2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

Design of Indirect Adaptive Fuzzy Control (IAFC) for Nonlinear Hysteretic Systems

Chi-Hsu Wang, *IEEE Fellow*, Jyun-Hong Wang, and Chun-Yao Chen
Department of Electrical and Computer Engineering
National Chiao Tung University
Hsinchu, Taiwan, 300, ROC
cwang@cn.nctu.edu.tw; jhwang.ece01g@nctu.edu.tw; 696700309@ntnu.edu.tw

***Abstract*—Nonlinear hysteretic phenomena occur in many physical systems, such as electronic throttle and solenoid valves in automobiles, piezoelectric sensors, and many other mechanical actuators. In order to handle the nonlinear properties of hysteretic systems, an indirect adaptive fuzzy controller (IAFC) is proposed in this paper. However, it is a hard task to directly identify unknown hysteretic effects. Firstly, to overcome this problem, a hysteretic function is employed to construct the nonlinear properties of backlash-like hysteretic systems. Then the existence of an indirect adaptive controller (IAFC) is derived in this paper. Unlike the existing fuzzy control methods, our proposed IAFC can deal with different kinds of hysteretic problems with adaptive and control laws. Based on the learning algorithm, the adaptive and control laws not only can be derived but the stability of the closed-loop system can also be guaranteed by the Lyapunov stability criterion. Finally, MATLAB software is used for simulations, and the results show that our proposed IAFC can effectively handle the nonlinear properties in some unknown hysteretic systems.**

Keywords—hysteretic effect; backlash-like hysteretic system; indirect adaptive fuzzy controller

I. INTRODUCTION

Nonlinear hysteretic phenomena exist in many physical systems and materials, such as ferroelectric and ferromagnetic materials, mechanical actuators, electronic throttles, and other related fields [1]-[5]. In fact, different types of hysteresis have totally different nonlinear properties. Thus, in this paper, we focus mainly on the hysteresis model called "backlash-like hysteresis." Backlash-like hysteresis is usually found in mechanical systems, which causes a delay between the input force and output response. To control the systems with unknown backlash-like hysteresis is quite important but typically challenging. Incidentally, conventional control methods are insufficient to deal with nonlinear systems with these non-smooth nonlinearities [6]. For simplicity, the hysteresis is sometimes ignored in the design of control systems. However, ignorance of nonlinear hysteresis will lead to obviously steady-state error, oscillation, and even instability. Hence, the development of alternate effective approaches is required and urging.

For research purpose, the foremost task is to find a model to describe the hysteretic nonlinearities, which helps us to design a proper controller. Until now the research on mathematical models for unknown hysteresis is still an on-going research topic. Thus, there are various models being proposed in past decades, and different hysteresis models will affect the effectiveness of the control algorithms. Generally, the existing hysteresis model can be roughly categorized into two types [7]: operator-based hysteresis models and differential equation-based hysteresis models. The operator-based models use integral equations which contain numerous hysteresis operators and can describe the shapes of hysteresis curves accurately. The popular operator-based models are Preisach model [8], Prandtl-Ishlinskii (PI) model [9], etc. For differential equation-based models, they have finite dimensions and can be extended to continuous inputs by using an approximation [7], which can reduce the computational complexity effectively. This kind of models like Bouc-Wen model [10], Duhem model [7], and Backlash-like model [17] are used widely in the controller design for hysteresis problem. Due to nonlinear hysteretic properties in our benchmark problems, the backlash-like model proposed in [17] is adopted throughout this paper to model our benchmark problems.

Based on the mathematical models, several alternate approaches have been proposed in [11]-[19] in past decades. The above methods used the adaptive control schemes to mitigate the nonlinear effects of hysteresis. In [11]-[13], an adaptive inverse operator was constructed to cancel the backlash nonlinearity, but the strict initial conditions were required. In [19], a smooth inverse function combined with backstepping technique was utilized to compensate the nonlinear effects of the backlash. Using the intelligent control schemes like fuzzy logic control (FLC) or neural network (NN) has been depicted in [14]-[16]. Those intelligent control methods have the advantage of excellent nonlinearity approximation, which can eliminate the inversion error [14]-[16]. Some experimental applications showed that backlash inverters would degrade the system control performance [20], [21]. Hence, a controller design scheme without constructing the inverse operator has been proposed in [15], [17], [18]. In [15], [17], a continuous

This work was supported by NSC Taiwan under Grant No:102-2221-E-009 -110 - .

dynamic backlash-like hysteresis model is defined. However, the backlash-like term multiplying the control in [15], [17] must be bounded, and the uncertain parameters must also be within known intervals . The backstepping adaptive control methods proposed in [18] strived to eliminate the above restrictions.

To propose a new way to mitigate the nonlinear properties of hysteretic phenomenon without the above restrictions, an indirect adaptive fuzzy controller (IAFC), which serves as a feedback controller in a "feedback + feed-forward" scheme, is proposed in this paper. Besides, a dynamic backlash-like hysteresis model is utilized in the nonlinear system with unknown nonlinear control gain which is more general than that in [17]. The existence of the IAFC for the unknown hysteretic system is first shown in *Theorem 1* in this paper. The adaptive laws of IAFC are constructed based on the Lyapunov stability theory; and the IAFC control law guarantees that all the signals of the closed-loop system are stable with excellent tracking performance. Two popular benchmark examples are adopted in this paper. The first one is the example widely used in [15], [17]-[20] and the other is the hysteretic Inverted Pendulum System (IPS) in [22]. Excellent tracking results are illustrated from these two examples via the proposed IAFC. It is noted that all the previous methods in [15], [17]-[20] cannot deal with these two benchmark examples with unknown nonlinear control gain.

