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Robust Adaptive Fuzzy Control of Uncertain Bilinear Systems with Unknown Dead-Zone

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***Abstract*—A robust adaptive fuzzy control approach is proposed in this paper for a class of uncertain bilinear systems with unknown dead-zone. Dead-zone is one of the most important nonsmooth nonlinearities encountered in actuators, such as DC servo systems, pressure control systems, machine tools, and power amplifiers. In most practical motion systems, the dead-zone parameters are poorly known and may severely limit system performance. Therefore, the design of the robust adaptive fuzzy controller in this paper provides robustness not only to uncertainties of the system, but also to the unknown dead-zone. Based on Lyapunov stability theorem, the proposed robust adaptive fuzzy controller would have the capability to ensure the successful achievement of the asymptotic stabilization of the whole close-loop system. Simulation results are included to illustrate the effectiveness of the proposed control scheme.**

Keywords—uncertain bilinear systems; dead zone; robust adaptive fuzzy control; Lyapunov stability theorem

I. INTRODUCTION

The research on the development of nonlinear process control methods has been examined in depth during the past few years. Throughout the research, bilinear systems are considered as the simplest nonlinear system that carries the important theoretical value. Many papers and monographs have been proposed and a variety of control designs, including adaptive control [1-2], robust control [3-4], output feedback control [5], and sliding mode control [6-7], have been used in practical systems.

Some nonsmooth nonlinearities, which include dead-zone, saturation and backlash are encountered in the actuators of real systems, such as DC servo systems, pressure control systems, machine tools, and power amplifiers [8-11, 23-24]. Because these nonsmooth nonlinearities are usually unacquainted and time-variant, the dead-zone traits in actuators will reduce the system performance such that the system output can not meet the requirements. In [8], the robust adaptive control was developed to cope with nonlinear systems with unknown dead-zone. The sliding mode controller was presented in [9] to robustly stabilize a nonlinear uncertain system, containing dead zone or backlash in the actuator devices. Lin *et al.* [10] presented a robust adaptive dead-zone compensation method for a DC servo-motor control system. Based on dead-zone compensation, the robust fuzzy logic control approach [12] was proposed to tackle the stabilization problem of a class of nonlinear uncertain systems in the presence of an unknown dead-zone. An adaptive control method was proposed by Su *et al.* [13] to treat nonlinear systems with non-symmetric dead-zone input. Many of existing control methods use a dead-zone inverse to handle the effects of dead-zone [14-15]. Therefore, in this paper, the controller will be constructed to cope with the robust control problem for bilinear systems including unknown dead-zone.

Due to the existence of uncertain elements, including parameter variations, modeling errors, unmodelled dynamics and external disturbances, it is difficult to describe a real system based on an exact mathematical model. Those uncertainties may affect the stability of the systems. The stabilization of a class of uncertain homogenous bilinear systems under the sliding mode control was given in [16]. Huang and Lam [17] provided the linear controller to cope with the uncertain bilinear system. In [18], the robust adaptive controller for nonlinear uncertain system was presented. In the case of the bilinear systems with high-order perturbation uncertainties, the robust adaptive controller was discussed in [19].

In recent years, the fuzzy control techniques have been successfully used in many control problems [20-22]. The fuzzy If-Then rules build up the fuzzy logic system to make it useful to approximate the unknown nonlinear functions and uncertainties in the nonlinear systems. Yang and Ren [20] designed the adaptive fuzzy robust tracking control to deal with uncertain nonlinear systems. The adaptive controller employs fuzzy systems to approximate the dynamics of nonlinear systems such that the tracking performance could be achieved in [21]. Takagi-Sugeno (T-S) fuzzy model [22] was utilized to approximate the unknown uncertain function in the nonlinear systems. In this paper, the fuzzy logic system is used to approximate the uncertainties in the bilinear systems.

The main focus of this paper is on the design of robust adaptive fuzzy control for a class of uncertain bilinear systems including unknown dead-zone. The description of a dead-zone feature is used to estimate the properties of the dead-zone model intuitively and mathematically. A common feature in

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previous works [14-15] is the construction of a dead-zone inverse to handle the effects of dead-zone. However, the dead-zone inverse can be applied only when the dead-zone nonlinearity are completely known. In this paper, the robust adaptive fuzzy controller is published without constructing the dead-zone inverse. The fuzzy logic systems can be applied to approximate the nonlinear uncertainties by means of the adaptive laws. Moreover, the proposed robust adaptive fuzzy control approach can guarantee the robust stability of the whole closed-loop system based on the Lyapunov stability theorem.

