2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

Multiperson Decision Making with Different Preference Representation Structures: A Selection Process based on Prospect Theory

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***Abstract*—In this study, we present a novel selection process to solve the multiperson decision making (MPDM) problems with different preference representation structures. This selection process is based on the prospect theory, which is one of the most influential psychological behavior theories, and seeks to maximize the satisfactory of all decision makers. Specifically, the individual selection methods associated with different preference structures are used to obtain individual preference orderings. Then, the preference-approval structures are used to determine the reference points of the prospect theory, according to the obtained individual preference orderings. Next, the gains and losses are calculated based on the prospect theory and the established reference points. Finally, the prospect values of the alternatives are obtained to rank the alternatives.**

Keywords—Multiperson decision making; Preference representation structures; Prospect theory; Preference-approval structures

I. Introduction

In multiperson decision making (MPDM) problems, there exists a natural phenomenon that different decision makers will use different preference representation structures to express their preference information due to their different educational backgrounds, experience and cognitive degrees.

Chiclana et al. [6][7] proposed the notable MPDM models with preference information being represented by means of preference orderings, utility functions, fuzzy preference relations, multiplicative preference relations. In the selection process of this MPDM model, the ordered weighted averaging (OWA) operator and the relative linguistic quantifier are used [25] [26]. Further, Herrera-Viedma et al. [12] and Dong et al. [9] investigated the consensus models for the MPDM with different preference representation structures.

Generally, the existing selection processes in MPDM (or group decision making) try to find the mathematical optimal solutions (in some sense) to decision problems and don't consider the decision maker's psychological behavior. However, much empirical evidence [5] [14] [22] has shown that the decision maker's psychological behavior would play an important role in decision analysis. Therefore, how to find the satisfactory solutions [19] involving the decision maker's psychological behavior has been an important problem in decision making. Particularly, in this study, the prospect theory, which is one of the most influential psychological behavior theories, has been presented in [11] [14], is introduced into MPDM problems. The key of the prospect theory is how to determine the *reference point* and how to calculate the *gains* and *losses*.

The aim of this study is to propose a novel selection process to solve the MPDM problems with different preference representation structures. This selection process seeks to maximize the satisfactory of all decision makers, based on the prospect theory. The rest arrangement of this study is as follows. Section 2 introduces the preliminary knowledge regarding four different preference representation structures and the prospect theory. A framework of the selection process for MPDM with different preference representation structures is proposed in Section 3. Following this, the selection process based on the prospect theory is designed in details in Section 4. Subsequently, Section 5 provides an illustrative example. Finally, concluding remarks are included in Section 6.

II. Preliminaries

This section introduces some basic knowledge regarding four kinds of preference representation structures and the prospect theory, which will provide a basis for this study.

Let $X=\{x_1,x_2,...,x_n\},(n\geq 2)$ be a finite set of alternatives, these alternatives have to be classified from best to worst, just according to the preference information given by the set of the decision makers, $E=\{e_1,e_2,...,e_m\},(m\geq 2)$.

A. Preference presentation structures

In this study, the decision maker's preference information over the set of X is assumed to be represented in one of the following formats.

1) Preference orderings of alternatives [17]. The decision maker e^k directly provides his individual preference information by a preference ordering $O^k=\{o_1^k,o_2^k,...,o_n^k\}$, where o_i^k indicates the positional order of alternative x_i in X.

2) Utility functions [20]. The decision maker e^k provides his individual preference information by means of a utility function $U^k=\{u_1^k,u_2^k,...,u_n^k\}$, where $u_i^k\in[0,1]$ denotes the utility evaluation value given by a decision maker to the alternative x_i.

3) Fuzzy preference relations [15]. The decision maker e^k 's individual preference information is described by a

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This work was supported by a grant (Nos. 71171160) from NSF of China, and a grant (No. skqx201308) from Sichuan University.

matrix $P^k = (p_{ij}^k)_{n\times n}$, where $p_{ij}^k \in [0,1]$ denotes the preference degree of the alternative x_i over x_j. A fuzzy relation satisfies the property of additive reciprocal, i.e. $p_{ij}^k + p_{ji}^k = 1, \forall i, j$.