This paper is organized as follows: Section II states the problem of this paper, where the nonlinear backlash-like model is introduced. In Section III, the proposed IAFC scheme is presented. In Section IV, the simulation results are presented to illustrate the effectiveness of the proposed approach. Finally, in Section V, conclusions are drawn.

II. Problem Specification

A. System Model

We consider the following nth-order SISO nonlinear system described by the differential equation, which is more general than that in [17], [18]:

$$\begin{cases} \dot{x}_i = x_{i+1} \ , \ i=1,\cdots,n-1 \\ \dot{x}_n = F(\mathbf{x}) + g(\mathbf{x})\omega(u) \\ y = x_1 \end{cases} \tag{1}$$

where $\mathbf{x}=[x_1,x_2,\cdots,x_n]^T=[x,\dot{x},\cdots,x^{(n-1)}]^T\in\mathbb{R}^n$ is the measurable state vector; $u\in\mathbb{R}$ and $y\in\mathbb{R}$ are the input and output of the system respectively; $F(\mathbf{x})=-\sum_{i=1}^{r}a_i f_i(\mathbf{x})$, and $f(\mathbf{x})$ and control gain $g(\mathbf{x})$ are unknown nonlinear functions. Parameters a_i are unknown but bounded constants. The function $\omega(u)$ is the nonlinear hysteretic function modeled by [17]. (1) has to be controllable so we require that the control gain $g(\mathbf{x})\neq 0$. Besides, without losing generality, it is assumed that $g(\mathbf{x})>0$. In [17], the control gain g is simply an unknown constant and functions f_i have to be known linear or nonlinear functions.

The control objective is to design a control law for $u(t)$ in (1) and an adaptive law for adjusting the parameter vector, such that the system output x can track the reference signal y_m , i.e. $x\rightarrow y_m$ as $t\rightarrow\infty$. Note that the reference signal y_m is assumed to be $(\mathrm{n}-1)^{th}$ differentiable.

B. Backlash Model and its Characteristics

A continuous dynamic model to simulate hysteresis phenomenon defined by [17] can be described by:

$$\frac{d\omega}{dt}=\alpha\left|\frac{du}{dt}\right|(cu-\omega)+B\frac{du}{dt} \tag{2}$$

where α,B,c are constants and $B>c>0$; $u(t)$ is the control signal and the input of $\omega(u(t))$. Then, the solution of (2) can be solved explicitly:

$$\omega(u(t))=cu(t)+d(u) \tag{3}$$

and

$$\begin{aligned} d(u)=&[\omega_0-cu_0]e^{-\alpha(u-u_0)sgn(\dot{u})} \\ &+e^{-\alpha u sgn(\dot{u})}\int_{u_0}^{u}[B-c]e^{\alpha\xi(sgn(\dot{u}))}d\xi \end{aligned} \tag{4}$$

where u_0 and $\omega(u_0)=\omega_0$ are initial conditions, which are set as $u_0=\omega_0=0$ in this paper. The relationship between ω and u are illustrated in Fig. 1. In Fig. 1, the sign of α in (4) determines the direction of the curves. When $\alpha>0$, the hysteresis loop will be in clockwise direction. Similarly, when $\alpha<0$, the hysteresis loop will be in counterclockwise direction. In this paper, we only take $\alpha>0$ into consideration.

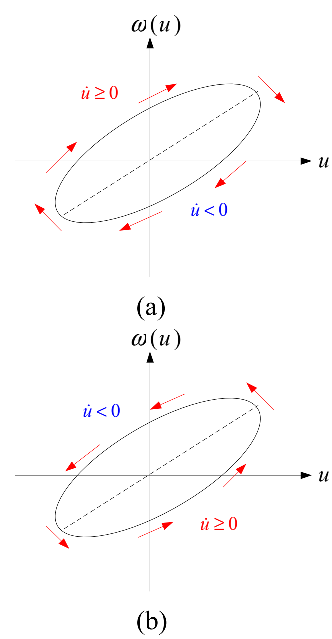

Fig. 1. The hysteretic illustrations: (a) $\alpha>0$; (b) $\alpha<0$

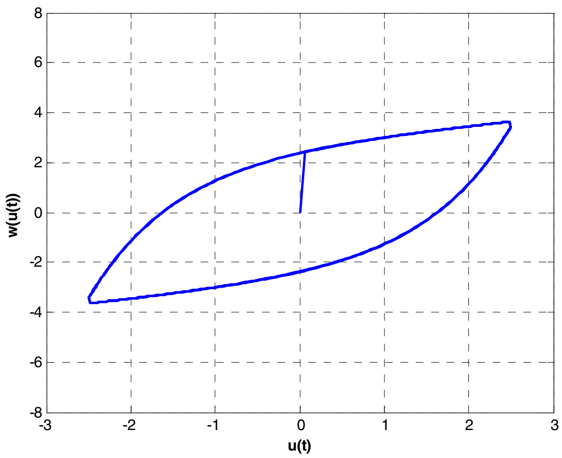

Fig. 2. Hysteresis curve

Solving for (3) and (4), we can get an explicit solution of backlash-like hysteretic function $\omega(u(t))$ as follows:

