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Tightly Coupled Fuzzy Rough Description Logic Programs under the Answer Set Semantics for the Semantic Web

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***Abstract*—The Semantic Web is an extension of the current World Wide Web, and aims to help computers to understand and process web information automatically. In recent years, the integration ontologies and rules has become a central topic in the Semantic Web. Therefore, significant research efforts have focused on integration description logic programs. However, description logic programs cannot well model a great deal of real-world problems because of the restriction of represented formalism. To address this problem, we further extend description logic programs such that they can deal with imprecise information, uncertain information and non monotonic reasoning at the same time. In this paper, we propose tightly coupled fuzzy rough description logic programs (or simply fuzzy rough dl-program) under the answer set semantics, which are tightly integrates fuzzy rough disjunctive programs under the answer set semantics with fuzzy rough description logics. To our knowledge, this is the first such approach. First of all, we define the syntax and semantics of fuzzy rough disjunctive logic programs, which is the rough extension of fuzzy disjunctive logic programs based on rough set theory. Then, we define the syntax and semantics of fuzzy rough dl-program. Finally, we show some semantic properties of fuzzy rough dl-program under the answer set semantics.**

Keywords—Semantic web; Description logics; Description logic programs; Answer set semantics

I. INTRODUCTION

The Semantic Web is an extension of the current World Wide Web by standards and technologies that help machines to understand the information on the Web in order to help computers to process Web information automatically and better enable computer and human beings to work in cooperation [1,2]. The main ideas behind it are to add a machine-readable meaning to Web pages, to use ontologies for a precise definition of share terms in Web resources, to use Knowledge Representation technology for automated reasoning from Web resources [3].

At present, the highest layer of the semantic web, which has reached a sufficient maturity, is the ontology layer in form of the OWL Web Ontology Language [4]. The next and ongoing step aims at sophisticated representation and reasoning capabilities of the Rules, Logic, and Proof layers of the Semantic Web [5,6].

As we have seen, the integration ontologies and rules has become a central topic in the Semantic Web. In fact, standard ontology language is based on Description Logic[7,8], and the existing proposals for a rule language for use in the Semantic Web originate from Logic Programming.

Description logics are a class of knowledge representation languages, which can model an application domain of interest by a structured and formally well-understood way[9]. Recently, with the rapid development of DLs, abundant DL systems have been provided, such as ALC[10], SHIN[11], SHIQ[12], SHOIQ[13,14], SROIQ [15]and so on.

Recently, significant research efforts have focused on integration description logics and logic programmings. Eiter et al introduce disjunctive description logics (for short, dl-programs), which is loose integration of description logic and logic programming [16,17]. In 2006, Rosatic presented tight integration of description logics and disjunctive Datalog [18]. Subsequently, Lukasiewicz propose tight integrated disjunctive description logic program under the answer set semantic [19]. In loose integration, the rules in disjunctive logic program knowledge of dl-program involved queries to description logic knowledge base, but the rules in disjunctive logic program knowledge involved concepts and roles from description logic knowledge base as unary resp. binary predicates in a tight integration. However, the above dl-programs can only represent and reason on precise or certain knowledge. Therefore some researchers extend the dl-programs allowing to express imprecise or uncertain knowledge. At this aspect, three kinds of dl-programs, i.e., fuzzy dl-programs, probabilistic dl-programs, and rough dl-programs, are proposed.

Regarding fuzzy dl-programs, Lukasiewicz introduced vagueness into dl-programs, and proposed fuzzy dl-program that combined fuzzy description logics and fuzzy disjunctive logic programs [20,21]. Subsequently, he presented tightly

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coupled fuzzy description logic programs under the answer set semantic, which extended tightly disjunctive description logic program by fuzzy vagueness in both the description logic and the logic program component [22,23]. Regarding probabilistic dl-programs, Lukasiewicz proposed the notion of probabilistic dl-programs, and described the syntax and semantic of probabilistic dl-programs [24,25]. Moreover, Andrea Calì present tightly coupled probabilistic dl-programs under the answer set semantics, which were a tight integration of disjunctive logic programs under the answer set semantics and Bayesian probabilities [26,27]. Furthermore, in 2009 Lukasiewicz and Straccia presented probabilistic fuzzy description logic programs, which combine fuzzy description logics, fuzzy logic programs, and probabilistic uncertainty in a uniform framework for the semantic web [28]. This novel approach allows for handling both probabilistic uncertainty and fuzzy vagueness. Regarding rough dl-programs, we propose tightly coupled rough description logic programs under the answer set semantics, which are a tight integration of disjunctive logic programs under the answer set semantics, rough set theory and rough description logics[29].

