuzzy

I. INTRODUCTION

The rule base is widely used in the artificial intelligent system [1-4]. Rule base model differs from non-symbolic model, mainly in that it can represent knowledge in an inspectable manner using if-then rules [5-6]. This facilitates validation and correction by human experts and provides a way of communicating with the users. Rule base model can be built by encoding expert knowledge into linguistic rules, giving a transparent system with knowledge that can be maintained and expanded by human experts [7]. However, knowledge acquisition is a tedious and complicated task. Experts are not always available and their knowledge is often incomplete, episodic and time varying [7-9]. When the new rules are added to a rule base, the reasoning performance of the intelligent system cannot improve consistently because of conflict and redundancy [10-12].

In some rule bases, the redundant rules not only enlarge the base, but also led to a wrong conclusion. In a fuzzy rule base system, if two rules r1, r2, whose conclusions are both Q, are met, the fuzziness of Q is[13]:

$$\mathrm{F(Q)} = \mathrm{F_1\,(Q)} + (1 - \mathrm{F_1\,(Q)})\,\mathrm{F_2\,(Q)} \qquad (1)$$

Here $\mathrm{F_1}$ (Q), $\mathrm{F_2}$ (Q) is the fuzziness of Q in rule 1 and rule 2 respectively and $F_1(Q) > F_2\,(Q)$. The conclusions enforce each other if the conclusions are derivate from different preconditions. However, if there are redundant rules in the base, then there are two or more reasoning paths from the same precondition to a conclusion [14][15]. It will cause the bias in confidence of conclusion. The redundant rules mean worthless or harmful duplication of a rule. For example there are some rules in the rule base:

r1: “x1 is A1”→”y1 is B1”
r2: “y1 is B1”→”y2 is B2”
r3: “x1 is A1”→”y2 is B2”

When the condition “x1 is A1” is met, we may get the conclusion “y2 is B2” by the path r1--r2 and path r3. It means there are redundant rules in the base.

Besides leading to a wrong conclusion, the redundant rules enlarge the rule base and reduce the reason speed. In order to shrink the rule base and accelerate reasoning, it is very important to revise the rule base [16-18]. One of the problems is to make sure that a knowledge base is consistent in the process of revising. Thus, the consistency should be checked [17]--[19]. This topic is also important for other artificial intelligent system.

Although less redundancy and conflicts can be superficially eliminated in form, more of them still exist at a deep layer when different rules work together [20]. The key problem is to find the combinatorial redundant rule in the applications of fuzzy logic.

In [20] the authors found the redundancy of fuzzy rule bases that derive from extensive sharing of a limited number of output membership functions among the rules and provided the means to detect and remove such kind redundancy.

In the [21], error driven method was presented. The rule base reduction can be fitted under two categories error-free reduction or degrading reduction. Error-free reduction searches for existing redundancies in the model. However the redundant rule is not clear and definite, so the reduction method cannot be applied automatic.

In [22] the authors explored the GA (Genetic Algorithm) to find redundancy in the fuzzy model for the purpose of base reduction. An aggregated similarity measure was applied to search for redundancy in the rule base description.

An algebraic approach was presented to revise the propositional rule-based knowledge bases in [23]. A way was introduced to transform a propositional rule-based knowledge base into a Petri net. A knowledge base is represented by a Petri net, and facts are represented by the initial marking. Thus, the consistency checking of a knowledge base is equivalent to the reach-ability problem of Petri nets. However the transfer process was complicated and difficult to be finished by the computer.

A frequency filter was used to delete redundant one while new acquired rules were added into rule base in [24]. The new optimization method was applied to a general MT system.

To revise the rule base, it is important to find out the redundant rules in the large scale fuzzy rules base automatically [25][26]. So that it is critical to define the redundant rule clearly and to design the algorithm to remove the redundancy. In the paper the general type of the fuzzy rule is discussed firstly, and then the redundant rule is defined in strictly formal criteria. Meanwhile the algorithms to find the redundant rule and to revise the rule are presented.