When $\dot{u} \geq 0$,

$$\omega(u) = cu(t) + (\omega_0 - cu_0)e^{-\alpha(u(t)-u_0)} + \frac{(B-c)}{\alpha}[1-(2e^{-\alpha u_s} - e^{-2\alpha u_s})e^{\alpha(u_0-u(t))}] \quad (5)$$

and when $\dot{u} < 0$,

$$\omega(u) = cu(t) + (\omega_0 - cu_0)e^{\alpha(u(t)-u_0)} + \frac{(c-B)}{\alpha}[1-(2e^{-\alpha u_s} - e^{-2\alpha u_s})e^{\alpha(u(t)-u_0)}] \quad (6)$$

where u_s is a positive value and the upper bound of u, i.e. $u \leq u_s$. As mentioned previously, we set $u_0 = \omega_0 = 0$, and (5) and (6) can be rewritten as follows:
When $\dot{u} \geq 0$,

$$\omega(u) = cu(t) + \frac{(B-c)}{\alpha}[1-(2e^{-\alpha u_s} - e^{-2\alpha u_s})e^{-\alpha(u(t))}] \quad (7)$$

and when $\dot{u} < 0$,

$$\omega(u) = cu(t) + \frac{(c-B)}{\alpha}[1-(2e^{-\alpha u_s} - e^{-2\alpha u_s})e^{\alpha(u(t))}] \quad (8)$$

A demonstrative nonlinear hysteretic curve generated from (7) and (8) is shown in Fig. 2, where $\alpha = 1$, $c = 0.345$, $B = 3.1635$, $u(t) = 2.5\sin(2.3t)$ and, $u_s = 2.5$.

III. Design of Indirect Adaptive Fuzzy Controller

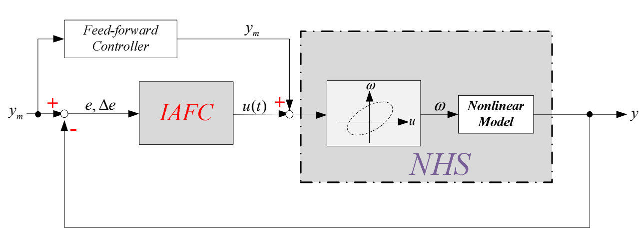

Fig. 3. The control strategy to control the *Nonlinear Hysteretic System*

To properly control the nonlinear systems with hysteresis phenomenon, we propose a feedback plus feed-forward closed loop configuration which is used widely in numerous industrial applications and is shown in Fig. 3.

In Fig. 3, y_m is input signal to the system and, in this paper, the feed-forward controller is only a transducer to convert the reference signal to our control signal, i.e. $u_{forward} = y_m$. Then, the feedback controller is the proposed indirect adaptive fuzzy controller which is presented step by step below.

A. Ideal Hysteresis Controller

Before presenting the indirect adaptive fuzzy controller (IAFC), the following Lemma must be proven:

<u>*Lemma 1*</u>:

In (3) and (4), the term $d(u)$ is bounded, i.e. $d(u) < \rho$.

Proof:

In the equation (4), if $\dot{u} > 0$ (with $sgn(\dot{u}) = 1$) and $u \to \infty$, then

$$\lim_{u\to\infty} d(u) = T_1 + T_2$$

where

$$T_1 = \lim_{u\to\infty}[\omega_0 - cu_0]e^{-\alpha(u-u_0)} \text{ and } T_2 = \lim_{u\to\infty}\int_{u_0}^{u}[B-c]e^{\alpha(\xi-u)}d\xi .$$

Obviously, $T_1 = 0$ and $T_2 = \lim_{u\to\infty}[(\frac{B-c}{\alpha})(e^{\alpha(u-u)} - e^{\alpha(u_0-u)})]$.

Thus,

$$\lim_{u\to\infty} d(u) = \frac{B-c}{\alpha} \quad (9)$$

Similarly, if $\dot{u} < 0$ (with $sgn(\dot{u}) = -1$) and $u \to -\infty$, then

$$\lim_{u\to\infty} d(u) = \frac{c-B}{\alpha} \quad (10)$$

From (9) and (10), we can imply that there exists a uniform bound ρ such that

$$d(u) < \rho \quad (11)$$

For the development of control law, the following assumptions are made.

<u>*Assumption 1*</u>:

For all $\boldsymbol{x} \in \mathbb{R}^n$, there exist unknown bound functions $\underline{g}(\boldsymbol{x})$ and $\overline{g}(\boldsymbol{x})$ such that $0 < \underline{g}(\boldsymbol{x}) \leq g(\boldsymbol{x}) \leq \overline{g}(\boldsymbol{x})$.

<u>*Assumption 2*</u>:

By (9), (10), and our experiments, we can assume that $d(u)$ is not only a bound function but also a uniform constant; thus, in the controller design, assume $d(u) = d$, where d is a constant.

With *Lemma 1* and *Assumption 1* and *Assumption 2*, the following *Theorem 1* can be presented:

<u>*Theorem 1*</u>:

Consider the system in (1) with hysteretic functions (5) and (6). There exists an ideal controller u_{ideal} such that the system output x_1 can track the reference signal y_m as close as possible.

Proof:

Let $\mathbf{y} = [y, \dot{y}, \cdots, y^{(n-1)}]^T$, and $\mathbf{y}_m = [y_m, \dot{y}_m, \cdots, y_m^{(n-1)}]^T$ denotes a bounded reference input which has the *n*th-order derivative.