4) Multiplicative preference relations [16]. The decision maker e^k gives his individual preference information by a matrix $A^k = (a_{ij}^k)_{n\times n}$, where a_{ij}^k indicates a ratio of the preference intensity of alternative x_i to that of x_j, and it must satisfy the condition of $a_{ij}^k \times a_{ji}^k = 1, \forall i, j$.

OWA operator [25] is used to aggregate the fuzzy preference relations in the selection process. Let $\{a_1, a_2, ..., a_n\}$ be a set of values to aggregate. And the OWA operator is expressed as:

$$OWA(a_1, a_2, ..., a_n) = \sum_{i=1}^{n} w_i b_i \tag{1}$$

where b_i is the i th largest value in $\{a_1, a_2, ..., a_n\}$, and the associated weight vector $W = (w_1, w_2, ..., w_n)^T$ satisfies that $w_i \in [0,1]$, $\sum_{i=1}^{n} w_i = 1$. Yager [26] proposed an effective way to calculate the value of w_i, according to the following expression:

$$w_i = Q(i/n) - Q((i-1)/n), i = 1,2,...,n \tag{2}$$

where $Q(r)$ can be represented as [27]:

$$Q(r) = \begin{cases} 0, & r < a \\ \frac{r-a}{b-a}, & a \le r \le b \\ 1, & r > b \end{cases} \tag{3}$$

with $a, b, r \in [0,1]$.

The linguistic quantifiers such as *most*, *at least half* and *as many as possible* are defined by the coefficients (a,b), which are (0.3, 0.8), (0, 0.5), and (0.5, 1), respectively. When a fuzzy linguistic quantifier Q is used to compute the weights of OWA operator, it is symbolized by OWA_Q.

B. Prospect theory

The prospect theory was initiated by Kahneman and Tversky [14] in 1979, and it is a descriptive theory for forecasting individual actual decision behavior under risk. A decision process of the prospect theory is comprised of the editing phase and the evaluation phase [14]. In the editing phase, outcomes of alternatives are coded as gains or losses relative to a reference point, if the value of the alternative is over the reference point, the part in excess can be regarded as 'gain'; If the value of the alternative is under the reference point, the different part can be regarded as 'loss'. In the evaluation phase, the edited prospects are evaluated by a prospect value function and a weighting function, and the prospect of highest value is chosen [11]. The prospect theory has three important principles as follows [14] [21] [22]:

1) *Reference dependence.* The decision makers perceive the gains and losses according to a reference point. For instance, under the same price of goods and services, you are facing two choices:
A). other colleagues make 60000 Dollars a year and your annual income is 70000 Dollars;
B). other colleagues' annual income is 90000 Dollars while you have a credit of 80000 Dollars a year.
In the option A, the reference point to you is 60000 Dollars, thus you have a "gain" of 10000 Dollars; however, in the option B, the reference point is 90000 Dollars, in this case, you have a "loss" of 10000 Dollars. Although, you have a higher income in the latter situation, the majority will choose the former for comparative psychological behavior [14]. Actually, the jealousy and comparisons between peers is the motive power of making money. People judge the gains and loss from comparison. This is the reference dependence. Thus, the prospect value function can be divided into the gain domain and the loss domain relative to the reference point.
2) *Diminishing sensitivity*. The decision makers exhibit risk-averse tendency for gains and risk-seeking tendency for losses. According to the principle of diminishing sensitivity, the prospect value function is concave in the loss domain and convex in the gain domain, i.e., the marginal value of both gains and losses is decreasing with the size.
3) *Loss aversion*. The decision makers are more sensitive to losses than to totally identical gains [1]. In accordance with the principle of loss aversion, the prospect value function is steeper in the loss domain than in the gain domain.

In accordance with the above three principles, an S-shaped value function of the prospect theory is illustrated in Fig.1. In Fig.1, x denotes the gain ($x > 0$) or the loss ($x < 0$) of the outcome relative to the reference point. This form of function is given by Kahneman and Tversky [22]:

$$v(x) = \begin{cases} x^{\alpha}, & x \ge 0 \\ -\lambda(-x)^{\beta}, & x < 0 \end{cases} \tag{4}$$

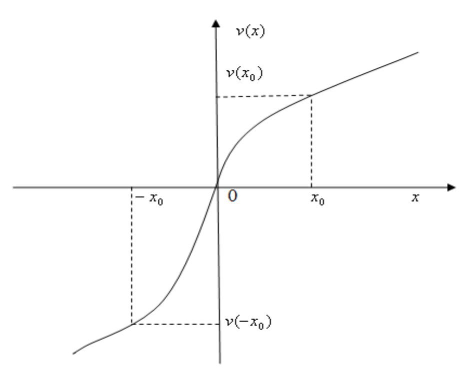

Fig. 1. The prospect value function

In the function, α and β are the two parameters which determine the concavity and convexity of the function, respectively, and $0 \le \alpha, \beta \le 1$. λ is the coefficient of loss aversion, $\lambda > 1$.