However, the above description logic programs can not deal with imprecise information, and uncertain information at the same time. Therefore, this paper aims to further extend description logic programs such that they can model imprecise information, and uncertain information. In this paper, we first present fuzzy rough disjunctive logic programs (for short, fuzzy rough programs) under the answer set semantic, which is the rough extension of fuzzy disjunctive logic programs based on rough set theory. Then, we propose tightly coupled fuzzy rough description logic programs (for short, fuzzy rough dl-programs) under the answer set semantics, which tightly integrates fuzzy rough disjunctive programs under the answer set semantics with fuzzy rough description logics. It is the generalization of the tightly coupled disjunctive dl-programs by fuzzy vagueness and rough in both the ontological and the rule component. Finally, we show that the new fuzzy rough dl-programs have nice semantic features. More concretely, all their answer sets are also minimal models, and the cautious answer set semantics faithfully extends both fuzzy rough programs and fuzzy rough description logics. Similarly, this approach also does not need the unique name assumption.

The rest of this paper is organized as follows. In section II, we recall fuzzy set theoretic operations and fuzzy rough description logic. Section defines fuzzy rough programs under the answer set semantics. In section , we present fuzzy rough dl-programs under the answer set semantics, and also propose some semantic properties. Section summarizes our main results.

II. PRELIMINARIES

In this section, we first recall some work related to fuzzy set theoretic operations. Then we introduce the syntax and semantic of fuzzy rough description logic.

A. Fuzzy set theoretic operations

In order to combine and modify the truth in [0,1], we assume fuzzy operations, namely, fuzzy conjunction, fuzzy disjunction, fuzzy implication and fuzzy complement, denoted by $\otimes$, $\oplus$, $\rhd$ and Θ, respectively, which are functions $\otimes$, $\oplus$, $\rhd : [0,1]\times[0,1]\to[0,1]$ and $\Theta:[0,1]\to[0,1]$ that generalize the ordinary Boolean operations to the set of truth values [0,1].

Several functions of fuzzy operations have been given in the literature. In the current paper we will use the following fuzzy functions:

$$a\otimes b=\min\{a,b\},$$

$$a\oplus b=\max\{a,b\},$$

$$a\rhd b=\max\{1-a,b\},$$

$$\Theta a=1-a.$$

B. Fuzzy rough description logics

Fuzzy rough description logics are fuzzy rough extensions classical description logics based on fuzzy rough theory, so fuzzy rough DLs can represent and reason on fuzzy and incomplete knowledge. We now recall the syntax and the semantics of fuzzy rough description logic FRSHIN [30].

Let A, R and I be pairwise disjoint sets of atomic concepts, roles and individuals, respectively. A role in FRSHIN is any element of R. FRSHIN-concepts(denoted by C or D) are composed inductively according to the following abstract syntax:

$$\begin{gathered}C,D\to\bot|\top|A\,|\,\neg C\,|\,C\sqcap D\,|\,C\sqcup D\,|\,\exists R.C\,|\\ \forall R.C\,|\geq nR\,|\leq nR\,|\,\overline{C}\,|\,\underline{C}\,|\end{gathered},$$

where A denotes atomic concept, C and D denote concepts (or concept descriptions), R denotes role name, and n denotes a natural number.

The semantics of FRSHIN is the extension of the semantics of the classical DL SHIN. A fuzzy rough interpretation $I=(\Delta^I,R^{\sim},\bullet^I)$ consists of a nonempty domain Δ^I, a fuzzy relation $R^{\sim}$ over Δ^I, and a mapping $\bullet^I$ that assigns to each individual an element of Δ^I, to each atomic concept $A\in\mathbf{A}$ a function $A^I:\Delta^I\to[0,1]$, to each $R\in\mathbf{R}$ a function $R^I:\Delta^I\times\Delta^I\to[0,1]$, $R\neq R^{\sim}$. The mapping $\bullet^I$ can be extended to all roles and concepts as follows: for all $a,b,d\in\Delta^I$,

1) $\top^I(d)=1$;, ,

2) $\perp^I(d) = 0$;

3) $(C \sqcap D)^I(d) = \min\{C^I(d), D^I(d)\}$;

4) $(C \sqcup D)^I(d) = \max\{C^I(d), D^I(d)\}$;

5) $(\neg C)^I(d) = 1 - C^I(d)$;