II. GENERAL TYPE OF FUZZY RULE

A. Definition

Generally fuzzy rules in an artificial intelligent system are based on the conjunctive rule format [2] :

$$\text{IF P1 AND P2 AND ... ... AND Pn THEN Q} \tag{2}$$

Simplified:

$$\text{P1 AND P2 AND ... ... AND Pn} \rightarrow \text{Q} \tag{3}$$

In the rule (3), the precondition, "P1 and P2...and Pn", is in conjunctive normal form, Pi ($1 \le i \le n$), Q are in disjunctive normal form. The Pi= pi1 or pi2 or pi3 or...or pim, Q= q1 or q2 or…or qm, are called clauses. Every pij in the Pi and q in Q, formed as "xij is Aij" (j=1…z), is an atom clause, such as "temperature is high", where the Aij (high) denotes the linguistic labels of the input variable xij (temperature). In the paper the Aij is the membership function of the fuzzy set high .

All rules may be transformed to the rules formed as rule (3) [17][18].

In rule (3), the fuzziness of pij is:

$$F(p_{ij}) = A_{ij}(x_{ij}) \tag{4}$$

In general case the fuzziness of $P_i= p_{i1}$ *or* p_{i2} *or* p_{i3}*...or* p_{im} is:

$$F(P_i) = \cup_{j=1}^{m}\left(F(p_{ij})\right) \tag{5}$$

Here the $\cup$ is the max operator.

The fuzziness of conclusion Q is:

$$F(Q)_2 = \cap_{i=1}^{n} F(P_i) \tag{6}$$

Here the $\cap$ is the min operator.

In the rule base if there are t rules which conclusion is Q, The fuzziness of conclusion Q:

$$F(Q) = \cup_{j=1}^{t}\left(F(Q)_j\right) \tag{7}$$

Here F(Q)j is fuzziness of Q in the j-th rule, which conclusion is Q.

In order to establish a clear definition of the redundant rule, we firstly present some theorems.

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Theorem1:

The rule:

r: P_1 and P_2 and … and P_{i-1} and (p_{i1} *or* p_{i2} *or …or* p_{im}) and P_{i+1} and ... and $P_n \rightarrow$ Q

May be replaced by the following rules:

r1: P_1 and P_2 and … and P_{i-1} and p_{i1} and P_{i+1} ... and $P_n \rightarrow$ Q
r2: P_1 and P_2 and … and P_{i-1} and p_{i2} and P_{i+1} ,... and $P_n \rightarrow$ Q
… …
rm: P_1 and P_2 and … and P_{i-1} and p_{im} and P_{i+1}... and $P_n \rightarrow$ Q

Proof:

In the rule r, $P_i = p_{i1}$ or p_{i2} or …or p_{im}

$$F(P_i) = \cup_{j=1}^{m}\left(F(p_{ij})\right) \tag{8}$$

$$\begin{aligned} F(Q) &= \cap_{j=1}^{n}\left(F(P_j)\right) \\ &= \cap_{j=1}^{i-1}\left(F(P_j)\right) \cap \mathrm{Pi} \cap_{j=i+1}^{n} F(P_j) \\ &= \cap_{j=1}^{i-1}\left(F(P_j)\right) \cap (\cup_{s=1}^{m} F(p_{is})) \cap_{j=i+1}^{n} F(P_j) \end{aligned} \tag{9}$$

In the rule r_1 - r_m :

r1: $F(Q)_1=\cap_{j=1}^{i-1}\left(F(P_j)\right) \cap p_{i1} \cap_{j=i+1}^{n} F(P_j)$
r2: $F(Q)_2=\cap_{j=1}^{i-1}\left(F(P_j)\right) \cap p_{i2} \cap_{j=i+1}^{n} F(P_j)$
… … …
rm: $F(Q)m=\cap_{j=1}^{i-1}\left(F(P_j)\right) \cap p_{im} \cap_{j=i+1}^{n} F(P_j)$