III. THE PROPOSED FRAMEWORK

This section presents a framework of the selection process based on prospect theory in the MPDM problem with different preference representation structures. The framework is composed of the following five-step procedure, which is described as Fig. 2.

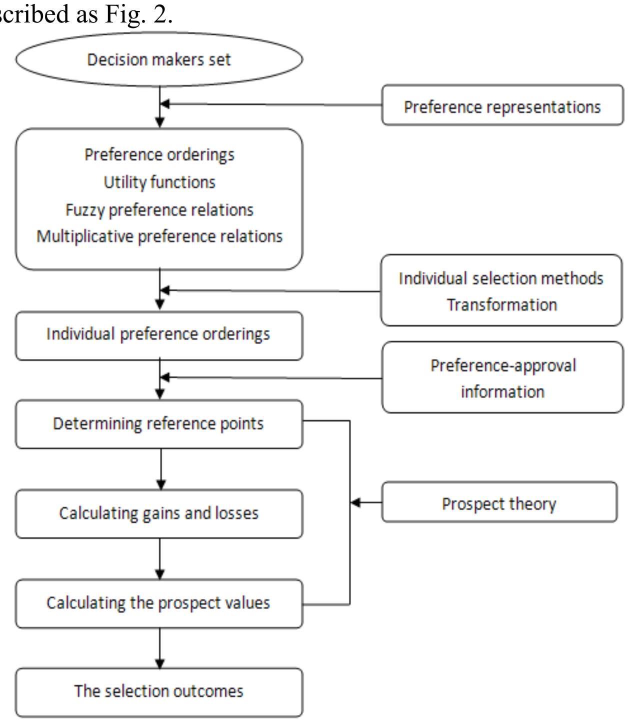

Fig. 2. The selection process based on prospect theory

1) Preference representations

Let X and E be as earlier. Let E^O, E^U , E^P and E^A be four subsets of E , representing the decision makers whose preference information on X is presented by means of preference orderings, utility functions, fuzzy preference relations and multiplicative preference relations, respectively. Without loss of generality, this study assume that $E^O = \{e_1, e_2, ..., e_{l_1}\}$, $E^U = \{e_{l_1+1}, e_{l_2+2}, ..., e_{l_2}\}$, $E^P = \{e_{l_2+1}, e_{l_2+2}, ..., e_{l_3}\}$, $E^A = \{e_{l_3+1}, e_{l_3+2}, ..., e_m\}$.

2) Obtaining the individual preference orderings

The individual selection methods associated with different preference structures are used to obtain individual preference orderings.

3) Determining the reference points according to the preference- approval information

In order to determine the reference points, the preference-approval structures proposed in [10] are introduced. Some collective decision making models [2] [3] [18] have assumed that the decision makers take a common language when they evaluate alternatives, and these models aggregate labels such as approved and disapproved. In [4], Brams and Sanver suggest a framework that can be considered as a compromise between standard and non-standard models by combining the information of ranking and approval in a hybrid system which they call preference-approval. In this study, the decision makers are assumed to give a cut-off line to distinguish satisfactory and dissatisfactory alternatives according to the preference orderings that have obtained in the previous step. An alternative which is ranked above the line is qualified as satisfactory. According to the cut-off line, a reference point is determined.

4) Calculating the gains and losses

In this step, we calculate the gains and losses relative to the reference points determined in the previous step. This calculating process uses the positional values of the alternatives.

5) Calculating the prospect values of the alternatives and obtaining the selection outcomes

Take the ‘gains’ and ‘losses’ into the prospect function to obtain the prospect values of the alternatives, and then use the simple additive method to obtain the overall prospect values to rank the alternatives.

IV. THE SELECTION PROCESS BASED ON PROSPECT THEORY

This section designs the selection process based on prospect theory and preference-approval structures in details.