Define the output tracking error as $e = y_m - y$ and the error vector as $\boldsymbol{e} = \boldsymbol{y_m} - \boldsymbol{y} = [e_1, e_2, \cdots, e_n]^T = [e, \dot{e}, \cdots, e^{(n-1)}]^T$. Choose a vector $\boldsymbol{k} = [k_n, \cdots, k_1]$ such that all roots of the polynomial $s^n + k_1 s^{n-1} + \cdots + k_n$ are in the left-half complex plane, and the polynomial is called Hurwitz polynomial [22, 23].

Then, apply (4) into (1), and we can get the following equation:

$$x^{(n)} = F(\mathbf{x}) + dg(\mathbf{x}) + cg(\mathbf{x})u(t) \tag{12}$$

where $F(\mathbf{x}) = -\sum_{i=1}^{r} a_i f_i(\mathbf{x})$.

From (12), if $F(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are known, we can assume that there exists an ideal controller shown as below:

$$u_{ideal} = \frac{1}{cg(\mathbf{x})}\left[-F(\mathbf{x}) + y_m^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right] - \frac{d}{c} \tag{13}$$

Then, we have to prove that (13) can control the output x_1 to track the reference signal y_m. Substitute (13) into (12), we can obtain the closed-loop system as:

$$\mathrm{e}^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e = 0 \tag{14}$$

With the Routh-Hurwitz stability criterion, via properly choosing the vector $\boldsymbol{k}$, we can acquire that $\lim_{t\to\infty} e(t) = 0$, which proves the theorem.

B. Indirect Adaptive Fuzzy Controller

Although we can prove the existence of the ideal hysteresis controller, $F(\mathbf{x})$ and $g(\mathbf{x})$ are usually unknown and we cannot actually derive the ideal controller. Hence, utilizing the adaptive fuzzy control strategy, the ideal controller can be approximated by an adaptive controller.

In order to design the indirect adaptive fuzzy controller, firstly, we have to construct the fuzzy logic system which can be expressed through singleton fuzzifier, center average defuzzifier, and product inference [22, 23]:

$$\mathrm{y}(\mathbf{x}) = \frac{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} = \boldsymbol{\theta}^T \xi(\mathbf{x}) \tag{15}$$

where $\mu_{F_i^l}(x_i)$ are the fuzzy input membership functions, $\bar{y}^l$ is the maximum point (or called *center*) of the output membership function $\mu_{G^l}(y)$, and, without lose generality, we assume that $\mu_{G^l}(\bar{y}^l) = 1$. $\boldsymbol{\theta} = [\bar{y}^1, \cdots, \bar{y}^M]^T$ is an adaptive parameter vector and $\xi(\boldsymbol{x}) = [\xi^1(\boldsymbol{x}), \cdots, \xi^M(\boldsymbol{x})]^T$ is a input fuzzy basis function and $\xi^l(\boldsymbol{x})$ can be defined as

$$\xi^l(\mathbf{x}) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} \tag{16}$$

Since $F(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are unknown, we replace them by fuzzy systems $\hat{f}(x \mid \theta_f)$ and $\hat{g}(x \mid \theta_g)$, respectively, which are in the form of (15) [22]. Then, the resulting control law is

$$u = u_{IAFC} = \frac{1}{c\hat{g}(\mathbf{x} \mid \theta_g)}\left[-\hat{f}(\mathbf{x} \mid \theta_f) + y_m^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right] \tag{17}$$

where $\hat{f}(x|\theta_f) = \theta_f^T \xi(\mathbf{x})$ and $\hat{g}(x|\theta_g) = \theta_g^T \eta(\mathbf{x})$ ($\eta(\mathbf{x})$ is another basis function with the same form in (16).)

Applying (17) to (1), we can obtain the error equation in the vector form [23]:

$$\dot{\boldsymbol{e}} = \Lambda\boldsymbol{e} + \boldsymbol{b}\begin{Bmatrix} \left[\hat{f}(\mathbf{x}|\theta_f) - F(\mathbf{x})\right] + \\ \left[\hat{g}(\mathbf{x}|\theta_g) - g(\mathbf{x})\right](cu_{IAFC} + d) \end{Bmatrix} \tag{18}$$

where $\Lambda = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ -k_n & -k_{n-1} & \cdots & \cdots & \cdots & \cdots & -k_1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 1 \end{bmatrix}$.

In [23], a minimum approximation error is defined as

$$w = \left[\hat{f}(\mathbf{x}|\theta_f^*) - F(\mathbf{x})\right] + \left[\hat{g}(\mathbf{x}|\theta_g^*) - g(\mathbf{x})\right](cu_{IAFC} + d) \tag{19}$$

and optimal parameters θ_f^* and θ_g^* are also defined in [23]. Using (19), we can rewrite (18) as

$$\dot{\boldsymbol{e}} = \Lambda\boldsymbol{e} + \boldsymbol{b}\begin{Bmatrix} \left[\hat{f}(\mathbf{x}|\theta_f) - \hat{f}(\mathbf{x}|\theta_f^*)\right] + \\ \left[\hat{g}(\mathbf{x}|\theta_g) - \hat{g}(\mathbf{x}|\theta_g^*)\right](cu_{IAFC} + d) + w \end{Bmatrix} \tag{20}$$

which is the same as

$$\dot{\boldsymbol{e}} = \Lambda\boldsymbol{e} + \boldsymbol{b}\begin{Bmatrix} \left[\theta_f - \theta_f^*\right]^T \xi(\mathbf{x}) + \\ \left[\theta_g - \theta_g^*\right]^T \eta(\mathbf{x})(cu_{IAFC} + d) + w \end{Bmatrix} \tag{21}$$

The adaptive laws are defined to minimize the tracking error $\boldsymbol{e}$, $\theta_f-\theta_f^*$, and $\theta_g-\theta_g^*$, and are shown in *Theorem 2*.