6) $(\exists R.C)^I(d) = \sup_{d' \in \Delta^I}\{\min\{R^I(d,d'), C^I(d')\}\}$;

7) $(\forall R.C)^I(d) = \inf_{d' \in \Delta^I}\{\max\{1 - R^I(d,d'), C^I(d')\}\}$;

8) $(\geq nR)^I(d) = \sup_{b_1,\ldots,b_n \in \Delta^I} \min_{i=1}^{n}\{R^I(a,b_i)\}$;

9) $(\leq nR)^I(d) = \inf_{b_1,\ldots,b_{n+1} \in \Delta^I} \max_{i=1}^{n+1}\{1 - R^I(a,b_i)\}$;

10) $(\overline{C})^I(d) = \sup_{d' \in \Delta^I}\{\min\{R^{\sim}(d,d'), C^I(d')\}\}$;

11) $(\underline{C})^I(d) = \inf_{d' \in \Delta^I}\{\max\{1 - R^{\sim}(d,d'), C^I(d')\}\}$;

12) $(R^{-})^I(b,a) = R^I(a,b)$;

A FRSHIN knowledge base consists of a TBox, a RBox and an ABox. A FRSHIN TBox is a finite (possibly empty) set of fuzzy rough inclusion introductions of the form $A \sqsubseteq C$ or fuzzy rough equivalence introductions of the form $A \equiv C$, where A is a concept name and C is a FRSHIN-concept.

A fuzzy rough interpretation I satisfies $A \sqsubseteq C$ iff $\forall d \in \Delta^I$, $A^I(d) \leq C^I(d)$, and it satisfies $A \equiv C$ iff $\forall d \in \Delta^I$, $A^I(d) = C^I(d)$.

A FRSHIN RBox is a finite (possibly empty) set of fuzzy (rough) transitive role axioms of the form $Trans(R)$ and fuzzy (rough) role inclusion axioms of the form $R \sqsubseteq S$, where R, S are FRSHIN-roles.

A fuzzy rough interpretation I satisfies $Trans(R)$ iff $\forall a,c \in \Delta^I$,

$$R^I(a,c) \geq \sup_{b \in \Delta^I}\{\min\{R^I(a,b), R^I(b,c)\}\}.$$

It satisfies $R \sqsubseteq S$ iff $\forall a,b \in \Delta^I$, $R^I(a,b) \leq S^I(a,b)$.

A FRSHIN ABox is a finite (possibly empty) set of fuzzy rough assertions of the form $\alpha\theta n$ and $a \neq b$, where α is an assertion of the form $a:C$, $<a,b>:R$, $\theta \in \{\geq,>,\leq,<\}$ and $n \in [0,1]$, C denotes a concept, R denotes a role, a and b denote individuals.

Formally, a fuzzy rough interpretation I satisfies $(a:C)\theta n$ (or $(<a,b>:R)\theta n$) iff $C^I(a^I)\theta n$ (or $R^I(a^I,b^I)\theta n$), and it satisfies $a \neq b$ iff $a^I \neq b^I$.

A fuzzy rough interpretation I satisfies a FRSHIN knowledge base Σ iff it satisfies all axioms in Σ; in this case, we say that I is a model of Σ.

A FRSHIN knowledge base Σ is satisfiable (unsatisfiable) iff there exists (does not exist) a fuzzy rough interpretation I which satisfies all axioms in Σ.

III. FUZZY ROUGH DISJUNCTIVE PROGRAMS UNDER THE ANSWER SET SEMANTIC

In this section, we definite fuzzy rough disjunctive program, and propose the syntax and semantic of fuzzy rough disjunctive program. First of all, we present some concepts that are useful of defining fuzzy rough disjunctive programs.

Logic program is based on first-order logic, and in first-order logic an unary predicate (i.e. R(t)) denotes whether a term (i.e. t) is an element of concept set expressed by predicate symbol (i.e. R). If it is, then the unary predicate is true (i.e. R(t)=1), otherwise unary predicate is false (i.e. R(t)=0). However, it may be impossible to define some concepts expressed by unary predicate symbol precisely, so we can't decide whether the unary predicate is true. Therefore, in this section we propose approximate unary predicate symbols. The basic idea is to approximate unary predicate symbol by bounding from below and above the concept set expressed by unary predicate symbol.

Let Q denotes unary predicate symbol, Con(Q) denotes concept set expressed by Q, and $R^{\sim}$ denotes equivalence relation on Con(Q). We define approximate predicate symbols as follows.