1) Obtaining the individual preference orderings

Let $O^k = \{o_1^k, o_2^k, ..., o_n^k\} (k = 1,2,..., m)$ be the basic preference representation structure to uniform the preference information.

Here, we apply the method proposed in [9] to unify the different preference representation structures, and this method can avoid internal inconsistency issue when using the transformation functions among different preference representation structures.

In order to obtain the O^k , three other cases are considered.

Case 1: $e^k \in E^U$

In this case, the decision maker gives his preference information over X by the utility function $U^k = \{u_1^k, u_2^k, ..., u_n^k\}$ .The lager value of u_i^k is, the better position for u_i^k is. The preference ordering of the alternatives from best to worst is obtained.

Case 2: $e^k \in E^P$

In this case, the decision maker provides his preference information on X by means of the fuzzy preference relations $P^k = (p_{ij}^k)_{n \times n}$. The quantifier-guided dominance degree $QGDD_i^k$ [6] is used to quantify the dominance that the alternative x_i has over all the others in a fuzzy majority sense Q^k as follows:

$$QGDD_i^k = OWA_{Q^k}(p_{i1}^k, p_{i2}^k, ..., p_{in}^k) \quad (5)$$

Then, according to $QGDD_i^k$, the preference ordering O^k is obtained.

Case 3: $e^k \in E^A$

In this case, the decision maker provides his preference information on X by the multiplicative preference relations $A^k = (a_{ij}^k)_{n\times n}$. Let $w^k = (w_1^k, w_2^k, ..., w_n^k)^T$ be the individual preference vector, where w_i^k denotes the weight of x_i. Here, use the row geometric mean method (RGMM) presented in [6] to obtain w_i^k from $A^k = (a_{ij}^k)_{n\times n}$.

$$\min \sum_{i=1}^{n}\sum_{j>i}^{n}[\ln(a_{ij}^k) - (\ln(w_i^k) - \ln(w_j^k))]^2$$

$$st\begin{cases}\sum_{i=1}^{n} w_i^k = 1 \\ w_i^k \geq 0\end{cases} \tag{6}$$

The model above can be simply found as the geometric means of the rows of matrix A^k [8]:

$$w_i^k = \frac{(\prod_{j=1}^{n} a_{ij}^k)^{1/n}}{\sum_{i=1}^{n}(\prod_{j=1}^{n} a_{ij}^k)^{1/n}} \tag{7}$$

According to w_i^k, the preference ordering O^k is obtained.

2) Determining the reference points

In this step, the decision makers need to provide their own preference-approval information according to the obtained preference orderings, then the preference-approval structures [4][10] are expressed as:

$$A^k(X) = \frac{x_{\sigma(1)}^k, x_{\sigma(2)}^k, ..., x_{\sigma(l)}^k}{x_{\sigma(l+1)}^k, x_{\sigma(l+2)}^k, ..., x_{\sigma(n)}^k} \tag{8}$$

where $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of $(1,2,...,n)$, and $x_{\sigma(i)}^k \in X$, $x_{\sigma(i+1)}^k \geq x_{\sigma(i)}^k$, i.e., if x_i is the j *th* largest based on the preference information provided by the decision maker e_k, then $x_{\sigma(j)}^k = x_i$

The alternatives above the dash line are satisfactory (good), and the alternatives below the dash line are dissatisfactory (bad). Obviously, the reference point of the decision maker e^k is between $x_{\sigma(l)}^k$ and $x_{\sigma(l+1)}^k$. In this study, we choose a median between the position of $x_{\sigma(l)}^k$ and the position of $x_{\sigma(l+1)}^k$ as the reference point of the decision maker e^k, and the expression can be presented as:

$$o^{k*} = \frac{2l+1}{2} \tag{9}$$

3) Calculating the gains and losses

Let G_i^k and L_i^k be the gain and loss of the alternative x_i associated with the decision maker e_k.

$$G_i^k = -(o_i^k - o^{k*}), \quad o_i^k < o^{k*} \tag{10}$$

$$L_i^k = -(o_i^k - o^{k*}), \quad o_i^k > o^{k*} \tag{11}$$

Further, a gain matrix $G = [G_{ij}]_{m\times n}$ and a loss matrix $L = [L_{ij}]_{m\times n}$ can be constructed, respectively.