According to the equation (7), so

$$\begin{aligned} F(Q) &= \cup_{j=1}^{m}\left(F(Q)_j\right) = \\ &\left(\cap_{j=1}^{i-1} F(P_j) \cap p_{i1} \cap_{j=i+1}^{n} F(P_j)\right) \cup \left(\cap_{j=1}^{i-1} F(P_j) \cap p_{i2} \cap_{j=i+1}^{n} F(P_j)\right) \\ &\cup \ldots \cup \left(\cap_{j=1}^{i-1}\left(F(P_j)\right) \cap p_{im} \cap_{j=i+1}^{n} F(P_j)\right) \\ &= \cap_{j=1}^{i-1}\left(F(P_j)\right) \cap (\cup_{s=1}^{m} F(p_{is})) \cap_{j=i+1}^{n} F(P_j) \end{aligned} \tag{10}$$

The fuzziness of Q in the rule r is equivalent to that in the rules r1-- rm. So the r is equivalent to the r1, r2,…and rm. ---QED.

According to the Theorem 1, all the rules formed as the rule (3) can be transformed into:

$$\text{p1 and p2 and … and pn} \rightarrow \text{q1 or q2 or …or qm} \tag{11}$$

Here each pi ($1 \le i \le n$) and qj ($1 \le j \le m$) is an atom clause, formed as "xi is Ai". In the rule (11) the precondition is conjunctive normal and the conclusion is disjunctive normal form and every sub clause is an atom clause. In the next sections all rules are formatted as (11) except special version.

B. Example

- Example 1:

In the chromatogram of transformer oil, we monitor the transformer by the online monitoring of dissolved gas in transformer oil. There is a rule in the system:

IF "CO-percentage(x1) is high(A1)" AND "CH4-percentage(x2) is very high (A2)" THEN "insulation resistance(y) is Low (B)."

Here:

p1 = "CO -percentage(x1) is high(A1)".

p2 = "CH4-percentage(x2) is very high (A2)".

q = "insulation resistance(y) is Low (B)".

The membership function "High(A1)" to parameter CO-percentage(x1), the "Very High(A2)" to CH4-percentage(x2), "low" to "insulation resistance(y)" ,are illustrated in Fig. 1.

If the input x1 is 16ppm, x2 is 18ppm then the fuzziness of p1, p2 is:

$$F(p_1)=A_1(x_1) = A_1(16) = 0.7 \tag{12}$$
$$F(p_2)=A_2(x_2) = A_2(18) = 0.8 \tag{13}$$

Then

$$F(q)= \cap(A_1(16), A_2(18)) = 0.7 \tag{14}$$

- Example 2:

r: IF "CO-percentage(x1) is high(A1)" AND "(CH4-percentage(x2) is very high(A2)" OR "C2H6-percentage(x3) is high (A3))" THEN "Insulation resistance(y) is Low(B)".

be transformed into:

r1 : IF "CO-percentage(x1) is high(A1)" AND "CH4-percentage(x2) is very high(A2)" then "Insulation resistance(y) is Low(B)".

r2: IF "CO-percentage(x1) is high(A1)" AND "C2H6-percentage(x3) is high(A3)" then "Insulation resistance(y) is Low(B)".

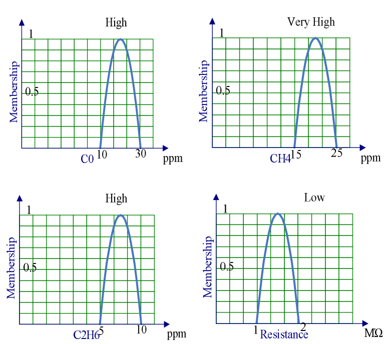

Fig.1. The membership function

Here p1=" CO -percentage(x1) is high(A1)" , p2= "CH4-percentage(x2) is very high(A2)" , p3= "C2H6-percentage(x3) is high(A3)", q= "insulation resistance(y) is Low(B)".