Definition 3.1 For unary predicate symbol Q, approximate predicate symbol is of the form $Q = (\underline{Q}, \overline{Q})$, where $\underline{Q}$ is lower approximate predicate symbol and $\overline{Q}$ is upper approximate predicate symbol. Moreover,

$$\mathrm{Con}(\underline{Q}) = \{x \in \mathrm{Con}(Q) \mid R^{\sim}(x) \subseteq \mathrm{Con}(Q)\},$$

$$\mathrm{Con}(\overline{Q}) = \{x \in \mathrm{Con}(Q) \mid R^{\sim}(x) \cap \mathrm{Con}(Q) \neq \varnothing\}.$$

Obviously, we can obtain the following properties.

1) $\mathrm{Con}(\underline{Q}) \subseteq \mathrm{Con}(Q) \subseteq \mathrm{Con}(\overline{Q})$;

2) $\forall x \in \mathrm{Con}(\underline{Q})$, $\underline{Q}(x) = 1$, otherwise $\underline{Q}(x) = 0$;

3) $\forall x \in \mathrm{Con}(\overline{Q})$, $\overline{Q}(x) = 1$, otherwise $\overline{Q}(x) = 0$;

Definition 3.2. For any approximate predicate symbol $Q = (\underline{Q}, \overline{Q})$ and x, unary predicate $Q(x)$ is definitely true iff lower approximate predicate $\underline{Q}(x) = 1$; unary predicate $Q(x)$ is possibly true iff upper approximate predicate $\overline{Q}(x) = 1$.

Now, we introduce the syntax and semantic of fuzzy rough disjunctive logic programs under the answer set semantics.

A. Syntax

Let Φ be a function-free first-order vocabulary with nonempty finite sets of constant and predicate symbols (include approximate predicate symbols). Let X be a set of variables. A term is either a variable from X or a constant symbol from Φ .

Definition 3.3. An approximate atom is of the form $\alpha = (\underline{\alpha}, \overline{\alpha})$, where $\underline{\alpha}$ is of the form $\underline{Q}(t)$ and $\overline{\alpha}$ is of the form $\overline{Q}(t)$, $Q = (\underline{Q}, \overline{Q})$ is an approximate predicate symbol from Φ , t is term.

An atom is either an approximate atom or of the form $p(t_1,\ldots,t_n)$, where p is a predicate symbol of arity $n \geq 0$ form Φ , and $t_1,\ldots,t_n$ are terms. A literal l is an atom α or a negated atom not α . If an atom α is not an approximate atom, then $\underline{\alpha} = \alpha$ and $\overline{\alpha} = \alpha$.

Definition 3.4. A disjunctive fuzzy rough rule (or simply fuzzy rough rule) r is of the form

$$\begin{aligned}\underline{\alpha}_1 \vee_{\oplus_1} \cdots \vee_{\oplus_{k-1}} \underline{\alpha}_k \leftarrow_{\otimes_0} \underline{\beta}_1 \wedge_{\otimes_1} \cdots \wedge_{\otimes_{l-1}} \underline{\beta}_l \wedge_{\otimes_l} \\ not_{\Theta_{l+1}} \underline{\beta}_{l+1} \wedge_{\otimes_{l+1}} \cdots \wedge_{\otimes_{n-1}} not_{\Theta_n} \underline{\beta}_n \geq u\end{aligned} \quad (1)$$

$$\begin{aligned}\overline{\alpha}_1 \vee_{\oplus_1} \cdots \vee_{\oplus_{k-1}} \overline{\alpha}_k \leftarrow_{\otimes_0} \overline{\beta}_1 \wedge_{\otimes_1} \cdots \wedge_{\otimes_{l-1}} \overline{\beta}_l \wedge_{\otimes_l} \\ not_{\Theta_{l+1}} \overline{\beta}_{l+1} \wedge_{\otimes_{l+1}} \cdots \wedge_{\otimes_{n-1}} not_{\Theta_n} \overline{\beta}_n \geq u\end{aligned} \quad (2)$$

where $k \geq 1$, $n \geq l \geq 0$, $\alpha_1,\ldots,\alpha_k$, $\beta_{l+1},\ldots,\beta_n$ are atoms, $\beta_1,\ldots,\beta_l$ are either atoms or truth values form [0,1], $\oplus_1,\ldots,\oplus_{k-1}$ are disjunction strategies, $\otimes_0,\ldots,\otimes_{n-1}$ are conjunction strategies, $\Theta_{l+1},\ldots,\Theta_n$ are negation strategies, and $u \in [0,1]$. The set $H(r) = \{\alpha_1,\ldots,\alpha_k\}$ is the head of r, while $B(r) = B^+(r) \cup B^-(r) = \{\beta_1,\ldots,\beta_l\} \cup \{\beta_{l+1},\ldots,\beta_n\}$ is the body of r.