4) Calculating the prospect values and ranking the alternatives

Based on the Eq. (4), the prospect value of the alternative x_i associated with the decision maker e_k is given by:

$$V_i^k = (G_i^k)^{\alpha} + [-\lambda(-L_i^k)^{\beta}], \quad \mathrm{i} = (1,2,...,\mathrm{n}) \tag{12}$$

where $0 \leq \alpha, \beta \leq 1$, and $\lambda > 1$. The smaller α is, the greater risk aversion in the gain domain is, and the smaller β is, the greater risk seeking in the loss domain is. About the values of α, β and λ, Tversky and Kahneman [22] have carried out a series of experiments to determine them. They discovered that the median values of α, β are both 0.88, and the value of λ is 2.25. Following that, some scholars obtained the same values for the parameters by experiments [13] [23] [24]. Therefore, in this study, we take the values given by Tversky and Kahneman [22].

Further, using the simple additive method to obtain the overall prospect values of the alternative x_i is computed as

$$V_i = \sum_{k=1}^{m} V_i^k, \qquad i = (1,2,...,n) \tag{13}$$

Obviously, the greater V_i is, the more satisfactory the alternative x_i is. Therefore, in accordance with a descending order of the overall prospect values of all alternatives, a ranking order of all alternatives is determined.

V. Illustrative Example

In order to demonstrate the selection process based on prospect theory, we use the example presented by Herrera-Viedma et al. in [12]. In this example, a set of eight decision makers $E = \{e_1, e_2, ..., e_8\}$ are asked to give their preference information on a set of six alternatives $X = \{x_1, x_2, ..., x_6\}$. The decision makers e_1 and e_2 give their opinions by preference orderings O^1 and O^2. The decision makers e_3 and e_4 provide their preference information using utility functions U^3 and U^4. The decision makers e_5 and e_6 provide their opinions using fuzzy preference relations P^5 and P^6. The decision makers e_7 and e_8 give their opinions using multiplicative preference relations A^7 and A^8. Their opinions are presented as follows:

e_1: $O^1 = \{2, 1, 3, 6, 4, 5\}$, $\qquad e_2$: $O^2 = \{1, 3, 4, 2, 6, 5\}$

e_3: $U^3 = \{0.3, 0.2, 0.8, 0.6, 0.4, 0.1\}$

$e_4: U^4 = \{0.3, 0.9, 0.4, 0.2, 0.7, 0.5\}$

$$e_5 : P^5 = \begin{bmatrix} 0.5 & 0.55 & 0.45 & 0.25 & 0.7 & 0.3 \\ 0.45 & 0.5 & 0.7 & 0.85 & 0.4 & 0.8 \\ 0.55 & 0.3 & 0.5 & 0.65 & 0.7 & 0.8 \\ 0.75 & 0.15 & 0.35 & 0.5 & 0.95 & 0.6 \\ 0.3 & 0.6 & 0.3 & 0.05 & 0.5 & 0.85 \\ 0.7 & 0.2 & 0.2 & 0.4 & 0.15 & 0.5 \end{bmatrix}$$

$$e_6 : P^6 = \begin{bmatrix} 0.5 & 0.7 & 0.75 & 0.95 & 0.6 & 0.85 \\ 0.3 & 0.5 & 0.55 & 0.8 & 0.4 & 0.65 \\ 0.25 & 0.45 & 0.5 & 0.7 & 0.6 & 0.45 \\ 0.05 & 0.2 & 0.3 & 0.5 & 0.85 & 0.4 \\ 0.4 & 0.6 & 0.4 & 0.15 & 0.5 & 0.75 \\ 0.15 & 0.35 & 0.55 & 0.6 & 0.25 & 0.5 \end{bmatrix}$$

$$e_7 : A^7 = \begin{bmatrix} 1 & 1/2 & 1/3 & 4 & 3 & 5 \\ 2 & 1 & 1/3 & 1/4 & 4 & 6 \\ 3 & 3 & 1 & 7 & 6 & 9 \\ 1/4 & 4 & 1/7 & 1 & 1/2 & 3 \\ 1/3 & 1/4 & 1/6 & 2 & 1 & 4 \\ 1/5 & 1/6 & 1/9 & 1/3 & 1/4 & 1 \end{bmatrix}$$

$$e_8 : A^8 = \begin{bmatrix} 1 & 1/5 & 1/4 & 1/2 & 3 & 1/6 \\ 5 & 1 & 2 & 4 & 6 & 1/3 \\ 4 & 1/2 & 1 & 3 & 5 & 4 \\ 2 & 1/4 & 1/3 & 1 & 3 & 6 \\ 1/3 & 1/6 & 1/5 & 1/3 & 1 & 8 \\ 6 & 3 & 1/4 & 1/6 & 1/8 & 1 \end{bmatrix}$$

The selection process based on prospect theory is presented as follows.

