Model-Based Takagi-Sugeno Fuzzy Approach for Vehicle Longitudinal Velocity Estimation during Braking

Haiping Du and Weihua Li

Abstract—Accurate vehicle longitudinal velocity estimation is important for wheel slip ratio control in antilock braking systems. To overcome the problem of nonlinear tyreroad friction characteristic when designing an observer for velocity estimation, this paper presents a novel approach by using the model-based fuzzy technique. The nonlinear vehicle braking system is modelled by a Takagi-Sugeno fuzzy model first. A fuzzy observer is then constructed by using the available measurements of wheel angular velocity and braking torque with the estimated premise variables. All the possible disturbances and uncertainties are considered so that the designed observer is robust under an H_{∞} performance index from the disturbances to the estimation error. The design of the observer is achieved by solving a set of linear matrix inequalities. Numerical simulations on a quartervehicle braking model are used to validate the effectiveness of the proposed approach.

Keywords: T-S fuzzy model, velocity estimation, longitudinal dynamics, antilock braking

I. INTRODUCTION

Vehicle antilock braking system (ABS) is one of the critical safety control systems of vehicles. The main purpose of ABS is to avoid the hard lock of wheels during braking so that the phenomenon of vehicle skid on the road surface can be avoided, and therefore, enhance vehicle control ability. In general, the wheel slip ratio, which is defined as the normalised difference between the vehicle longitudinal speed and the wheel linear speed, should be properly regulated such that the maximum friction force between the wheel and the road surface can be produced and thus maximally reduce the stopping distance of vehicle during braking, even on a possibly slippery road. Therefore, the wheel slip ratio control is an important research objective for implementing ABS [1], [2], [3].

To realise the wheel slip ratio control, the real time information of vehicle longitudinal speed and the wheel speed should be known so that the information about the slip ratio can be updated. The wheel angular velocity can be easily measured with available sensors such as encoders. The vehicle longitudinal velocity can be directly measured by using the sensors like GPS [4], however, there is reliability problem with GPS signals, and GPS is relatively expensive to passenger vehicles. To overcome this problem, many researchers proposed different estimation methods to estimate the vehicle speed through the use of observers [5], [6]. The main difficulty for velocity estimation is due to the nonlinear characteristic between tyre-road friction and slip ratio, which further affects the tyre-road friction force, therefore modelling and considering the nonlinear characteristic in the observer design procedure will effectively improve the estimation performance. To this end, a new approach that uses the model-based Takagi-Sugeno (T-S) fuzzy technique [7], [8], which has been applied to modelling and control of many nonlinear systems such as vehicle suspensions [9] and permanent-magnet synchronous motors [10], is proposed in this study to estimate the vehicle longitudinal velocity during braking.

In this paper, the T-S fuzzy modelling of vehicle longitudinal dynamics is discussed first. Then conditions for designing a T-S fuzzy model-based observer with estimated premise variables are derived and expressed as linear matrix inequalities (LMIs), which can be efficiently solved by using available software like Matlab LMI Toolbox. At last, simulation results on a quarter-vehicle braking model are used to validate the effectiveness of the proposed approach. The main contribution of the paper is to apply the model-based fuzzy technique to design an observer for vehicle velocity estimation during braking.

This paper is organised as follows. In section II, the fuzzy modelling of the vehicle braking model is introduced. The conditions for designing a model-based fuzzy observer are derived in section III. In section IV, the simulation results on a quarter-vehicle braking model are discussed. Finally, conclusions are presented in section V.

The notation used throughout the paper is fairly standard. For a real symmetric matrix W, the notation of W > 0 (W < 0) is used to denote its positive-(negative-) definiteness. $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. I is used to denote the identity matrix of appropriate dimensions. To simplify notation, * is used to represent a block matrix which is readily inferred by symmetry.

This research was supported under Australian Research Council's Discovery Projects funding scheme (project number DP140100303). H. Du is with School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Wollongong, NSW 2552, Australia (e-mail: hdu@uow.edu.au). W. Li is with School of Mechanical, Materials, and Mechatronics Engineering, University of Wollongong, Wollongong, NSW 2552, Australia (e-mail: weihuali@uow.edu.au).



Fig. 1. Quarter-vehicle braking model.

II. FUZZY MODELLING OF VEHICLE LONGITUDINAL DYNAMICS IN BRAKING

A. Vehicle Longitudinal Dynamics Model

To simplify the problem formulation, a simple but effective quarter-vehicle braking model as shown in Fig. 1, which is widely used for the preliminary design and testing of braking control strategies [11], [12], [2], is used in this study. This model is obtained from a straight-line braking event on a flat road, where the wind force, hill climbing force, and rolling resistance are ignored.