This paper is organized as follows. In Section II, the form of the uncertain bilinear system with unknown dead-zone is described, and a detailed description of fuzzy logic systems and fuzzy basis functions are introduced. Section III presents the robust adaptive fuzzy control and its stability analysis. Simulation results of two examples are illustrated to show the performance of the proposed robust adaptive fuzzy controller in Section IV. Finally, a conclusion is given in Section V.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. Problem Statement

Consider a class of the following uncertain bilinear system with an unknown dead-zone of the form:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{BZ}(\boldsymbol{u}(t)) + \sum_{i=1}^{q} \mathbf{N}_i \mathbf{x}\, \mathrm{Z}_i(u_i(t)) + \mathbf{\Delta\Phi}(\mathbf{x}), \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the system state vector which is assumed to be available for measurement, $\boldsymbol{u}(t) = [u_1(t), \cdots, u_q(t)]^T \in R^q$ is the input of the system, $\mathbf{Z}(\boldsymbol{u}(t)) = [\mathrm{Z}_1(u_1(t)), \cdots, \mathrm{Z}_q(u_q(t))]^T \in R^q$ is the output of the dead-zone model with input $\boldsymbol{u}(t)$, $\boldsymbol{A} \in R^{n\times n}$, $\boldsymbol{B} \in R^{n\times q}$, and $\boldsymbol{N}_i \in R^{n\times n}, \forall i$ are assumed to be known constant matrices and $\mathbf{\Delta\Phi}(\mathbf{x}) \in \boldsymbol{R^n}$ is the unknown uncertainty.

Eq. (1) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{Ax} + \sum_{i=1}^{q} (\mathbf{b}_i + \mathbf{N}_i \mathbf{x}) \mathrm{Z}_i(u_i(t)) + \mathbf{\Delta\Phi}(\mathbf{x}), \tag{2}$$

where $\mathbf{b}_i$ is the ith column of matrix $\mathbf{B}$ .That is, $\mathbf{B} = [\mathbf{b}_1 \ldots \mathbf{b}_q]$. $\mathrm{Z}_i(u_i(t)) : R \to R$ is the output of the dead-zone model with the input $u_i(t)$.

***Assumption 1*:** $\|\mathbf{\Delta\Phi}(\mathbf{x})\| \le h(\mathbf{x})$, where $h(\mathbf{x})$ is an unknown positive smooth continuous function and can be estimated by an adaptive law in the later.

To clarify the dead-zone nonlinear input function $Z_i(\cdot)$, the dead-zone with input $u_i(t)$ and output $w_i(t)$ is described by

$$w_i(t) = Z_i(u_i(t)) = \begin{cases} m_{ir}\left(u_i(t) - c_{ir}\right) & \text{for } u_i(t) \ge c_{ir}, \\ 0 & \text{for } c_{il} < u(t) < c_{ir}, \\ m_{il}\left(u_i(t) - c_{il}\right) & \text{for } u_i(t) \le c_{il}, \end{cases} \tag{3}$$

where $c_{ir} > 0$, $c_{il} < 0$ and $m_{ir} > 0$, $m_{il} > 0$ are parameters and slopes of the dead-zone, respectively. In order to investigate the key features of the dead-zone in the control problems, we have the following assumptions:

Assumption 2: The dead-zone output $w_i(t)$ is not available.

Assumption 3: The dead-zone slopes are same, i.e. $m_{ir} = m_{il} = m = m_i$, for i=1,$\cdots$,q.

Assumption 4: There exist known constants $c_{r\ \min}$, $c_{r\ \max}$, $c_{l\ \min}$, $c_{l\ \max}$, $m_{\min}$, $m_{\max}$ such that the unknown dead-zone parameters c_{ir} , c_{il} , and m_i are bounded, i.e. $c_{ir} \in [c_{r\ \min}, c_{r\ \max}]$, $c_{il} \in [c_{l\ \min}, c_{l\ \max}]$, and $m_i \in [m_{\min}, m_{\max}]$.

Based on the above assumptions, the expression (3) can be represented as

$$w_i(t) = Z_i(u_i(t)) = mu_i(t) + z_i(u_i(t)), \tag{4}$$

where $z_i(u_i(t))$ can be calculated from (3) and (4) as

$$z_i(u_i(t)) = \begin{cases} -mc_{ir} & \text{for } u_i(t) \ge c_{ir}, \\ -mu_i(t) & \text{for } c_{il} < u_i(t) < c_{ir}, \\ -mc_{il} & \text{for } u_i(t) \le c_{il}. \end{cases}$$

(5)

From ***Assumptions 3*** and ***4***, one can conclude that $z_i(u_i(t))$ is bounded, and satisfies

$$|z_i(u_i(t))| \le \rho ,$$

where ρ is the upper-bound, which can be chosen as

$$\rho = \max\{m_{\max} c_{r\ \max}, -m_{\max} c_{l\ \min}\}, \tag{6}$$

where $c_{l\ \min}$ is a negative value.