1) Obtaining individual preference orderings

Since $e^3, e^4 \in E^U$, we can easily get the preference orderings for e^3 and e^4 as:

$e^3: O^3 = \{4, 5, 1, 2, 3, 6\}$, $e^4: O^4 = \{5, 1, 4, 6, 2, 3\}$

Since $e^5, e^6 \in E^P$, use the Eq. (1), Eq. (2), Eq. (3) and Eq. (5) with fuzzy quantifier "most", with the pair (0.3, 0.8) to obtain $QGDD_i^5$, $QGDD_i^6$, respectively. And then the preference orderings for e^5 and e^6 are obtained as:

$e^5: O^5 = \{4, 1, 2, 3, 5, 6\}$ $e^6: O^6 = \{1, 2, 3, 6, 4, 5\}$

Since $e^7, e^8 \in E^A$, use Eq. (7) to obtain $(w_1^7, w_2^7, ..., w_6^7)$ and $(w_1^8, w_2^8, ..., w_6^8)$ from $A^{(7)}$ and $A^{(8)}$, respectively. Then, based on $(w_1^7, w_2^7, ..., w_6^7)$ and $(w_1^8, w_2^8, ..., w_6^8)$, the individual preference orderings of e^7 and e^8 are obtained as:

$e^7: O^7 = \{2, 3, 1, 4, 5, 6\}$ $e^8: O^8 = \{6, 2, 1, 3, 5, 4\}$

2) Determining the reference points

There often exists a "reference point" in every decision maker's mind in actual decision making. The alternatives ranking before it are approval while the rest are disapproval. In other words, only these alternatives, whose ranking are high in one' mind, are supported by the decision maker.

For example, in the election, every voter must have a list of candidates who are qualified. The number of the candidates of the list can be one or several, and the candidates of the list must have rank in the front in the voter's own heart. That is the decision makers' psychological behavior and we can use the prospect theory to measure the satisfaction degree of the alternative set of every decision maker.

According to the decision makers' individual psychological behavior and the before obtained individual preference orderings, the decision makers may give their preference-approval structures (similar to Eq. (8)) as follows:

Note 1: In this example, the reference points are set by assumption arbitrarily, and in the real decision making process, the reference points are given by the decision makers, it is a dynamic decision making process. The example here is just to clarify the process.

$$e_1 : A(X^1) = \frac{x_2, x_1, x_3}{x_5, x_6, x_4}, \qquad e_2 : A(X^2) = \frac{x_1, x_4, x_2, x_3}{x_6, x_5}$$

$$e_3 : A(X^3) = \frac{x_3, x_4}{x_5, x_1, x_2, x_6}, \qquad e_4 : A(X^4) = \frac{x_2, x_5}{x_6, x_3, x_1, x_4}$$

$$e_5 : A(X^5) = \frac{x_2, x_3, x_4}{x_1, x_5, x_6}, \qquad e_6 : A(X^6) = \frac{x_1, x_2, x_3, x_5, x_6}{x_4}$$

$$e_7 : A(X^7) = \frac{x_3, x_1, x_2, x_4}{x_5, x_6}, \qquad e_8 : A(X^8) = \frac{x_3, x_2, x_4}{x_6, x_5, x_1}$$

In this study, we choose a median position as the reference point, based on the Eq. (9), the values of the reference points of the decision makers are:

$o^{1*} = 3.5$, $o^{2*} = 4.5$, $o^{3*} = 2.5$, $o^{4*} = 2.5$
$o^{5*} = 3.5$, $o^{6*} = 5.5$, $o^{7*} = 4.5$, $o^{8*} = 3.5$