The A3 membership function is illustrated in Fig. 1.

If the input x1 is 16 ppm , x2 is 18 ppm and x3 is 8 ppm then

$$F(p_1) = A_1(x_1) = A_1(16) = 0.7 \tag{15}$$
$$F(p_2) = A_2(x_2) = A_1(18) = 0.8 \tag{16}$$
$$F(p_3) = A_3(x_3) = A_3(8) = 0.6 \tag{17}$$

In the rule r, the fuzziness of Q:

$$\begin{aligned} F(Q) &= \cap\left(A_1(16), \left(\cup\left(A_2(18), A_3(8)\right)\right)\right) \\ &= \cap(0.7, (\cup(0.8, 0.6))) \\ &= 0.7 \end{aligned} \tag{18}$$

In the rule r1, the fuzziness of Q:

$$\begin{aligned} F(Q)_1 &= \cap(A_1(16), A_2(18)) \\ &= \cap(0.7, 0.8) \\ &= 0.7 \end{aligned} \tag{19}$$

In the rule r2, the fuzziness of Q:

$$\begin{aligned} F(Q)_2 &= \cap(A_1(16), A_3(8)) \\ &= \cap(0.7, 0.6) \\ &= 0.6 \end{aligned} \tag{20}$$

According to the equation (10):

$$\begin{aligned} F(Q) &= \cup(F(Q)_1, F(Q)_2) \\ &= \cup(0.7, 0.6) \\ &= 0.7 \end{aligned} \tag{21}$$

The rule r may be replaced by the r1 and r2.

III. ABSTRACTION RELATIONSHIP

Definition 1: P, Q is assert statements (disjunctive normal form or conjunctive normal form), if *P* is the abstraction class of *Q*, noted as $Q \propto P$.

Definition 2: As to the atom clause p_1=" *x* is A_1" and p_2="*x* is A_2" , if $A_1 \subseteq A_2$ then $p_1 \propto p_2$.

For example：

p1= "speed(x) is Higher(A1)".

p2= "speed(x) is High(A2)".

The membership function of A_1 and A_2 are illustrated in Fig 2.

$\because$ $A_1 \subseteq A_2$

$\therefore$ $p_1 \propto p_2$

Definition 3: As to the conditions of the rules, the clause $Q_1=q_{11}$ or $q_{12}\ldots$, $Q_2=q_{21}$ or q_{22} or … . For every sub-clause q_{1j} in Q_1, if there is a sub-clause q_{2j} in Q_2 and $q_{1i} \propto q_{2j}$, then $Q_1 \propto Q_2$.

For example：

Q_1= "y_1 is B_1"or "y_2 is B_2"

Q_2= "y_1 is B_1"or "y_2 is B'_2"or"y_3 is B_3"

Here

q_{11}="y_1 is B_1", q_{12}="y_2 is B_2";

q_{21}="y_1 is B_1", q_{22}= "y_2 is B'_2" q_3="y_3 is B_3".

if $B_2 \subseteq B'_2$ then $q_{12} \propto q_{22}$

and

$\because q_{11} \propto q_{21}$

$\therefore Q_1 \propto Q_2$

Definition 4: As to the preconditions of the rules, the $P_1=p_{11}$ and p_{12} and ... and p_{1n}. $P_2=p_{21}$ and p_{22} and … and p_{2m}. For every p_{2i} in P_2, if there is a sub-clause p_{1j} in P_1 and $p_{1j} \propto p_{2i}$, then $P_1 \propto P_2$.