Control Objective: Design the controller for (2) such that the system states $\mathbf{x}(t)$ would converge to zero.

B. Description of Fuzzy Logic Systems

The fuzzy logic system performs a mapping from $U \subset R^n$ to $V \subset R$. Let $U = U_1 \times \cdots \times U_n$ where $U_i \subset R$, $i = 1, 2, \cdots, n$. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules:

$$R^{(l)}: \text{IF } x_1 \text{ is } F_1^l, \text{ and } x_2 \text{ is } F_2^l, \text{ and } \cdots \text{ and, } x_n \text{ is } F_n^l \quad \text{THEN } y \text{ is } G^l, \quad \text{for } l = 1, 2, \cdots, M. \tag{7}$$

in which $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T \in U$ and $y \in V \subset R$ are the input and output of the fuzzy logic system, F_i^l and G^l are fuzzy sets in U_i and V , respectively. The fuzzifier maps a crisp point $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T$ into a fuzzy set in U . The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V , based upon the fuzzy IF-THEN rules in the fuzzy rule base and the compositional rule of inference. The defuzzifier maps a fuzzy set in V to a crisp point in V .

The fuzzy systems with center-average defuzzifier, product inference and singleton fuzzifier are of the following form:

$$y(\mathbf{x})=\frac{\sum_{l=1}^{M}\theta^{l}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}, \tag{8}$$

where θ^l is the point at which fuzzy membership function $\mu_{G^l}(\theta^l)$ achieves its maximum value, and we assume that $\mu_{G^l}(\theta^l)=1$. Eq. (8) can be rewritten as

$$y(\mathbf{x})=\boldsymbol{\theta}^T\xi(\mathbf{x}) \tag{9}$$

where $\boldsymbol{\theta}=\left[\theta^1,\theta^2,\cdots,\theta^l\right]^T$ is a parameter vector, and $\xi(\mathbf{x})=\left[\xi^1(\mathbf{x}),\cdots,\xi^M(\mathbf{x})\right]^T$ is a regressive vector with the regressor $\xi^l(\mathbf{x})$, which is defined as fuzzy basis function

$$\xi^l(\mathbf{x})=\frac{\prod_{i=1}^{n}\mu_{F_i^l}(x_i)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}. \tag{10}$$

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, we using expression (4), system (2) becomes

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\sum_{i=1}^{q}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})mu_i(t)+\sum_{i=1}^{q}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})z_i(u_i(t))+\Delta\boldsymbol{\Phi}(\mathbf{x}) \tag{11}$$

where the state variables of the control problem become linear to the input signal $u_i(t)$. It is very important to note that $z_i(u_i(t))$ is uniformly bounded, and $\mathbf{A}$, $\mathbf{b}_i$ and $\mathbf{N}_i$ are known constant matrices, $\Delta\boldsymbol{\Phi}$ are unknown uncertainties with unknown upper bound functions and satisfies the *assumption 1*, i.e., $\|\Delta\boldsymbol{\Phi}\|\le h(\mathbf{x})$.

First, let the nonlinear function $h(\mathbf{x})$ can be approximated, over a compact set $\Omega_{\mathbf{x}}$, by the fuzzy logic systems as follows:

$$\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)=\boldsymbol{\theta}_h^T\boldsymbol{\xi}(\mathbf{x}), \tag{12}$$

where $\boldsymbol{\xi}(\mathbf{x})$ is the fuzzy basis vectors and $\boldsymbol{\theta}_h$ is the corresponding adjustable parameter vectors of each fuzzy logic system. It is assumed that $\boldsymbol{\theta}_h$ belong to compact set $\Omega_{\boldsymbol{\theta}_h}$, which is defined as

$$\Omega_{\boldsymbol{\theta}_h}=\left\{\boldsymbol{\theta}_h\in R^M:\|\boldsymbol{\theta}_h\|\le N<\infty\right\}, \tag{13}$$

where N is the designed parameter, and M is the number of fuzzy inference rules. Let us define the optimal parameter vectors $\boldsymbol{\theta}_h^*$ as follows:

$$\boldsymbol{\theta}_h^*=\arg\min_{\boldsymbol{\theta}_h\in\Omega_{\boldsymbol{\theta}_h}}\left\{\sup_{\mathbf{x}\in\Omega_{\mathbf{x}}}\left|h(\mathbf{x})-\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)\right|\right\}, \tag{14}$$

where $\boldsymbol{\theta}_h^*$ is bounded in the suitable closed set $\Omega_{\boldsymbol{\theta}_h}$. The parameter estimation errors can be defined as

$$\tilde{\boldsymbol{\theta}}_h=\boldsymbol{\theta}_h-\boldsymbol{\theta}_h^*, \tag{15}$$

and

$$|\omega_1|\le\omega \tag{16}$$

where

$$\omega_1=\left(h(\mathbf{x})-h(\mathbf{x}|\boldsymbol{\theta}_h^*)\right) \tag{17}$$

as the minimum approximation errors, which correspond to approximation errors obtained when optimal parameters are used.

Secondly, we define

$$\tilde{\phi}=\hat{\phi}-\phi \tag{18}$$

$$\tilde{\omega}=\hat{\omega}-\omega \tag{19}$$

where $\hat{\phi}$ is an estimate of ϕ, which is defined as $\phi=(m)^{-1}$. $\hat{\omega}$ is an estimate of ω.

Based on the given plant and dead-zone models under the ***Assumptions 1-4,*** consider the following controller:

$$u_i=u_{i1}+u_{i2}+u_{i3}+u_{i4}+u_{i5}, \tag{20}$$

where

$$u_{i1}=\frac{-\left(\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\left\|\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}\right\|}{m_{\min}\cdot q\left\|\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right\|^2}, \tag{21}$$

$$u_{i2}=\frac{-k^*\cdot\left(\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)}{\left\|\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right\|}, \tag{22}$$

$$u_{i3}=\frac{-\left(\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\cdot\hat{\phi}\cdot\left\|\mathbf{x}^T\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)}{q\left\|\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right\|^2}, \tag{23}$$

$$u_{i4}=\frac{-\left(\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\cdot\left\|\mathbf{x}^T\mathbf{P}\right\|\cdot\hat{\omega}}{m_{\min}\cdot q\left\|\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right\|^2}, \tag{24}$$

$$u_{i5}=\frac{-\mu\left(\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\cdot\|\mathbf{x}\|}{q\left\|\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right\|^2}, \tag{25}$$

where $k^*\ge\rho/m_{\min}$, ρ is defined in (6), and $\mu>0$ is a positive constant, $\mathbf{P}$ is a symmetric positive definite matrix, and the parameter update laws are as follows:

$$\dot{\boldsymbol{\theta}}_h=\gamma_h\cdot\left\|\mathbf{x}^T\mathbf{P}\right\|\cdot\xi(\mathbf{x}), \tag{26}$$

$$\dot{\hat{\omega}}=\gamma_\omega\cdot\left\|\mathbf{x}^T\mathbf{P}\right\|, \tag{27}$$

$$\dot{\hat{\phi}}=\eta\cdot\left\|\mathbf{x}^T\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h), \tag{28}$$

where the scalar γ_h, γ_ω and η are positive constants, determining the rates of adaptations.

Remark 1: Without loss of generally, the adaptive laws used in this thesis are assumed that the parameter vectors are within the constraint sets or on the boundaries of the constraint sets but moving toward the inside of the constraint sets. If the parameter vectors are on the boundaries of the constraint sets but moving toward the outside of the constraint sets, we have to use the projection algorithm [25] to modify the adaptive laws such that the parameter vectors will remain inside of the constraint sets. The proposed adaptive law (26) can be modified as the following form:

$$\dot{\boldsymbol{\theta}}_h=\begin{cases}\gamma_h\left\|\mathbf{x}^T\mathbf{P}\right\|\boldsymbol{\xi}(\mathbf{x}(t)), & \text{if } \left(\|\boldsymbol{\theta}_h\|<N\right) \text{ or } \\ & \left(\|\boldsymbol{\theta}_h\|=N \text{ and } \left\|\mathbf{x}^T\mathbf{P}\right\|\boldsymbol{\theta}_h^T\boldsymbol{\xi}(\mathbf{x}(t))\le 0\right), \\ P\left\{\gamma_h\left\|\mathbf{x}^T\mathbf{P}\right\|\boldsymbol{\xi}(\mathbf{x}(t))\right\}, & \text{if } \left(\|\boldsymbol{\theta}_h\|=N \text{ and } \left\|\mathbf{x}^T\mathbf{P}\right\|\boldsymbol{\theta}_h^T\boldsymbol{\xi}(\mathbf{x}(t))>0\right),\end{cases} \tag{29}$$