Theorem 2: Consider the backlash-like system (1) satisfying *Assumption 1* and *2*. The control law is designed in (18), and the adaptive laws are

$$\dot{\theta}_f=-\gamma_1 e^T P\boldsymbol{b}\xi(\mathbf{x}) \tag{22}$$

$$\dot{\theta}_g=-\gamma_2 e^T P\boldsymbol{b}\eta(\mathbf{x})(cu_{IAFC}+d) \tag{23}$$

where γ_1 and γ_2 are learning rate parameters and P is a positive definite matrix satisfying the following Lyapunov algebraic equation:

$$\Lambda^T P+P\Lambda=-\mathrm{Q} \tag{24}$$

Proof:

Define a Lyapunov equation

$$\begin{aligned}\mathrm{V}=&\frac{1}{2}\boldsymbol{e}^T P\mathbf{e}+\frac{1}{2\gamma_1}\left(\theta_f-\theta_f^*\right)^T\left(\theta_f-\theta_f^*\right)\\&+\frac{1}{2\gamma_2}\left(\theta_g-\theta_g^*\right)^T\left(\theta_g-\theta_g^*\right)\end{aligned} \tag{25}$$

Then, differentiate (25), and we can get

$$\begin{aligned}\dot{V}=&-\frac{1}{2}\boldsymbol{e}^T Q\boldsymbol{e}+\boldsymbol{e}^T P\boldsymbol{b}w+\frac{1}{\gamma_1}\left(\theta_f-\theta_f^*\right)^T\left[\dot{\theta}_f+\gamma_1\boldsymbol{e}^T P\boldsymbol{b}\xi(\mathbf{x})\right]\\&+\frac{1}{\gamma_2}\left(\theta_g-\theta_g^*\right)^T\left[\dot{\theta}_g+\gamma_2\boldsymbol{e}^T P\boldsymbol{b}\eta(\mathbf{x})\left(cu_{IAFC}+d\right)\right]\end{aligned} \tag{26}$$

In order to minimize $\boldsymbol{e}$, $\theta_f-\theta_f^*$, and $\theta_g-\theta_g^*$, we have to make $\dot{V}<0$. The first term $-\frac{1}{2}\boldsymbol{e}^T Q\boldsymbol{e}<0$. In the second term, the minimum approximation error w can be small enough by designing the fuzzy systems. If we choose $\dot{\theta}_f=-\gamma_1 e^T P\boldsymbol{b}\xi(\mathbf{x})$, the third term becomes zero. However, if we choose $\dot{\theta}_g=-\gamma_2 e^T P\boldsymbol{b}\eta(\mathbf{x})(cu_{IAFC}+d)$, the fourth term can also be zero. Hence substituting (23) into (26), the final term becomes negative. This completes the proof.

Finally, for industrial applications, the overall control signal with feed-forward signal in Fig. 3 can be shown as below

$$u=u_{IAFC}+y_m \tag{27}$$

IV. Simulation Studies

In this section, two benchmark hysteretic examples controlled by the proposed IAFC are presented and compared to the backstepping adaptive control method in [18]. *Example 1* uses the hysteresis example in [15, 17-20] with little modification: the control gain in *Example 1* is an unknown nonlinear function; however, the control gain in [15, 17-20] is just an unknown constant. *Example 2* is an Inverted Pendulum System (IPS) [22] with hysteresis, which is a standard benchmark used in numerous researches. These two examples, which are simulated based on MATLAB platform, can fully examine the effectiveness of our novel indirect adaptive fuzzy controller, and the results are shown below:

Example 1: Consider the following second-order backlash-like hysteresis nonlinear system:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=a\dfrac{1-e^{-x_2}}{1+e^{-x_2}}+g(x_1,x_2)\omega(u)\\ y=x_1\end{cases} \tag{28}$$

where ω represents the output of the hysteresis nonlinearity as in (7) and (8), the actual parameter $a=1$ and, for simulation purpose, $g(x_1,x_2)=2+\sin(x_1 x_2)>0$.

Without control, i.e. $u(t)=\omega(u(t))=0$, the system (28) is unstable. The control objective is to control the system output $y=x_1$ with initial state $\boldsymbol{x}_0=[0.3,0]$ to follow a desired trajectory $y_m=0.5sin(2.3t)$.

The backlash-like hysteresis is described by (7) and (8) with parameters $\alpha=1, c=0.345, B=3.1635, and\ u_s=10$.

The initial 25 values of vector $\theta_f(0)$ and the initial 9 values of vector $\theta_g(0)$ can be randomly chosen in $\begin{bmatrix}0 & 1\end{bmatrix}$.

Construct the IAFC for the nonlinear system:

Step 1: Select the design parameters: the learning rates $\gamma_1=35$ and $\gamma_2=1$ for (22) and (23) respectively; control parameters $\boldsymbol{k}=[k_1,k_2]^T=[10\quad 1]^T$, positive definite matrix $\boldsymbol{P}=\begin{bmatrix}255 & 25\\ 25 & 5\end{bmatrix}$, and $\boldsymbol{b}=[0\quad 0\quad\cdots\quad 1]^T$.