During braking, the governing equations for the vehicle motion and wheel motion are expressed as

$$M\dot{v} = -F_x, \tag{1}$$

$$J\dot{\omega} = RF_x - T_b, \qquad (2)$$

where R is the effective wheel radius, J is the total moment of inertia of the wheel, v is the longitudinal velocity of the vehicle, ω is the angular velocity of the wheel, T_b is the braking torque, F_x is the longitudinal tyre force, M is the total mass of the quarter-vehicle. Note that the braking torque T_b can be simply related to the pressure of the master cylinder with the formula $T_b = K_b P_b$, where K_b is the braking system gain. As this study is focusing on the estimation of vehicle longitudinal velocity, the braking torque is simply assumed to be measurable without discussing how to generate it.

B. Tyre Force Model

The longitudinal type force F_x is often modelled as

$$F_x = F_z \mu(\lambda, \varsigma), \tag{3}$$

where F_z is the vertical load and $\mu(\lambda, \varsigma)$ is the tyre longitudinal friction coefficient, which is a function of slip ratio λ and a set of parameters ς . The slip ratio is defined as

$$\lambda = \frac{v - \omega R}{v}.\tag{4}$$

Note that several empirical formulae have been used to describe the type friction model, like the Magic Type



Fig. 2. Friction coefficient vs slip ratio for different surface.

Formula and the second order rational fractions model [2]. In this paper, the Burckhardt tyre friction model [13] will be applied. This tyre model gives the tyre-road friction coefficient as a function of wheel slip ratio as

$$\mu(\lambda,\varsigma) = (\varsigma_1(1 - e^{-\lambda\varsigma_2}) - \lambda\varsigma_3)e^{-\varsigma_4\lambda v}, \tag{5}$$

where ς_1 , ς_2 and ς_3 are constants for characterising different road conditions. ς_1 is the maximum value of the friction curve, ς_2 gives the friction curve shape and ς_3 represents the difference between the maximum value of the friction curve and the value when slip ratio is one. ς_4 is the wetness characteristic value and is in the range 0.02 - 0.04 s/m. Fig. 2 shows a plot of the friction coefficient according to this model for some often used road surfaces.

C. Fuzzy Modelling

Substituting (3) into (1) and (2) yields

$$\dot{v} = -\frac{1}{M} F_z \mu(\lambda, \varsigma), \tag{6}$$

$$\dot{\omega} = \frac{R}{J} F_z \mu(\lambda, \varsigma) - \frac{1}{J} T_b.$$
(7)

Equations (6) and (7) are further written into a statespace model as

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{1}{Mv}F_z\mu(\lambda,\varsigma) & 0 \\ 0 & \frac{R}{J\omega}F_z\mu(\lambda,\varsigma) \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} -\frac{1}{M}F_z \\ \frac{R}{J}F_z \end{bmatrix} \Delta\mu(\lambda,\varsigma) + \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix} T_b, \quad (8)$$

where $\Delta \mu(\lambda, \varsigma)$ is introduced to describe the uncertainty of $\mu(\lambda, \varsigma)$. In fact, $\mu(\lambda, \varsigma)$ is just an empirical model, which cannot accurately represent the actual friction coefficient. Introducing a norm-bounded uncertainty term $\Delta \mu(\lambda, \varsigma)$ may cover the possible discrepancy between the empirical model and the actual value. This uncertainty will be regarded as an external disturbance when designing an observer in the next section such that the designed observer is robust to all the possible uncertainties.

By defining $f_v = \frac{1}{v}\mu(\lambda,\varsigma)$ and $f_\omega = \frac{1}{\omega}\mu(\lambda,\varsigma)$, (8) is further written as

$$\dot{x} = A(f_v, f_\omega)x + B_1w + B_2u, \tag{9}$$

where

$$x = \begin{bmatrix} v \\ \omega \end{bmatrix}, \ A(f_v, f_\omega) = \begin{bmatrix} -\frac{F_z}{M} f_v & 0 \\ 0 & \frac{RF_z}{J} f_\omega \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -\frac{1}{M} F_z \\ \frac{R}{J} F_z \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix},$$
$$w = \Delta \mu(\lambda, \varsigma), \ u = T_b.$$