where $P\left\{\gamma_h \left\|\mathbf{x}^T\mathbf{P}\right\|\xi(\mathbf{x}(t))\right\}$ is defined as

$$P\left\{\gamma_h \left\|\mathbf{x}^T\mathbf{P}\right\|\xi(\mathbf{x}(t))\right\} = \gamma_h \left\|\mathbf{x}^T\mathbf{P}\right\|\xi(\mathbf{x}(t)) - \gamma_h \left\|\mathbf{x}^T\mathbf{P}\right\| \frac{\boldsymbol{\theta}_h \boldsymbol{\theta}_h^T}{\left\|\boldsymbol{\theta}_h\right\|^2} \xi(\mathbf{x}(t)). \quad (30)$$

The main result of the robust adaptive fuzzy control scheme is summarized in the following theorem.

Theorem 1: Consider the uncertain bilinear system (2) with an unknown dead-zone input (4) and unmatched uncertainties. If ***Assumptions 1-4*** are satisfied, then the proposed robust adaptive fuzzy controller defined by (21)-(25) with adaptation laws (26)-(28) ensures that all signals of the closed-loop system are bounded, and the system states can converge to zero.

Proof: Consider the Lyapounov function candidate

$$V = \frac{1}{2}\left[\frac{1}{m}\mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{x} + \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\tilde{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}^2 + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}^2\right] \quad (31)$$

Differentiating the Lyapounov function V with respect to time, we can obtain

$$\dot{V} = \frac{1}{2m}\dot{\mathbf{x}}^{\mathrm{T}}\mathbf{P}\mathbf{x} + \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\mathbf{P}\dot{\mathbf{x}} + \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\tilde{\boldsymbol{\theta}}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\tilde{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\tilde{\omega}}. \quad (32)$$

From (11) and by the fact $\dot{\tilde{\boldsymbol{\theta}}}_h = \dot{\boldsymbol{\theta}}_h$, $\dot{\tilde{\phi}} = \dot{\hat{\phi}}$ and $\dot{\tilde{\omega}} = \dot{\hat{\omega}}$ the above equation becomes

$$\begin{aligned}
\dot{V} &= \frac{1}{2m}\left[\mathbf{A}\mathbf{x} + \sum_{i=1}^{q}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})mu_i(t) + \sum_{i=1}^{q}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})z_i(u_i(t)) + \boldsymbol{\Delta\Phi}(\mathbf{x})\right]^{\mathrm{T}}\mathbf{P}\mathbf{x} \\
&+ \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\mathbf{P}\left[\mathbf{A}\mathbf{x} + \sum_{i=1}^{q}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})mu_i(t) + \sum_{i=1}^{q}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})z_i(u_i(t)) + \boldsymbol{\Delta\Phi}(\mathbf{x})\right] \\
&+ \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}, \\
&= \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T u_i(t) \\
&+ \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T \frac{z_i(u_i(t))}{m} + \frac{1}{m}\mathbf{x}^{\mathrm{T}}\mathbf{P}\boldsymbol{\Delta\Phi}(\mathbf{x}) \\
&+ \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}. \quad (33)
\end{aligned}$$

Applying ***Assumption 1*** to (33) yields

$$\begin{aligned}
\dot{V} &\le \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T u_i(t) \\
&+ \sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T\right\| \cdot \frac{\left|z_i(u_i(t))\right|}{m} + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\boldsymbol{\Delta\Phi}(\mathbf{x})\right\| \\
&+ \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}, \\
&\le \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T u_i(t) \\
&+ \sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T\right\| \cdot \frac{\left|z_i(u_i(t))\right|}{m} + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot h(\mathbf{x}) \\
&+ \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}. \quad (34)
\end{aligned}$$

Substituting (12) into (34), we have

$$\begin{aligned}
\dot{V} &\le \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T u_i(t) \\
&+ \sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T\right\| \cdot \frac{\left|z_i(u_i(t))\right|}{m} + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \hat{h}(\mathbf{x}|\boldsymbol{\theta}_h) \\
&+ \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \left(h(\mathbf{x}) - \hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)\right) + \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} \\
&+ \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}, \\
&= \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T u_i(t) \\
&+ \sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T\right\| \cdot \frac{\left|z_i(u_i(t))\right|}{m} + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \hat{h}(\mathbf{x}|\boldsymbol{\theta}_h) \\
&+ \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \left(\boldsymbol{\theta}_h^{*T}\xi(\mathbf{x}) - \boldsymbol{\theta}_h^T\xi(\mathbf{x})\right) + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \left(h(\mathbf{x}) - h(\mathbf{x}|\boldsymbol{\theta}_h^*)\right) \\
&+ \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}, \quad (35)
\end{aligned}$$