Step 2: The adaptive fuzzy membership function can be defined as Gaussian functions as follows

$$\mu_{F_i^l}(x_i)=e^{(-\frac{(x_i-c_{fl})^2}{2})},\ for\ \ i=1,2;l=1,2,\cdots,5$$

$$\mu_{G_i^m}(x_i)=e^{(-\frac{(x_i-c_{gl})^2}{2})},\ for\ \ i=1,2;m=1,2,3$$

where $[c_{f1},c_{f2},c_{f3},c_{f4},c_{f5}]=[-10,-5,0,5,10]$ and $[c_{g1},c_{g2},c_{g3}]=[1,1.5,3]$. Then, compute the fuzzy basis functions.

Step 3: Compute the adaptive laws in (22), (23) and control law in (17), and then obtain the IAFC control signal in (27).

The simulation results are shown in Figs. 4 - 6. The system output x_1 tracks the desired reference signal y_m is shown in Fig. 4. The control input $u(t)$ is shown in Fig. 5 and tracking error $e_1(t)$ is shown in Fig. 6.

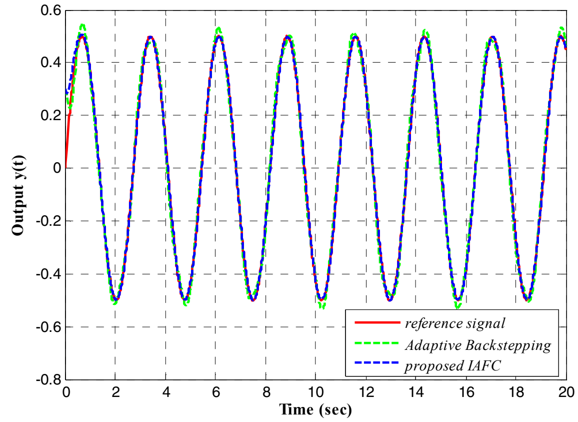

Fig. 4. Output y tracks reference signal y_m (red solid line)

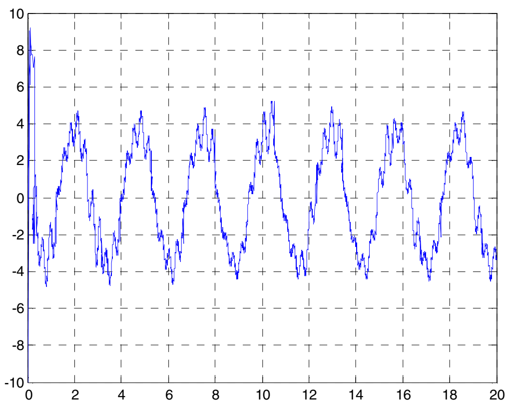

Fig. 5. Control input $u(t)$

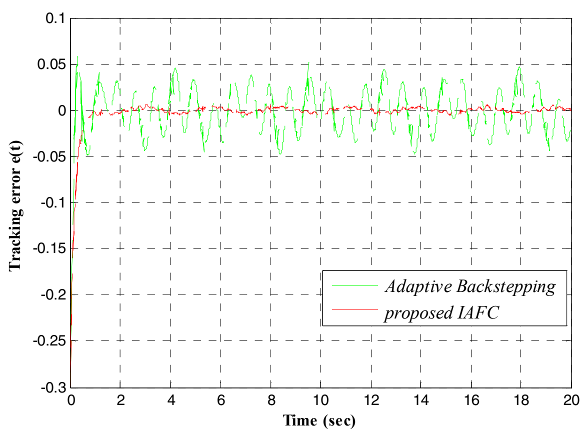

Fig. 6. Tracking error e

The system reaches the steady state at about 1 second by using proposed IAFC. In Fig. 6, it states that the steady-state error of IAFC is smaller than the error of the traditional adaptive backstepping approach.

<u>*Example 2*</u>: Consider a second-order inverted pendulum system with hysteresis, which is shown in below:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{g\sin x_1 - \dfrac{mlx_2^2\cos x_1 \sin x_1}{m_c+m}}{l(\dfrac{4}{3}-\dfrac{m(\cos x_1)^2}{m_c+m})} + \dfrac{\dfrac{\cos x_1}{m_c+m}}{l(\dfrac{4}{3}-\dfrac{m(\cos x_1)^2}{m_c+m})}\omega(u) \\ y = x_1 \end{cases} \quad (29)$$

where x_1 is the angular position of the pendulum, x_2 is the angular velocity of the pendulum, $g = 9.8\,m/s^2$ is the gravitational acceleration, $m_c = 1\,kg$ is the mass of the cart, $m = 0.1\,kg$ is the mass of the pole, $l = 0.5\,m$ is the half-length of the pendulum. The control objective is to ensure the system output x_1 with initial state $\boldsymbol{x}_0 = [\pi/12, 0]$ can track the desired reference signal $y_m = (\pi/30)\sin(t)$. The backlash-like hysteresis parameters are the same as the previous example.

The initial values of $\theta_f(0)$ and $\theta_g(0)$ are shown in TABLE I and TABLE II.

Construct the IAFC for the nonlinear system:

Step 1: Select the design parameters: the learning rates $\gamma_1 = 50$ and $\gamma_2 = 0.1$ for (22) and (23) respectively; control parameters are the same as in the previous example.