Since v, ω , and $\mu(\lambda, \varsigma)$ are actually bounded during braking, the nonlinear functions f_v and f_ω are also limited in operation. Suppose the nonlinear function f_v is bounded by its minimum value $f_{v \min}$ and its maximum value $f_{v \max}$, using the "sector nonlinearity" approach [7], it is not difficult to represent the nonlinear function f_v by

$$f_v = M_1(\xi_1) f_{v \max} + M_2(\xi_1)) f_{v \min}, \qquad (10)$$

where $\xi_1 = f_v$ is a premise variable, $M_1(\xi_1)$ and $M_2(\xi_1)$ are membership functions, and

$$M_1(\xi_1) = \frac{\xi_1 - f_{v\min}}{f_{v\max} - f_{v\min}}, \ M_2(\xi_1(t)) = \frac{f_{v\max} - \xi_1}{f_{v\max} - f_{v\min}}.$$
(11)

Similarly, the nonlinear function f_{ω} is bounded by its minimum value $f_{\omega \min}$ and its maximum value $f_{\omega \max}$ so that it can be represented by

$$f_{\omega} = N_1(\xi_2) f_{\omega \max} + N_2(\xi_2) f_{\omega \min},$$
 (12)

where $\xi_2 = f_{\omega}$ is also a premise variable, $N_1(\xi_2)$ and $N_2(\xi_2)$ are membership functions which are defined as

$$N_1(\xi_2) = \frac{\xi_2 - f_{\omega \min}}{f_{\omega \max} - f_{\omega \min}}, \ N_2(\xi_2) = \frac{f_{\omega \max} - \xi_2}{f_{\omega \max} - f_{\omega \min}}.$$
(13)

By using the above defined four membership functions, the vehicle longitudinal dynamics model (9) can be represented by the following models:

Model Rule 1:

IF
$$\xi_1$$
 is M_1 and ξ_2 is N_1 .
THEN $\dot{x} = A_1 x + B_1 w + B_2 u$.

Model Rule 2:

IF	ξ_1	is	M_1	and	ξ_2	\mathbf{is}	N_2
THEN	ż :	= _	4.2x	$+B_1$	w ·	+ 1	$B_{2}u$

Model Rule 3:

IF	ξ_1 is M_2 and ξ_2 is N_1 ,
THEN	$\dot{x} = A_3 x + B_1 w + B_2 u,$

Model Rule 4:

IF
$$\xi_1$$
 is M_2 and ξ_2 is N_2 ,
THEN $\dot{x} = A_4 x + B_1 w + B_2 u$,

where matrices A_i , i = 1, 2, ..., 4, are obtained by replacing f_v and f_ω in matrix $A(f_v, f_\omega)$ of equation (9) with $f_{v \min}$, $f_{v \max}$, $f_{\omega \min}$, and $f_{\omega \max}$, respectively, according to the above defined rules. Then, the T-S fuzzy model that exactly represents the nonlinear vehicle longitudinal dynamics model (9) under the assumption of bounded v, ω , and $\mu(\lambda, \varsigma)$ is obtained as:

$$\dot{x} = \sum_{i=1}^{4} \mu_i(\xi) (A_i x + B_1 w + B_2 u),$$

$$y = C x + n,$$
(14)

where y is the measurement output, C is a constant matrix used to define the measurement output, n is the measurement noise, and

$$\begin{array}{rcl} \mu_1(\xi) &=& M_1(\xi_1)N_1(\xi_2), & \mu_2(\xi) = M_1(\xi_1)N_2(\xi_2), \\ \mu_3(\xi) &=& M_2(\xi_1)N_1(\xi_2), & \mu_4(\xi) = M_2(\xi_1)N_2(\xi_2), \\ \mu_i(\xi) &\geqslant& 0, \ i=1,2,...,4, \ {\rm and} \ \sum_{i=1}^4 \mu_i(\xi) = 1. \end{array}$$

Note that this T-S fuzzy model can exactly represent the vehicle longitudinal dynamics with the nonlinear friction model (5). It is also possible to extend this method to other nonlinear tyre models such as Pacejka Magic Formula tyre model. Furthermore, considering the possible normal load variation due to suspension dynamics during a manoeuvre and the possible modelling error between a simplified vehicle model and a real vehicle model, the uncertainties will be introduced in the model as

$$\dot{x} = \sum_{i=1}^{4} \mu_i(\xi) (A_i + \Delta A_i) x + (B_1 + \Delta B_1) w + B_2 u,$$

$$y = Cx + n,$$
(15)

where the uncertain matrices ΔA_i and ΔB represent the possible parameter and modelling uncertainties and are assumed to be bounded.