Applying (15)-(17) to (35) yields

$$\begin{aligned}
\dot{V} &\le \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T u_i(t) \\
&+ \sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T\right\| \cdot \frac{\left|z_i(u_i(t))\right|}{m} + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \hat{h}(\mathbf{x}|\boldsymbol{\theta}_h) \\
&+ \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \left(\boldsymbol{\theta}_h^{*T}\xi(\mathbf{x}) - \boldsymbol{\theta}_h^T\xi(\mathbf{x})\right) + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \omega \\
&+ \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}, \\
&\le \frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + \sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T u_i(t) \\
&+ \sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i + \mathbf{N}_i\mathbf{x})\right)^T\right\| \cdot \frac{\left|z_i(u_i(t))\right|}{m} + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \hat{h}(\mathbf{x}|\boldsymbol{\theta}_h) \\
&- \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \tilde{\boldsymbol{\theta}}_h^T\xi(\mathbf{x}) + \frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\| \cdot \omega \\
&+ \frac{1}{m\cdot\gamma_h}\tilde{\boldsymbol{\theta}}_h^T\dot{\boldsymbol{\theta}}_h + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}. \quad (36)
\end{aligned}$$

According to adaptive laws (26), (36) can be rewritten as

$$\dot{V}\le\frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}+\sum_{i=1}^{q}\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}u_i(t)$$
$$+\sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\right\|\cdot\frac{|z_i(u_i(t))|}{m}+\frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)$$
$$+\frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\omega+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}},\tag{37}$$

Using the control law (21)-(25), (37) can be rewritten as

$$\dot{V}\le\frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}$$
$$-\sum_{i=1}^{q}\frac{\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\mathbf{x}^{T}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\cdot\left\|\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}\right\|}{m_{\min}\cdot q\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\right\|^{2}}$$
$$-\sum_{i=1}^{q}\frac{k^{*}\cdot\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\mathbf{x}^{T}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})}{\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\right\|}$$
$$-\sum_{i=1}^{q}\frac{\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\mathbf{x}^{T}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\cdot\hat{\phi}\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)}{q\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\right\|^{2}}$$
$$-\sum_{i=1}^{q}\frac{\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\mathbf{x}^{T}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{\omega}}{m_{\min}\cdot q\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\right\|^{2}}$$
$$-\sum_{i=1}^{q}\frac{\mu\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\cdot\left(\mathbf{x}^{T}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\cdot\|\mathbf{x}\|}{q\left\|\mathbf{x}^{T}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right\|^{2}}$$
$$+\sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\right\|\cdot\frac{|z_i(u_i(t))|}{m}+\frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)$$
$$+\frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\omega+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}},$$
$$=\frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}\right\|$$
$$-k^{*}\sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\right\|+\sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)^{T}\right\|\cdot\frac{|z_i(u_i(t))|}{m}$$
$$-\mu\|\mathbf{x}\|-\hat{\phi}\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)+\phi\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{\omega}$$
$$+\frac{1}{m}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\omega+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}.\tag{38}$$

According to (18)-(19), we obtain

$$\dot{V}\le\frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}\right\|$$
$$-\sum_{i=1}^{q}\left\|\left(\mathbf{x}^{\mathrm{T}}\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)\right\|\left(k^{*}-\frac{|z_i(u_i(t))|}{m}\right)-\tilde{\phi}\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)$$
$$-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\tilde{\omega}-\mu\|\mathbf{x}\|+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}},$$
$$\le\frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}\right\|$$
$$-\tilde{\phi}\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)-\mu\|\mathbf{x}\|-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\tilde{\omega}_h$$
$$+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}.\tag{39}$$

According to adaptive laws (27)-(28), we have

$$\dot{V}\le\frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}\right\|$$
$$-\tilde{\phi}\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)-\mu\|\mathbf{x}\|-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\tilde{\omega}$$
$$+\tilde{\phi}\cdot\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\hat{h}(\mathbf{x}|\boldsymbol{\theta}_h)+\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\mathbf{P}\right\|\cdot\tilde{\omega}$$
$$\le\frac{1}{2m}\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}-\frac{1}{m_{\min}}\left\|\mathbf{x}^{\mathrm{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}\right\|-\mu\|\mathbf{x}\|$$
$$<0.\tag{40}$$

Therefore, it can be concluded that $\dot{V}\le 0$ from (40), and the closed-loop system is asymptotically stable based on Lyapunov synthesis approach. This completes the proof.