Step 2: Compute the fuzzy basis functions with the adaptive fuzzy membership function as below

$$\mu_{F_i^l}(x_i) = \mu_{G_i^l}(x_i) = e^{(-\frac{(x_i-c_l)^2}{2})}, \; for\; i = 1,2; l = 1,2,\cdots,5$$

where $[c_1, c_2, c_3, c_4, c_5] = [-\dfrac{\pi}{6}, -\dfrac{\pi}{12}, 0, \dfrac{\pi}{12}, \dfrac{\pi}{6}]$.

Step 3: Compute the adaptive laws in (22), (23) and control law in (17), and then obtain the IAFC control signal in (27).

The simulation results are shown in Fig. 7 and Fig. 8. The system output x_1 tracks the desired reference signal y_m is shown in Fig. 7. The tracking error $e_1(t)$ is shown in Fig. 8. As we can see, with faster rise time and smaller steady-state error, the proposed IAFC outperforms the traditional adaptive backstepping approach.

TABLE I. INITIAL VALUE OF $\theta_f(0)$

μ_{F_2} \ μ_{F_1}	$-\pi/6$	$-\pi/12$	0	$\pi/12$	$\pi/6$
$-\pi/6$	-5	-1	0	1	5
$-\pi/12$	-5	-1	0	1	5
0	-5	-1	0	1	5
$\pi/12$	-5	-1	0	1	5
$\pi/6$	-5	-1	0	1	5

TABLE II. INITIAL VALUE OF $\theta_g(0)$

μ_{G_2} \ μ_{G_1}	$-\pi/6$	$-\pi/12$	0	$\pi/12$	$\pi/6$
$-\pi/6$	0.126	0.136	0.146	0.136	0.126
$-\pi/12$	0.126	0.136	0.146	0.136	0.126
0	0.126	0.136	0.146	0.136	0.126
$\pi/12$	0.126	0.136	0.146	0.136	0.126
$\pi/6$	0.126	0.136	0.146	0.136	0.126

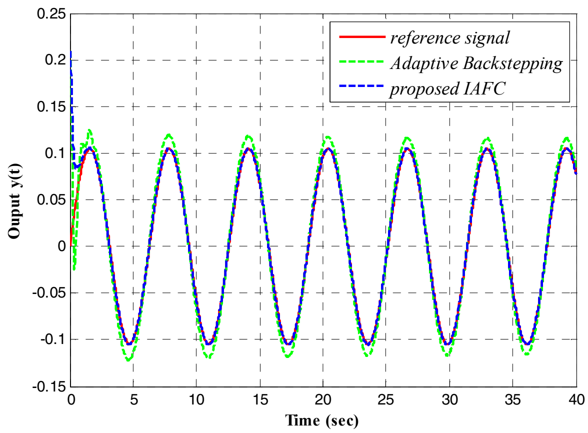

Fig. 7. Output y tracks reference signal y_m (red solid line)

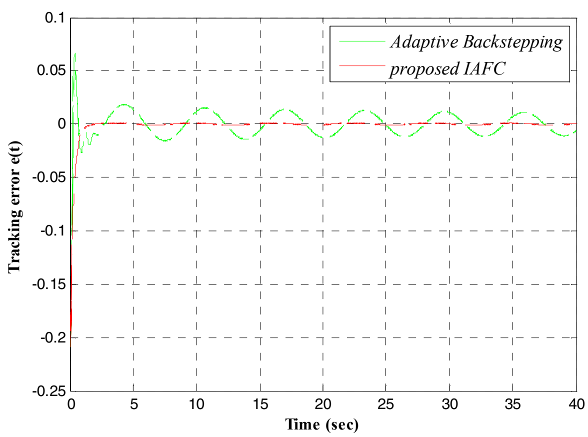

Fig. 8. Tracking error e

V. CONCLUSIONS

In this paper, a new "feedback + feed-forward" control strategy for real applications is presented to deal with nonlinear hysteresis problem. The existence of an ideal controller for unknown backlash-like hysteresis system is proposed and proven. However, ideal controller cannot be derived directly; hence, an indirect adaptive fuzzy controller (IAFC) is proposed to approximate the ideal controller. Then, by using Lyapunov theory, the stability of the closed-loop system and the tracking performance can be guaranteed. Finally, the simulation results demonstrated that the proposed approach can achieve excellent tracking performance for the hysteretic IPS with unknown nonlinear gain, which cannot be handled by previous approach. Further studies would focus on real industrial applications to verify the effectiveness of our proposed approach.

REFERENCES

[1] D.C. Jiles and J. B. Thoelke, "Theory of Ferromagnetic Hysteresis: Determination of Model Parameters from Experimental Hysteresis Loops," *IEEE Transactions on Magnetics*, v 25, n 5, p 3928-3930, Sep 1989.

[2] Y. B. Li, Shuangxia Niu, S. L. Ho, Yanhai Li, and W. N. Fu, "Hysteresis Effects of Laminated Steel Materials on Detent Torque in Permanent Magnet Motors," *IEEE Transactions on Magnetics*, v 47, n 10, p 3594-3597, October 2011, *SELECTED PAPERS FROM THE 2011 INTERNATIONAL MAGNETICS (INTERMAG) CONFERENCE - Taipei, Taiwan, April 25-29, 2011*.