III. OBSERVER DESIGN

Assume that the measurement output is the wheel angular speed, which can be measured by available sensor in practice, the C matrix is given as $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$. Based on this measurement, we need to construct an observer based on the model (15) to estimate the vehicle speed. Note that the premise vector ξ in (15) is actually immeasurable. Instead, the observer will be constructed by using the estimated premise vector $\hat{\xi}$, which can be obtained from the estimated state vector \hat{x} . Therefore, the proposed observer is defined as

$$\dot{\hat{x}} = \sum_{i=1}^{4} \mu_i(\hat{\xi}) \left[A_i \hat{x} + L_i (y - \hat{y}) \right] + B_2 (u + \tilde{n})
\hat{y} = C \hat{x}$$
(16)

where \hat{x} is the observer state vector, \hat{y} is the estimated output, \tilde{n} is the measurement noise on the braking torque, and L_i is the observer gain matrix to be designed.

Considering (15) and (16), the error dynamics model is defined as

$$\dot{e} = \dot{x} - \dot{\hat{x}}
= \sum_{i=1}^{4} \mu_i(\hat{\xi})(A_i - L_iC)e + \sum_{i=1}^{4} (\mu_i(\xi) - \mu_i(\hat{\xi}))A_ix
+ B_1w - \sum_{i=1}^{4} \mu_i(\hat{\xi})L_in + \sum_{i=1}^{4} \mu_i(\xi)\Delta A_ix
+ \Delta B_1w + B_2\tilde{n},$$
(17)

where $e = x - \hat{x}$ is the estimation error.

To deal with some terms in (17) which are related to the immeasurable premise variables [14] and uncertain matrices, different approaches will be considered. A norm-bounded approach can be used to describe the uncertain matrix ΔB_1 as $\Delta B_1 = HFE$, where H and Eare known matrices, F is unknown matrix with $F^T F \leq I$. As the uncertain matrices ΔA_i are norm-bounded, we can define $\bar{w} = \sum_{i=1}^{4} \mu_i(\xi) \Delta A_i x$ as an external disturbance. In addition, we define $d = \sum_{i=1}^{4} (\mu_i(\xi) - \mu_i(\hat{\xi}))A_i x$. By referring to [15], [16], there exist some bounded function vectors Λ_i^T in terms of the membership functions defined in (11) and (13) such that $\mu_i(\xi) - \mu_i(\hat{\xi}) = \Lambda_i^T e$ can be obtained. Therefore, the term d can be written as $d = \left(\sum_{i=1}^{4} A_i x \Lambda_i^T\right) e$, which is bounded by

$$d = \left(\sum_{i=1}^{4} A_i x \Lambda_i^T\right) e, \text{ which is bounded by}$$
$$d^T d \le e^T U e,$$

for a nonsingular matrix U which is dependent on Λ_i^T and x. Then, (17) is further written as

$$\dot{\hat{e}} = \sum_{i=1}^{4} \mu_i(\hat{\xi}) \left[(A_i - L_i C) e - L_i n \right] + d + (HFE + B_1) w + \bar{w} + B_2 \tilde{n} = \sum_{i=1}^{4} \mu_i(\hat{\xi}) \left[\bar{A}_i e - L_i n \right] + d + (HFE + B_1) w + \bar{w} + B_2 \tilde{n}, \quad (18)$$

where $\bar{A}_i = A_i - L_i C$.

Consider the Lyapunov function candidate as

$$V(e) = e^T X e, (19)$$

where $X = X^T > 0$, the time derivative of V(e) is

$$\begin{split} \dot{V}(e) &= \sum_{i=1}^{4} \mu_i(\hat{\xi}) \\ & \begin{bmatrix} e^T (\bar{A}_i^T X + X \bar{A}_i) e + e^T X \delta + \delta^T X e \\ + w^T E^T F H^T X e + e^T X H F E w \\ - n^T L_i^T X e - e^T X L_i n + \bar{w}^T X e \\ + e^T X \bar{w} + w^T B_1^T X e + e^T X B_1 w \\ + \tilde{n}^T B_2^T X e + e^T X B_2 \tilde{n} \end{bmatrix} \\ & \leq \sum_{i=1}^{4} \mu_i(\hat{\xi}) \end{split}$$