Definition 3.5. A fuzzy rough disjunctive program (or simply fuzzy rough program) P is a finite set of disjunctive fuzzy rough rules of the form (3.1) and (3.2). Moreover, P is normal fuzzy rough program iff $k = 1$ for all fuzzy rough rules in P; P is a positive fuzzy rough program iff $n = l$ for all fuzzy rough rules in P.

Example 1. A fuzzy rough disjunctive program P contains the following fuzzy rough rules:

$$\begin{aligned}\underline{Q}(x) \leftarrow_{\otimes} \underline{C_1}(x) \wedge_{\otimes} R_1(x,y_1) \wedge_{\otimes} R_2(x,y_2) \\ \wedge_{\otimes} \underline{C_2}(y_1) \wedge_{\otimes} \underline{C_3}(y_2) \geq 1\end{aligned} ,$$

$$\begin{aligned}\overline{Q}(x) \leftarrow_{\otimes} \overline{C_1}(x) \wedge_{\otimes} R_1(x,y_1) \wedge_{\otimes} R_2(x,y_2) \\ \wedge_{\otimes} \overline{C_2}(y_1) \wedge_{\otimes} \overline{C_3}(y_2) \geq 1\end{aligned} .$$

B. Semantics

Now, we define the answer set semantics of fuzzy rough disjunctive programs based on finite sets of ground atoms, which represent Herbrand interpretations.

More formally, a term is ground iff it includes only constant symbols. An atom is ground iff all terms in it are ground. A fuzzy rough rule is ground iff all atoms in it are ground.

The Herbrand universe of a fuzzy rough program P, namely HUp, denotes the set of all constant symbols appearing P. If HUp is empty, then we let HUp={c}, where c is an arbitrary constant symbol from Φ . The Herbrand base of a fuzzy rough program P, namely HBp, denotes the set of all ground atoms that can be made from the predicate symbols contained by P and the constant symbols contained by HUp. A ground instance of a rule $r \in P$ is get through substituting constant symbol coming from HUp for every variable appearing in r. We use ground(P) to denote the set of all ground instances of rules in P.

An interpretation I relative to a fuzzy rough program P is a mapping $I : HBp \to [0,1]$. For any interpretations I and J, for all $\alpha \in HBp$, the inclusion relation $I \subseteq J$ is true iff $I(\alpha) \leq J(\alpha)$; the intersection relation $I \cap J$ is true iff $I \cap J(\alpha) = \min\{I(\alpha), J(\alpha)\}$.

Definition 3.6. Let I be an interpretation of a fuzzy rough program P, and α be a ground atom of HBp. α is definitely satisfiable under interpretation I iff $I(\underline{\alpha}) > 0$; α is possible satisfiable under interpretation I iff $I(\overline{\alpha}) > 0$.

Definition 3.7. Let I be an interpretation of a fuzzy rough program P, r be a ground fuzzy rough rule of the form (1) and (2). I is a model of r, denoted $I \models r$, iff

$$I(\underline{\alpha}_1) \oplus_1 \cdots \oplus_{k-1} I(\underline{\alpha}_k) \geq \begin{cases} I(\underline{\beta}_1) \otimes_1 \cdots \otimes_{l-1} I(\underline{\beta}_l) \otimes_l \Theta_{l+1} I(\underline{\beta}_{l+1}) \\ \quad \otimes_{l+1} \cdots \otimes_{n-1} \Theta_n I(\underline{\beta}_n) \otimes_0 I(u), \quad if\ n \geq 1 \\ u, \quad otherwise \end{cases} \tag{3}$$

$$I(\overline{\alpha}_1) \oplus_1 \cdots \oplus_{k-1} I(\overline{\alpha}_k) \geq \begin{cases} I(\overline{\beta}_1) \otimes_1 \cdots \otimes_{l-1} I(\overline{\beta}_l) \otimes_l \Theta_{l+1} I(\overline{\beta}_{l+1}) \\ \quad \otimes_{l+1} \cdots \otimes_{n-1} \Theta_n I(\overline{\beta}_n) \otimes_0 I(u), \quad if\ n \geq 1 \\ u, \quad otherwise \end{cases} \tag{4}$$

Definition 3.8. Let I be an interpretation of a fuzzy rough program P. I is a model of P, denoted $I \models P$, iff $I \models r$ for every $r \in ground(P)$.