IV. AN EXAMPLE AND SIMULATION RESULTS

In this section, simulation results of the second-order bilinear system are illustrated to demonstrate the effectiveness of the proposed control method.

Consider the second-order system, described by

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{Z}(u(t))+\sum_{i=1}^{2}\mathbf{N}_i\mathbf{x}\,Z_i(u_i(t))+\boldsymbol{\Delta\Phi}(\mathbf{x})\tag{41}$$

where

$$A=\begin{bmatrix}3 & 2\\ 2 & 1/6\end{bmatrix},\qquad B=\begin{bmatrix}1 & 0\\ 0 & 100\end{bmatrix},$$
$$N_1=\begin{bmatrix}1 & 0\\ 0 & 10\end{bmatrix},\qquad N_2=\begin{bmatrix}1 & 0\\ 0 & 10\end{bmatrix},$$
$$\boldsymbol{\Delta\Phi}=\begin{bmatrix}\Delta\phi_1\\ \Delta\phi_2\end{bmatrix},$$

and

$$\Delta\phi_1(\mathbf{x})=0.1\cdot x_1\cdot\sin(t),$$
$$\Delta\phi_2(\mathbf{x})=0.3\cdot x_2\cdot\sin(t),$$

$Z(u(t))$ is an output of a dead-zone. $\Delta\phi_1(\mathbf{x})=0.1x_1\sin(t)$, $\Delta\phi_2(\mathbf{x})=0.3x_2\sin(t)$ are unknown uncertainties. The control objective is to maintain the system states x_1 and x_2 converge to zero.

In the simulation, the parameters of the dead-zone are $m_i=1$, $c_{ir}=0.5$, $c_{il}=-0.6$, for $i=1,2$. And their bounds are chosen as $m_{\max}=1.5$, $m_{\min}=0.6$, $c_{r\max}=0.9$, $c_{r\min}=0.1$, $c_{l\max}=-0.1$, and $c_{l\min}=-0.8$. In the implementation, six fuzzy sets are defined over interval [-3, 3] for both x_1 and x_2, with labels *F1*, *F2*, *F3*, *F4*, *F5*, and *F6*, and their membership functions are

$$\mu_{F1}(x_i) = \frac{1}{1+\exp\left(5\left(x_i+2\right)\right)},\ \mu_{F2}(x_i) = \exp\left(-\left(x_i+1.5\right)^2\right),$$

$$\mu_{F3}(x_i) = \exp\left(-\left(x_i+0.5\right)^2\right),\ \mu_{F4}(x_i) = \exp\left(-\left(x_i-0.5\right)^2\right),$$

$$\mu_{F5}(x_i) = \exp\left(-\left(x_i-1.5\right)^2\right),$$

$$\mu_{F6}(x_i) = \frac{1}{1+\exp\left(-5\left(x_i-2\right)\right)}, \quad \text{for } i=1,\ 2.$$

In this section, we apply the robust adaptive fuzzy control approach in Section 3 to deal with an uncertain second-order bilinear system with unknown dead-zone.

Choose a symmetric positive definite matrix as follows:

$$\mathbf{P} = \begin{bmatrix} 5.25 & 0.25 \\ 0.25 & 0.275 \end{bmatrix}.$$

Let the sampling time be 0.01, and the initial values are chosen as $\mathbf{x}(0) = [5,-3]^{\mathrm{T}}$. The other values are selected as $\hat{\phi}(0) = 0.85$, $\hat{\omega}(0) = 0$, $\gamma_h = 2.0$, $\gamma_\omega = 0.1$, $\eta = 0.5$, $q = 2$, $k^* = 2.5$, $\mu = 1$. The control scheme is shown to suffer from chattering which is an expected behavior due to the presence of the switching function $\operatorname{sgn}\left(\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right)$. The effect of chattering can be abated by replacing the switching function with the continuous approximation $\dfrac{\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})}{\left|\mathbf{x}^T\mathbf{P}(\mathbf{b}_i+\mathbf{N}_i\mathbf{x})\right|+\varepsilon}$ where $\varepsilon > 0$. The controller $u_i(t)$ with $\varepsilon = 0.02$ is shown to be able to stabilize the system in Fig. 1. Fig. 2 illustrates the effectiveness of the proposed control law, where it is clear to see the trajectories towards the origin. Finally, Figs. 3-4 explain that the chattering is mitigated in the control action when the system is in steady state.