$$\begin{bmatrix} e^T (\bar{A}_i^T X + X \bar{A}_i + \varepsilon_1^{-1} X X + \varepsilon_1 U \\ + \varepsilon_2^{-1} X H H^T X) e + \varepsilon_2 w^T E^T E w \\ - n^T L_i^T X e - e^T X L_i n + \bar{w}^T X e \\ + e^T X \bar{w} + w^T B_1^T X e + e^T X B_1 w \\ + \tilde{n}^T B_2^T X e + e^T X B_2 \tilde{n} \end{bmatrix}$$
(20)

where $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and the matrix inequalities $e^T X \delta + \delta^T X e \leq e^T (\varepsilon_1^{-1} X X + \varepsilon_1 U) e$ and $w^T E^T F H^T X e + e^T X H F E w \leq \varepsilon_2^{-1} e^T X H H^T X e + \varepsilon_2 w^T E^T E w$ are applied.

To eliminate the effects of disturbances, such as measurement noise, uncertainties, and the term d, on the estimation error, the observer will be designed to achieve a minimum H_{∞} -norm level on the estimation error.

By defining an objective output as

$$z = x - \hat{x} = C_z e, \tag{21}$$

and adding $z^T z - \gamma^2 w^T w - \gamma^2 n^T n - \gamma^2 \bar{w}^T \bar{w} - \gamma^2 \tilde{n}^T \tilde{n}$, $\gamma > 0$, to the two sides of (20), we have

$$\begin{split} \dot{V}(e) + z^{T}z - \gamma^{2}w^{T}w - \gamma^{2}n^{T}n - \gamma^{2}\bar{w}^{T}\bar{w} - \gamma^{2}\tilde{n}^{T}\tilde{n} \\ \leq & \sum_{i=1}^{4} \mu_{i}(\hat{\xi}) \begin{bmatrix} e^{T}(\bar{A}_{i}^{T}X + X\bar{A}_{i} + \varepsilon_{1}^{-1}XX \\ +\varepsilon_{1}U + \varepsilon_{2}^{-1}XHH^{T}X)e \\ +\varepsilon_{2}w^{T}E^{T}Ew - n^{T}L_{i}^{T}Xe \\ -e^{T}XL_{i}n + \bar{w}^{T}Xe + e^{T}X\bar{w} \\ +w^{T}B^{T}Xe + e^{T}XBw \end{bmatrix} \\ + e^{T}C^{T}Ce - \gamma^{2}w^{T}w - \gamma^{2}n^{T}n - \gamma^{2}\bar{w}^{T}\bar{w} \\ = & \sum_{i=1}^{4} \mu_{i}(\hat{\xi}) \left(\begin{bmatrix} e \\ w \\ n \\ \bar{w} \\ \bar{n} \end{bmatrix}^{T} \Psi_{i} \begin{bmatrix} e \\ w \\ n \\ \bar{w} \\ \bar{n} \end{bmatrix} \right), \end{split}$$
(22)

where

$$\Psi_{i} = \begin{bmatrix} \Phi_{i} & XB_{1} & -XL_{i} & X & XB_{2} \\ * & -\gamma^{2}I + \varepsilon_{2}E^{T}E & 0 & 0 & 0 \\ * & * & -\gamma^{2}I & 0 & 0 \\ * & * & * & -\gamma^{2}I & 0 \\ * & * & * & * & -\gamma^{2}I \end{bmatrix}$$

 $\Phi_i = \bar{A}_i^T X + X \bar{A}_i + \varepsilon_1^{-1} X X + \varepsilon_1 U + \varepsilon_2^{-1} X H H^T X + C^T C.$ It can be seen from (22) that if the following inequality is satisfied

$$\begin{bmatrix} \Phi_{i} & XB_{1} & -XL_{i} & X & XB_{2} \\ * & -\gamma^{2}I + \varepsilon_{2}E^{T}E & 0 & 0 & 0 \\ * & * & -\gamma^{2}I & 0 & 0 \\ * & * & * & -\gamma^{2}I & 0 \\ * & * & * & * & -\gamma^{2}I \end{bmatrix} < 0,$$
(23)

the error dynamic system (17) is stable with an H_{∞} disturbance attenuation level less than γ . By the Schur

complement equivalence, (23) is equivalent to

$$\begin{bmatrix} \Theta_i & XB_1 & -Y_i & X & XB_2 & X & XH \\ * & \Lambda & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix}$$

$$< 0, i = 1, 2, \dots, 4,$$

where $\Theta_i = \bar{A}_i^T X + X \bar{A}_i + \varepsilon_1 U + C_z^T C_z, \Lambda = -\gamma^2 I + \varepsilon_2 E^T E, Y_i = X L_i.$

In summary, for a given scalar $\gamma > 0$, the error dynamic system (17) is stable with an H_{∞} disturbance attenuation level less than γ if there exist matrices $X > 0, Y_i, i = 1, 2, ..., 4$, scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$, satisfying LMIs (24) and the observer gains can be obtained as $L_i = Y_i X^{-1}$. Moreover, to make the estimation error as small as possible, the following optimisation problem is carried out:

min
$$\gamma$$
 s.t. LMIs (24). (25)

This is a convex optimisation problem which can be solved efficiently by means of available software.