The Gelfond-Lifschitz reduct of a fuzzy rough program P relative to an interpretation I, denoted P^I, is the ground positive fuzzy rough program obtained form ground(P) by substituting the truth value $\Theta_j I(\beta_j)$ for all default-negated atoms $not_{\Theta_j} I(\beta_j)$.

Example 2. Consider again the fuzzy rough disjunctive program P of Example 1. It is not difficult to verify that P has a model I, which defined as follows:

$$I(\underline{Q}(a)) = 0.38, I(\overline{Q}(a)) = 0.45,$$

$$I(\underline{C_1}(a)) = 0.21, I(\overline{C_1}(x)(a)) = 0.35,$$

$$I(\underline{C_2}(b)) = 0.27, I(\overline{C_2}(x)(b)) = 0.42,$$

$$I(\underline{C_3}(c)) = 0.39, I(\overline{C_1}(x)(c)) = 0.55,$$

$$I(R_1(a,b)) = 0.45, I(R_2(a,c)) = 0.56.$$

Definition 3.9. Let P be a fuzzy rough program. An interpretation I is an answer set of P iff I is a minimal model of P^I. P is consistent iff P has an answer set.

Definition 3.10. Let P be a fuzzy rough program, α be a ground atom of HBp and $n \in [0,1]$. Then, $\alpha \geq n$ is a cautious (resp., brave) consequence of P under the answer set semantics iff $I(\alpha) \geq n$ for every (resp., some) answer set I of P.

IV. FUZZY ROUGH DESCRIPTION LOGIC PROGRAMS UNDER THE ANSWER SET SEMANTIC

In this section, we propose a tightly coupled approach to fuzzy rough description logic programs (or simply fuzzy rough dl-programs) under the answer set semantics, which extends tightly coupled fuzzy description logic programs under the answer set semantics with a simple mechanism to handle approximate knowledge.

A. The syntax and semantics

The basic idea of this tightly couple is presented as follows. Let P be a fuzzy rough disjunctive program, it is known that P is equivalent to its grounding ground(P) under the answer set semantics. If some of the ground atoms in ground(P) are additionally related to each other by a fuzzy rough description logic knowledge base L, that is to say that some of the ground atoms in ground(P) actually present concepts and role memberships relative to L, then we must consider L when dealing with ground(P). However, we only to consider it when we actually need for dealing with ground(P). Therefore, for a fuzzy rough Herbrand interpretation I, we need to prove that I represents a valid truth value assignment relative to L. In other words, the Herbrand interpretation I which satisfies P, is also satisfies L, while L is interpreted relative to general interpretations over a first-order domain.

The first-order vocabulary Φ is defined as section Ⅲ, and the sets **A**, **R** and **I** is defined as section Ⅱ. Let Φ_c be the set of all constant symbols in Φ. Suppose that Φ_c is a subset of **I**. This hypothesis ensures that every ground atom made from **A**, **R** and **I** can be interpreted in the description logic knowledge base.

A fuzzy rough description logic program (for short, fuzzy rough dl-program) KB=(L,P) includes a fuzzy rough description logic knowledge base L and a fuzzy rough disjunctive program P. It is a positive fuzzy rough dl-program iff P is positive fuzzy rough disjunctive program. It is a normal fuzzy rough dl-program iff P is normal fuzzy rough disjunctive program.

Example 3. A fuzzy rough description logic program KB=(L,P) contains a fuzzy rough disjunctive program P in Example 1 and fuzzy rough description logic knowledge base L, which is given by the following axioms.

$$C_1 \sqcup C_5 \sqcup C_6 \sqsubseteq C_7, \quad C_7 \sqsubseteq \exists R_1.C_2 \sqcap \exists R_2.C_3,$$

$$C_8 \sqsubseteq \overline{C_9} \sqcap \overline{C_{10}}, \qquad C_{10} \sqsubseteq \exists R_3.C_{11},$$

$$a_1 : \underline{C_1} \sqcap \exists R_1.\underline{C_2} \sqcap \exists R_2.\underline{C_3} \geq 0.21,$$

$a_1 : \overline{C_1} \sqcap \exists R_1.\overline{C_2} \sqcap \exists R_2.\overline{C_3} \geq 0.35$,

$a_2 : \underline{C_2} \geq 0.28, \quad a_2 : \overline{C_2} \geq 0.42$,

$a_3 : \underline{C_3} \geq 0.37, \quad a_2 : \overline{C_3} \geq 0.51$,

$< a_1, a_2 >: R_1 \geq 0.45$, $< a_1, a_3 >: R_2 \geq 0.56$.