For Example:

P_1= "x_1 is A_1" and "x_2 is A_2" and "x_3 is A_3"

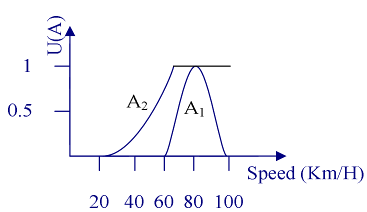

Fig.2. The Fuzzy set of A_1 and A_2

P_2= "x_1 is A_1" and "x_2 is A'_2"

Here

p11="x1 is A1", p12="x2 is A2", p13="x3 is A3".
p21="x1 is A1", p22= "x2 is A'2"
if A2 ⊆ A'2 then p12∝ p22
∴ p11∝ p21
∵ P1∝ P2

Theorem 2: The "abstraction" relationship is reflexive, anti-symmetric and transitive:

Reflexive: $P_1 \propto P_1$.

Anti-symmetric: if $P_1 \propto P_2$ and $P_2 \propto P_1$ then $P_1=P_2$.

Transitive: if $P_1 \propto P_2$ and $P_2 \propto P_3$ then $P_1 \propto P_3$.

Proof: (omit)

The abstraction relationship is partial order based on the *Theorem 2.*

IV. Redundant Rule

A. Definition

In order to revise the rule base, the redundant rule should be defined in strictly formal firstly.[26]--[28]

Theorem 3:

Transitive Law: $P_1 \rightarrow P_2$, $P_2 \rightarrow P_3$ => $P_1 \rightarrow P_3$.
Precondition Specific Law: $P_1 \rightarrow P_2$, $P_1' \propto P_1$ => $P_1' \rightarrow P_2$.
Conclusion Abstraction Law: $P_1 \rightarrow P_2$, $P_2 \propto P_2'$ => $P_1 \rightarrow P_2'$.
Proof: (omit) .

Definition 5: *F* is a rule base. The conclusion set of a clause *P* in *F* is P_F^+={P'| P→P' can be deduced by the three laws in Theorem 3 }.

Definition 6: For the rule r: O→O' in rule base F, let rule base G=F-{r}, if $O' \in O_G^+$ then *r* is a redundant rule in F.

B. Example

- Transitive Law

  Supposed there are the following rules in a rule base F

  r1: "x_1 is A_1"→"y_1 is B_1"

  r2: "y_1 is B_1"→"y_2 is B_2"

  r3: "x_1 is A_1"→"y_2 is B_2"

  G=F-r3; P="x_1 is A_1", Q1="y_1 is B_1", Q="y_2 is B_2".

  So $Q \in P_G^+$ (Transmit Law).

  In the rule base G, the fuzziness of Q is:

  $$F(Q) = F(Q_1) = F(P) = A_1(x_1) \tag{22}$$

  (path: *r1 →r2*)

  In the rule base F, The fuzziness of Q is :

  $$F(Q)=\cup(A_1(x_1), A_1(x_1))=A_1(x_1) \tag{23}$$

  (path1: *r1 →r2*)

  (path2: *r3*)

  The *r3* do not affects the fuzziness of Q. The rule *r3* is a redundant rule in rule base F.

- Precondition Specific Law

  In the rule base F, there two rues:

  r1: "x_1 is A_1" →"*y* is B"

  r2: "x_1 is A_2" →"*y* is B"

  Here P_1= "x_1 is A_1" ,P_2= "x_1 is A_2", Q="*y* is B", $A_2 \subseteq A_1$ so $P_2 \propto P_1$;

$G=F-\{r_2\}$;

$Q \in (P_2)_G^+$

In rule base G: $F(y) = A_1(x_1)$; (path : *r1*)

In rule base *F*, when we input x_1, the fuzziness of P_1, P_2 is $A_1(x_1)$ and $A_2(x_1)$.

$$F(y) = A_1(x_1) \cup A_2(x_1) \quad (24)$$

Because $A_2 \subseteq A_1$

So $A_2(x_1) < A_1(x_1)$, $B(y)= A_1(x_1)$

The *r2* is a redundant rule.

The following example is another "Precondition Specific Law" style redundant rule.