V. CONCLUSION

The dead-zone characteristics in the actuators of practical control systems are often poorly known and severely limit system performance. In this paper, a robust adaptive fuzzy control scheme is proposed for a class of uncertain bilinear systems with unknown dead-zone. By utilizing a description of a dead-zone feature and by investigating the properties of the dead-zone model intuitively, this paper suggests a feasible robust adaptive fuzzy control scheme without constructing a dead-zone inverse. Based on Lyapunov stability theorem, the proposed robust adaptive fuzzy controller is able to ensure the successful achievement of the asymptotic stabilization of the whole close-loop system. The simulation results are provided to demonstrate the validity of the proposed control method.

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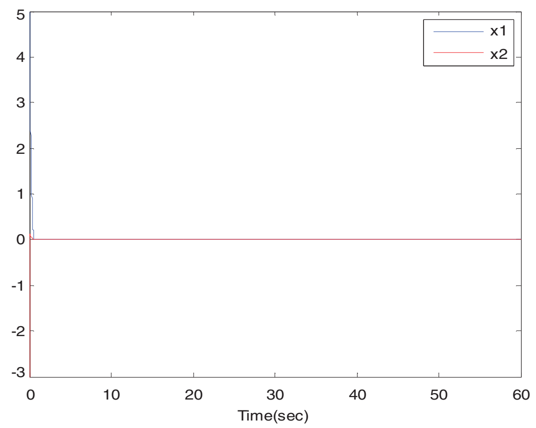

Fig. 1. The trajectories of states x_1 and x_2 with $u_i(t)$ for $\varepsilon = 0.02$.

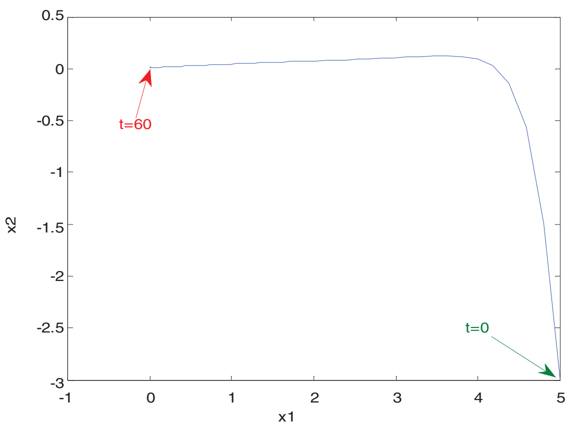

Fig. 2. The phase plane plot of states x_1 and x_2 with $u_i(t)$ for $\varepsilon = 0.02$.

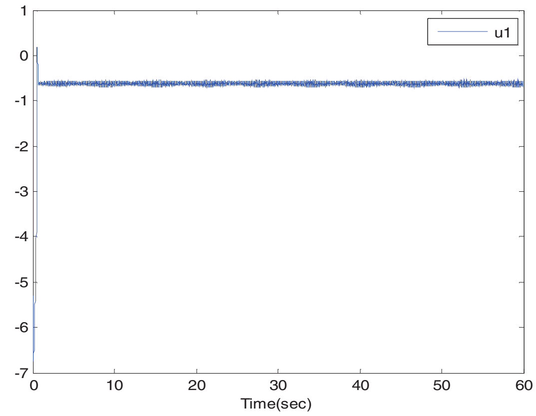

Fig. 3. The control input u1(t) for ε=0.02.

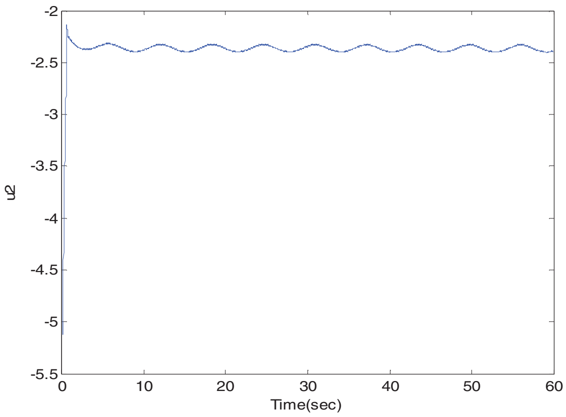

Fig. 4. The control input u2(t) for ε=0.02.