3) Calculating the gains and losses

After obtaining the reference points, according to the Eq. (10) and Eq. (11), we obtain a gain matrix G and a loss matrix L as:

$$G = \begin{bmatrix} 1.5 & 2.5 & 0.5 & 0 & 0 & 0 \\ 3.5 & 1.5 & 0.5 & 2.5 & 0 & 0 \\ 0 & 0 & 1.5 & 0.5 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0.5 & 0 \\ 0 & 2.5 & 1.5 & 0.5 & 0 & 0 \\ 4.5 & 3.5 & 2.5 & 0 & 1.5 & 0.5 \\ 2.5 & 1.5 & 3.5 & 0.5 & 0 & 0 \\ 0 & 1.5 & 2.5 & 0.5 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 0 & -2.5 & -0.5 & -1.5 \\ 0 & 0 & 0 & 0 & -1.5 & -0.5 \\ -1.5 & -2.5 & 0 & 0 & -0.5 & -3.5 \\ -2.5 & 0 & -1.5 & -3.5 & 0 & -0.5 \\ -0.5 & 0 & 0 & 0 & -1.5 & -2.5 \\ 0 & 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & -1.5 \\ -2.5 & 0 & 0 & 0 & -1.5 & -0.5 \end{bmatrix}$$

4) Calculating the prospect values and ranking the alternatives

Based on the Eq. (12), where the values of α and β are both 0.88, and the value of λ is 2.25.

$$V_i^k = (G_i^k)^{0.88} + [-2.25(-L_i^k)^{0.88}],\ \ \mathrm{i} = (1,2,...,\mathrm{n})$$

A prospect value matrix is obtained as:

$$V = \begin{bmatrix} 1.43 & 2.24 & 0.54 & -5.04 & -1.22 & -3.21 \\ 3.01 & 1.43 & 0.54 & 2.24 & -3.21 & -1.22 \\ -3.21 & -5.04 & 1.43 & 0.54 & -1.22 & -6.78 \\ -5.04 & 1.43 & -3.21 & -6.78 & 0.54 & -1.22 \\ -1.22 & 2.24 & 1.43 & 0.54 & -3.21 & -5.04 \\ 3.76 & 3.01 & 2.24 & -1.22 & 1.43 & 0.54 \\ 2.24 & 1.43 & 3.01 & 0.54 & -1.22 & -3.21 \\ -5.04 & 1.43 & 2.24 & 0.54 & -3.21 & -1.22 \end{bmatrix}$$

Using Eq. (13), the overall prospect values of the alternatives $x_1, x_2, ..., x_6$ are obtained，respectively.

$V_1 = -4.07$， $V_2 = 8.17$， $V_3 = 8.22$，
$V_4 = -4.14$， $V_5 = -11.32$， $V_6 = -21.36$

Therefore, a ranking ordering of the six alternatives is obtained：

$x_3 \succ x_2 \succ x_1 \succ x_4 \succ x_5 \succ x_6$

Obviously, the assumed preference-approval structures will affect the final result due to their effects on the reference points. In the process, the decision makers give their preference-approval information according to their own orderings, so the satisfactory solution won't betray the preference representation structures.

Here, we make some comparisons between our selection outcome and the selection outcome in [12]. The collective order of [12] is $x_2 \succ x_3 \succ x_1 \succ x_5 \succ x_4 \succ x_6$, which is different from our $x_3 \succ x_2 \succ x_1 \succ x_4 \succ x_5 \succ x_6$. Such differences are easy to understand because they are two totally different selection processes. There are probably two main reasons to result in these differences: 1) in the aggregation phase, we use the method proposed in [9] different from [12] to avoid internal inconsistency issue when using the transformation functions among difference preference representation structures, and the difference of the coefficients of OWA operator will have some influence; 2) in the exploitation phase, we use the prospect theory with reference points and preference-approval structures, however, in [12], the authors use *Quantifier Guided Dominance Degree* to measure the collective values of each alternative.

VI. CONCLUSION

In this study, we present a novel selection process to solve the MPDM problems with different representation structures (preference orderings, utility functions, fuzzy preference relations, multiplicative preference relations). The prospect theory and the preference-approval structures are used as the basis of the proposed selection process. The main improvement of this selection process is to seek the satisfactory solutions, which is more consistent to human's actual behavior. In our further research, we will continue to develop the consensus process based on the prospect theory to solve the MPDM problems with different presentation structures.

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