[3] Qingsong Xu, "Identification and Compensation of Piezoelectric Hysteresis Without Modeling Hysteresis Inverse," *IEEE Transactions on Industrial Electronics*, v 60, n 9, p 3927-3937, 2013.

[4] Jianping Cai, Changyun Wen, Hongye Su, Zhitao Liu, and Xiangbin Liu, "Robust Adaptive Failure Compensation of Hysteretic Actuators for Parametric Strict Feedback Systems," *Proceedings of the IEEE Conference on Decision and Control*, p 7988-7993, 2011, *2011 50th IEEE Conference on Decision and Control and European Control Conference, CDC-ECC 2011*.

[5] Chi-Hsu Wang and De-Yu Huang, "A New Intelligent Fuzzy Controller for Nonlinear Hysteretic Electronic Throttle in Modern Intelligent Automobiles," *IEEE Transactions on Industrial Electronics*, v 60, n 6, p 2332-2345, 2013.

[6] Xinkai Chen, Takeshi Hisayama, and Chun-Yi Su, "Adaptive Control for Continuous-Time Systems Preceded by Hysteresis," *Proceedings of the IEEE Conference on Decision and Control*, p 1931-1936, 2008, *Proceedings of the 47th IEEE Conference on Decision and Control, CDC 2008*.

[7] Ying Feng, Camille Alain Rabbath, Tianyou Chai, and Chun-Yi Su, "Robust Adaptive Control of Systems with Hysteretic Nonlinearities: A Duhem Hysteresis Modelling Approach," *IEEE AFRICON Conference*, 2009.

[8] C. Natale, F. Velardi, and C. Visone, "Modelling and compensation of hysteresis for magnetostrictive actuators," *IEEE/ASME International Conference on Advanced Intelligent Mechatronics, AIM*, v 2, p 744-749, 2001.

[9] M. Brokate, J. Sprekels, *Hysteresis and Phase Transitions*. Berlin: Springer-Verlag, 1996.

[10] L. O. Chua and K. A. Stromsmoe, "Mathematical model for dynamic hysteresis loops," *International Journal of Engineering Science*, v 9, n 5, p 435-450, May 1971.

[11] G. Tao and P. V. Kokotovic, "Adaptive Control of Plants with Unknown Hystereses," *IEEE Transactions on Automatic Control*, v 40, p 200-212, 1995.

[12] G. Tao and P. V. Kokotovic, "Continuous-Time Adaptive Control of Systems with Unknown Backlash," *IEEE Transactions on Automatic Control*, v 40, p 1083-1087, 1995.

[13] N. J.Ahmad and F. Khorrami, "Adaptive Control of Systems with Backlash Hysteresis at the Input," in *Proc. Amer. Control Conf.*, 1999, p 3018-3022.

[14] J. O. Jang, M. K. Son, and H. T. Chung, "Friction and Output Backlash Compensation of Systems Using Neural Network and Fuzzy Logic," *Proceedings of the American Control Conference*, v 2, p 1758-1763, 2004, *Proceedings of the 2004 American Control Conference (AAC)*.

[15] Chun-Yi Su, Masahiro Oya, and Henry Hong, "Stable Adaptive Fuzzy Control of Nonlinear Systems Preceded by Unknown Backlash-Like Hysteresis," *IEEE Transactions on Fuzzy Systems*, v 11, n 1, p 1-8, February 2003.

[16] J. Campos, F. L. Lewis, and R. Selmic, "Backlash Compensation in Discrete Time Nonlinear Systems Using Dynamic Inversion by Neural Networks," *Proceedings of the IEEE Conference on Decision and Control*, v 4, p 3534-3540, 2000.

[17] Chun-Yi Su, Yury Stepanenko, Jaroslav Svoboda, and T. P. Leung, "Robust Adaptive Control of a Class of Nonlinear Systems with

Unknown Backlash-Like Hysteresis," *IEEE Transactions on Automatic Control*, v 45, n 12, p 2427-2432, Dec 2000.

[18] Jing Zhou, Changyun Wen,and Ying Zhang, "Adaptive Backstepping Control of a Class of Uncertain Nonlinear Systems with Unknown Backlash-Like Hysteresis," *IEEE Transactions on Automatic Control*, v 49, n 10, p 1751-1757, October 2004.

[19] Jing Zhou, Chengjin Zhang, and Changyun Wen, "Robust Adaptive Output Control of Uncertain Nonlinear Plants with Unknown Backlash Nonlinearity," *IEEE Transactions on Automatic Control*, v 52, n 3, p 503-509, March 2007.

[20] Jian Guo, Bin Yao, Qingwei Chen, and Jun Jiang, "Adaptive Robust Control for Nonlinear Systems with Input Backlash or Backlash-Like Hysteresis," *2009 IEEE International Conference on Control and Automation, ICCA 2009*, p 1962-1967, 2009.

[21] Stephen R. H. Dean, Brian W. Surgenor, and Harry N. Iordanou, "Experimental Evaluation of a Backlash Inverter as Applied to a servomotor with Gear Train," *IEEE Conference on Control Applications - Proceedings*, p 580-585, 1995.

[22] Li-Xin Wang, "Stable Adaptive Fuzzy Controllers with Application to Inverted Pendulum Tracking," *IEEE Trans. on Syst., Man, Cyber.-Part B:Cyber.,*v 26. n 5, October 1996.

[23] Li-Xin Wang, *A Course in Fuzzy Systems and Control*. Prentice Hall PTR, Prentice-Hall, Inc., 1997.