In the rule base F,

r1: "x_1 is A_1" →"y is B"

r2: "x_1 is A_1" and "x_2 is A_2" →"y is B"

Here P_1= "x_1 is A_1", P_2= "x_1 is A_1" and "x_2 is A_2", Q = "y is B",

$P_2 \propto P_1$

$G=F-\{r2\}$;

$Q \in (P_2)_G^+$

In rule base G: $B(y) = A_1(x_1)$;

In rule base F: $B(y) = A_1(x_1) \cup (A_1(x_1) \cap A_2(x_2)) = A_1(x_1)$

The *r2* is a redundant rule.

In the rule *r1*, if the precondition "x_1 is A_1" is true then the conclusion "y is B" is true. However in rule *r2* in order to deduce the conclusion "y is B", both precondition "x_1 is A_1" and "x_2 is A_2" must be true. The rule *r2* means the superfluous preconditions.

- Conclusion Abstraction Law

In the rule base F,

r1: "x is A" →"y is B_1"

r2: "x is A" →"y is B_2"

Here p= "x is A"

q_1= "y is B_1" , q_2="y is B_2", $B_1 \subseteq B_2$, $q_1 \propto q_2$

$G=F-\{r2\}$;

$Q \in (p)_G^+$

The *r2* is a redundant rule.

The following example is another "Conclusion Abstraction Law" style redundant rule.

In the rule base F,

r1: "x is A" → "y_1 is B1"

r2: "x is A"→" y_1 is B1" or "y_2 is B2"

Here p= "x is A"

q1= " y_1 is B_1" or "y_2 is B_2" , q_2="y_2 is B_2", $q_1 \propto q_2$

$G=F-\{r2\}$;

$Q \in (p)_G^+$

The *r2* is a redundant rule.

In the rule r2, if "x is A" is true, one or both of the "y_1 is B_1" and "y_2 is B_2" is true. In the rule r1, if "x is A" is true, only "y_1 is B_1" is true.

V. LEAST RULE BASE

In order to revise the rule base *F*, all the redundant rules should be removed from *F* and then the base becomes the least base F_m [20][21]. F_m is the rule base which has not the redundant rules. In the revising process, the new version rule base should be equivalent to the original rule base. It means $F_m \equiv F$ should be ensured.

Firstly the concept of equivalence between the two rule bases is defined in the following Definition 7 and Definition 8.

Definition 7: The close set of a rule base F is F^+ ={G| every rule in G that can be deduced by the rules in F according to the Theorem 3}.

Definition 8: If the two close sets of the rule base: $F^+=G^+$, then G is equivalent to F, signed as $G \equiv F$.

For example in the fuzzy rule base F_1 and F_2, there are the rules:

F_1 : r11: p_1 and $p_2 \to p_3$
r12: $p_1 \to p_3$
r13: $p_3 \to p_4$

F_2 : r21: $p_1 \to p_3$
r22: $p_3 \to p_4$
r23: $p_1 \to p_4$

The rule r11 may be deduced from the rule r21 (Precondition Specific Law). The rule r23 may be deduced from the rule r12 and r13 (Transitive Law). So the close sets of the rule base F1 and F2 :

$F_1^+= F_2^+$={r11,r12,r13,r23}.

Then the rule base F_1 is equivalence to F_2, $F_1 \equiv F_2$;

Definition 9: F is the least rule base, if

There is not a rule *r*: P→Q in F which is equivalent to G = F-{r} and $Q \in P_G^+$.

There is not rule r: P→Q in F which is equivalent to (F-{ P→ Q})∪{ P_1→Q} and $P \propto P_1$.

There is not rule r: $P \rightarrow O$ in F which is equivalent to (F-{ $P \rightarrow O$})∪{ $P \rightarrow O_1$} and $Q_1 \propto O$.

Theorem 4: Every rule base F is equivalent to a least rule base $\mathbf{F}_{\boldsymbol{m}}$.

Proof: The following is a constructive proof. Follow the following steps, the least rule base is constructed and the theorem is proved meanwhile.