IV. NUMERICAL SIMULATIONS

In this section, numerical simulations are used to validate the effectiveness of the proposed approach. To show the improvement on the estimation performance, another two existing methods are also used for comparison purpose. One method estimates the velocity by using the equation of [11]

$$\dot{\hat{v}} = \frac{J}{R^2 M} (\dot{v}_m - R\dot{\omega})$$

where $\dot{v}_m = -\frac{R}{J}T_b$. For description brevity, this method is called Linear Observer thereafter.

Another method estimates the velocity by using the equation of [5]

$$\hat{v} = a + K(R\omega - \hat{v})$$

where a is the measured longitudinal acceleration, K is the observer gain that depends on the longitudinal acceleration measurement. For description brevity, this method is called Nonlinear Observer thereafter.

First, the vehicle is running on a dry asphalt road surface with an initial speed of 33.33 m/s during braking. The brake torque is applied according to the desired slip ratio, 0.2, with a bang-bang control strategy together with a first-order actuator model. In the simulations, both the wheel angular velocity and the braking torque are measured with measurement noises. The parameter values for the quarter-vehicle braking model are given in Table I [1].

The actual vehicle speed and wheel linear speed are shown in Fig. 3. Fig. 4 shows the braking torque.

Parameter	Value			
M	275 kg			
J	$12.891 \ {\rm kg} \cdot {\rm m}^2$			
R	0.25 m			
TABLE I				

Parameter values of the quarter-vehicle braking model



Fig. 3. Vehicle speed and wheel linear speed when braking on a dry asphalt road surface.



Fig. 4. Braking torque on a dry asphalt road surface.



Fig. 5. Comparison of estimated longitudinal velocity when braking on a dry asphalt road surface.

Fig. 5 shows the estimated velocity using difference methods, and Fig. 6 shows the estimation error by using different methods, where the proposed method is denoted as Fuzzy Observer. From Fig. 6 it can be seen the proposed method shows the best estimation performance compared to other two methods. It is noted that Linear Observer is actually a direct integration method without using feedback. It is based on the assumption of no slip. When there is measurement noise with sensor bias and when slip ratio is not zero, big estimation error will happen. The accumulated estimation error due to bias and none zero slip can be clearly observed from Fig. 6. Nonlinear Observer is an integration method with a feedback of the difference between the measured wheel speed and the estimation. The performance of this method depends on the adaptive tuning of the observer gain, which however, is a non-trivial task. The estimated velocity is more closing to the wheel linear speed.

To further validate the effectiveness of the proposed method, the braking is applied when the vehicle is running on a cobble wet road surface. Fig. 7 shows the actual vehicle speed and wheel linear speed, Fig. 8 shows the braking torque, Fig. 9 shows the estimated velocity using difference methods, and Fig. 10 shows the estimation error by using different methods. Similarly, from Fig. 10, it can be seen the proposed method shows the best estimation performance compared to other two methods. This further confirms the improved



Fig. 6. Comparison of estimation error of longitudinal velocity when braking on a dry asphalt road surface.

performance of the proposed method.



Fig. 7. Vehicle speed and wheel linear speed when braking on a cobble wet road surface.



Fig. 8. Braking torque on a cobble wet road surface.



Fig. 9. Comparison of estimated longitudinal velocity when braking on a cobble wet road surface.



Fig. 10. Comparison of estimation error of longitudinal velocity when braking on a cobble wet road surface.

V. CONCLUSIONS

In this paper, a T-S fuzzy modelling approach is adopted to represent a quarter-vehicle braking model. Based on this T-S fuzzy model, an observer is designed to estimate the longitudinal velocity with the available measurements of wheel angular velocity and braking torque. The H_{∞} performance is applied when designing the observer with estimated premise variables. The conditions are expressed in terms of LMIs which can be solved easily with available software. Numerical simulations are used to demonstrate the effectiveness of the proposed approach. Further study on the estimation based slip ratio control will be investigated.

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