Let KB=(L,P) be a fuzzy rough dl-program. The Herbrand base relative to Φ , namely HB_{Φ} , denotes the set of all ground atoms made from the predicate symbols and the constant symbols from Φ . A ground instance of a rule $r \in P$ is getted through substituting constant symbol coming from Φ_c for every variable appearing in *r*. We use ground(P) to denote the set of all ground instances of rules in *P*.

An interpretation *I* relative to a fuzzy rough dl-program KB=(L,P) is a mapping $I : HB_{\Phi} \to [0,1]$. In this way, *I* is a model of a fuzzy rough description logic knowledge base *L*, denoted $I \models L$, iff $L \cup \{\alpha = I(a) \mid a \in HB_{\Phi}\}$ is satisfiable; *I* is a model of fuzzy rough disjunctive knowledge base *P*, denoted $I \models P$, iff $I \models r$ for every $r \in ground(P)$.

Definition 4.1. Let *I* be an interpretation of a fuzzy rough dl-program *KB=(L,P)*. Then *I* is a model of KB, denoted by $I \models KB$, iff $I \models L$ and $I \models P$. KB is satisfiable iff it has a model.

The Gelfond-Lifschitz reduct of a fuzzy rough dl-program KB=(L,P) relative to an interpretation I, denoted KB^I , is defined as the fuzzy rough dl-program (L, P^I), where P^I is the Gelfond-Lifschitz reduct of P.

Definition 4.2. Let KB=(L,P) be a fuzzy rough dl-program. An interpretation *I* is an answer set of KB iff *I* is a minimal model of KB^I . KB is consistent iff KB has an answer set.

Example 4. Consider again the fuzzy rough dl-program KB=(L,P) of Example 3. It is not difficult to verify that KB has an answer set, and so is consistent.

$$M(\underline{Q}(a_1)) = 0.21 ,$$

$$M(\overline{Q}(a_1)) = 0.42 .$$

Definition 4.3. Let KB=(L,P) be a fuzzy rough dl-program, $\alpha \in HB_{\Phi}$ and $n \in [0,1]$. Then, $\alpha \geq n$ is a cautious (resp., brave) consequence of KB under the answer set semantics iff $I(\alpha) \geq n$ for every (resp., some) answer set I of KB.

Example 5. Consider again the fuzzy rough dl-program KB=(L,P) of Example 3. According to Example 4,

$M(\underline{Q}(a_1)) \geq 0.21$ and $M(\overline{Q}(a_1)) \geq 0.42$

are both cautious and brave consequences of KB.

B. The semantic properties

In this section, we introduce some semantic properties of fuzzy rough dl-program under the answer set semantics, i.e., minimal models, faithfulness and unique name assumption.

Theorem 4.1. Let KB=(L,P) be a fuzzy rough dl-program. Then,

1) Every answer set of KB is a minimal model of KB;

2) If KB is positive, then the set of all answer sets of KB is the set of all minimal models of KB.

Proof. 1) Suppose that I is an arbitrary answer set of KB, then I is a minimal model of $KB^I = (L, P^I)$. More formally, (a) $I \models L$ and (b) $I \models r$ for every $r \in P^I$. In other words, (a) $I \models L$ and (b) $I \models r$ for every $r \in ground(P)$. Therefore, according Definition 4.1, I is a model of KB. Now, we need to prove that I is a minimal model of KB. Towards a contradiction, we assume that there exists a model $J \subset I$ of KB. In this way, (a) $J \models L$ and (b) $J \models r$ for every $r \in ground(P)$. According to the monotonicity and antitonicity of conjunction and negation strategies, this is equivalent to (a) $J \models L$ and (b) $J \models r$ for every $r \in P^I$. Thus J is also a model of KB^I . But this contradicts I being a minimal model of KB^I . Therefore, I is a minimal model of KB. In summary, every answer set of KB is a minimal model of KB.

2) Because KB is a positive fuzzy rough dl-program, then $KB^I = (L, P^I) = (L, ground(P))$. Therefore, the set of all answer sets of KB is the set of all minimal models of KB^I that coincides with the sets of all minimal models of KB.