Step 1: For every rule r: $P \rightarrow Q$, Let G=F-{r}, if $Q \in P_G^+$, then F is replaced by G.

That the rule base F is equivalent to G is proven through the following steps:

Suppose the rule r satisfies the Condition 1:

r: $P \rightarrow Q \in F$

$G=F-\{r\}$ and $Q \in P_G^+$

∴ $G \subseteq F$

∵ $G+ \subseteq F^+$

(b) For every rule r 'in F which does not meet the condition 1. (r' $\neq$ r)

∴ r' ∈ F and r' ∈ G

∵ r' ∈ G+ and r' ∈ F^+.

(c) For every rule r' in F which meets the condition 1

∴ r' ∈ F

∵ r' ∈ F^+

and

∴ $Q \in P_G^+$

∵ r' ∈ G^+.

(d) According to (b) and (c), $F^+ \subseteq G+$ is true .

(e) According to (a) and (d), $F^+ = G^+$ is true,

(f) According to (a) and (d), $F \equiv G$.

Step 2: For every rule r: $P \rightarrow Q$, let B={Pi |$P \propto P_i$ }. For every $P_i \in B$, if $Q \in (P_i)_F^+$ then let P_i replace P, viz. G=(F-{ r: $P \rightarrow Q$})∪($P_i \rightarrow Q$).

That original F is equivalent to G is proven through the following steps:

Suppose the rule *r* satisfies the condition 2:

Condition 2:

r: $P \rightarrow Q$, B={P_i |$P \propto P_i$ }.

For every $P_i \in B$, let G=(F-{ r: $P \rightarrow Q$})∪($P_i \rightarrow Q$) and $Q \in (P_i)_F^+$.

For every rule r' in F which does not meet the condition 2

∴ r' ∈ F and r' ∈ G

∵ r' ∈ F^+ and r' ∈ G^+ .

For every rule r': $P \rightarrow Q$ in F which meet the condition 2,

∴ r' ∈ F

∵ r' ∈ F^+.

∴ $Pi \rightarrow Q \in G$ and $P \propto P_i$

∵ $P \rightarrow Q \in G^+$. (Precondition Specific Law)

According to (a) and (b), $F^+ \subseteq G^+$ is true.

For the rule r' in G except the rule: $P_i \rightarrow Q$ in condition 2.

r' ∈ G^+ and r' ∈ F^+.

For rule r': $P_i \rightarrow Q$ in condition 2, $r \in G^+$.

∴ $Q \in (P_i)_F^+$

∵ $r \in F^+$.

(d) and (e) $G^+ \subseteq F^+$.

(c) and (f) $F^+=G^+$, viz. $F \equiv G$.

Step 3: For every rule r: $P \rightarrow Q$, let B={ P_i | $P_i \propto Q$}. If $P_i \in B$ and $P_i \in (P)_F^+$ then let P_i replace Q, viz. G=(F-{ r: $P \rightarrow Q$})∪($P \rightarrow P_i$).

That original F is equivalent to G is proven through the following steps:

Suppose:

Condition 3: there are rule *r*: $P \rightarrow Q$, and B={ P_i | $P_i \propto Q$}.

$P_i \in B$, P_i replace Q, viz. G=(F-{ r: $P \rightarrow Q$})∪($P \rightarrow P_i$) and $P_i \in (P)_F^+$.

For every rule r' in F which does not meet the condition 3,

∴ r' ∈ F and r' ∈ G

∵ r' ∈ F^+ and r' ∈ G^+.

For every rule r':$P \rightarrow Q$ in F which meets the condition 3, $r \in F^+$.

∴ $P_i \rightarrow P_i \in G$ and $P_i \propto Q$

∵ $r \in G^+$.

(a) and (b) => $F^+ \subseteq G^+$

For every rule r' in G except r:$P_i \rightarrow Q$ in condition 3 then r' ∈ G^+ and $r \in F^+$.

For every rule r' in G, so $r \in G^+$.