Theorem 4.2. Let KB=(L,P) be a fuzzy rough dl-program with $L = \varnothing$. Then, the set of all answer sets of KB is consistent with the set of all answer sets of the fuzzy rough program P.

Proof. I is a model of $KB^I = (L, P^I)$ iff (a) $I \models L$ and (b) $I \models r$ for every $r \in P^I$. Since $L = \varnothing$, then this is equivalent to $I \models r$ for every $r \in P^I$. Moreover, I is a minimal model of KB^I iff I is a minimal model of P^I . In other words, I is an answer set of KB iff I is an answer set of P. Therefore, the set of all answer sets of KB is consistent with the set of all answer sets of the fuzzy rough program P.

Theorem 4.3. Let KB=(L,P) be a positive fuzzy rough dl-program, $\alpha \in HB_{\Phi}$ and $n \in [0,1]$. Then, $\alpha \geq n$ is true in all answer set of KB iff $\alpha \geq n$ is true in all fuzzy rough first-order models of $L \cup ground(P)$.

Proof. Since KB is a positive fuzzy rough dl-program, then according to Theorem 4.1, the set of all answer sets of KB is the set of all minimal models of KB. Observe that for $\alpha \in HB_{\Phi}$, $\alpha \geq n$ is true in all minimal models of KB iff $\alpha \geq n$ is true in all models of KB. Therefore, the conclusion can be rewritten into the following form, $\alpha \geq n$ is true in all models of KB iff $\alpha \geq n$ is true in all fuzzy rough first-order models of $L \cup ground(P)$. Now, we prove this conclusion.

($\Rightarrow$) Suppose that $\alpha \geq n$ is true in all models of KB. Let J be any fuzzy rough first-order models of $L \cup ground(P)$, I be defined by $I(\beta) = J(\beta)$ for all $\beta \in HB_{\Phi}$. In this way, J is a model of $L' = L \cup \{\alpha = I(\alpha) \mid \alpha \in HB_{\Phi}\}$, and thus L' is satisfiable. Therefore, I is a model of L. On the other hand, because J is a model of ground(P), then I is a model of ground(P). Thus, I is a model of KB. Therefore, $\alpha \geq n$ is true in I, and $\alpha \geq n$ is also true in J. In summary, $\alpha \geq n$ is true in all fuzzy rough first-order models of $L \cup ground(P)$.

($\Leftarrow$) Suppose that $\alpha \geq n$ is true in all fuzzy rough first-order models of $L \cup ground(P)$. Let I be any model of KB. Then $L' = L \cup \{\alpha = I(\alpha) \mid \alpha \in HB_{\Phi}\}$ is satisfiable. Let J be a first-order model of L'. In this way, J is a especial model of L. On the other hand, because I is a model of ground(P), then J is a model of ground(P). Thus, J is a model of $L \cup ground(P)$. Therefore, $\alpha \geq n$ is true in J, and $\alpha \geq n$ is also true in I. In summary, $\alpha \geq n$ is true in all models of KB.

Corollary 4.4. Let KB=(L,P) be a fuzzy rough dl-program such that $P = \varnothing$, $\alpha \in HB_{\Phi}$ and $n \in [0,1]$. Then, $\alpha \geq n$ is true in all answer set of KB iff $\alpha \geq n$ is true in all fuzzy rough first-order models of L.

Another aspect that we do not hope to consider in the Semantic Web is the unique name assumption. Therefore, in our approach, we do not need to make this assumption. The reason is that the fuzzy rough description logic knowledge base of a fuzzy rough dl-program includes or implies equalities between individuals, hence we have no unique name assumption in L and also have no unique name assumption in P.

V. CONCLUSION

We have proposed tightly coupled fuzzy rough description logic programs (fuzzy rough dl-programs) under the answer set semantics, which generalize the tightly coupled description logic programs by fuzzy rough set theory in both the logic program and the description logic.

In this paper, we first provide the syntax and semantics of fuzzy rough disjunctive logic programs (for short, fuzzy rough programs) under the answer set semantic. Then we present the syntax and semantics of fuzzy rough dl-programs, and give some reasoning problems of fuzzy rough dl-program. Finally we show that the answer set of fuzzy rough dl-program has a close relation with the minimal model, and the fuzzy rough dl-program faithfully extends both fuzzy rough disjunctive logic program and fuzzy rough description logic. In a word, fuzzy rough dl-program can well represent and reason a great deal of real-word problems.

An interesting topic of future research is to implement of the presented approach. Another interesting issue is to extend fuzzy rough dl-programs by a new semantics.

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