∴ $P_i \in P_F^+$

∵ $r \in F^+$.

(d) and (e) =>$G^+ \subseteq F^+$.

(c) and (f) =>$G^+=F^+$.

$G \equiv F$

Do Step 1 to Step 3 until there are not any rules that can be removed from F. The G is the least rule base which is equal to the original base F.

VI. ALGORITHM

In order to revise the rule base and get the least rule base, it is important to find the redundant rules. The algorithm to find the redundant rules is presented in this section.

Definition 10: The conclusion set of a clause *P* on F sinned as P_F^+ , P_F^+ is the set of conclusion in F if *P* is true. The following is the algorithm to calculate P_F^+.

Algorithm 1:

Output: P_F^+

Input: P, F

Steps:

1) let P(0)=P , I =0， ;
2) B=Ø ;
3) For every P' in P(*I*) do
4) begin
   a) B'={A| (X)(Y)((X→Y∈F) and (P'∝X) and (Y∝A))}
   b) B=B∪B'
5) end
6) P(I+1) =B∪P(I)
7) if P(I+1) <> P(I), then I=I+1 go to step 2
8) else P(I) is the P_F^+ .
9) end

Suppose M(I) is the number of elements in P(I). If the set B=Ø in step (8) then P(I+1)=P(I) and the algorithm ends. If B≠Ø, the M(I+1) is larger than the M(I)， so {M(I)} is an increase sequence. The sequence {M(I)} has the upper limit because the number of rules in the rule base is finite. So the algorithm can terminate.

the following algorithm 2 may determine whether the rule r: O→O' is a redundant rule in the rule base F. The step(1)—step(7) is same with the algorithm 1.

Algorithm 2:

Steps 1)-7) (Algorithm 1)

8） P(I+1) =B ∪ P(I), if Q ∈ P(I+1) then r is a redundant rule in F , end.

9） If P(I+1) <> P(I), I=I+1 go to step 2

10） r is not a redundant rule in F.

11） end

The time complexity of the Algorithm 2 is also $n \times m^2$ in the worst case.

VII. APPLICATION

The hidden troubles of power transformer are indicated by different characteristic signal. Due to the complexity of fault reason and phenomenon, the synthetic disposal and cooperative analysis for multi-characteristic signal of power transformer are needed. Power Transformer Fault Diagnosing Expert System is a fuzzy AI system to help the user to find out the hidden troubles of the power transformer. For it is an open system, the rule base may be revised by the experts, the rule base become bigger and bigger in the maintain process. And the experts can define their own states variable by the membership function. Such as some experts define 3 states in the content of CH_4 -- low, middle, high. And the others define 5 states-- very low, low, middle, high and very high.

Because the rule base is modified by many experts at different times, many redundant rules are inserted into the rule base. In order to remove the redundant rules, the "Auto Detect and Remove Redundant Rules Sub system" is designed in the expert system on the basis of the algorithm discussed above. The subsystem can check whether the rule is redundancy when the expert inserts a rule. The other model of the subsystem is batch processing which detects the redundant rules and revises the rule base automatically.

The time complexity of the algorithm2 is proportional to the square of the number of states in the rule base F. In order to accelerate the process, the rule base can be divided into several sub-bases by the topic or scope which the rules concerns, the revise algorithm will only run in the sub rule base. In the Transformer Fault Diagnosis System, the rule base is also divided into three parts by the signal domain -- transformer oil chromatographic detection, partial discharge test and regular experiment (measuring the direct current resistances of inductive coils and iron core insulation resistance). The algorithm to find the redundant rules runs in every sub-rule base independently. This method accelerates finding and removing the redundant rules.

VIII. CONCLUSION

In order to find and remove the redundant rules automatically, a series of definitions and algorithms about the redundant rule and least rule base are pretended. A synthetical process is established to revise the fuzzy rule base. However some problems are open in the process. We will focus on the efficient mechanism of optimization.

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