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Optimal Finite-Horizon Control with Disturbance Attenuation for Uncertain Discrete-Time T-S Fuzzy Model Based Systems

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Abstract—**In this paper, the sufficiency condition for disturbance attenuation level for uncertain discrete-time T-S fuzzy model-based system is derived by non-quadratic Lypaunov function (NQLF) and is expressed in terms of LMIs. And the quadratic finite horizon performance index optimal robust control with disturbance attenuation level for uncertain T-S fuzzy system can be formulated into static constrained optimization problem. Then, for static constrained optimization problem, the genetic algorithm is employed to search feedback gain for optimal finite quadratic performance index of uncertain discrete-time TS fuzzy model. Thus, the problem solving can be greatly simplified**

***Keywords—** finite horizon optimal control, H_∞ control, T-S fuzzy models, hybrid-Taguchi genetic algorithm, non-quadrtaic Lyapunov function, LMI.*

I. INTRODUCTION

A method is proposed to design robust controller to minimize finite horizon summation of quadratic cost with prescribed level of disturbance attenuation for discrete-time TS fuzzy-model-based systems with a class of uncertainty. To achieve robust disturbance attenuation, i.e. H_∞ control, we derive the relax conditions by employing non-quadratic Lyapunov function and non-PDC state feedback controller , which give more degree of freedom, as the forms in [1]. The conditions are expressed in terms of LMIs. Then, Hybrid Taguchi-genetic algorithm (HTGA)[2,3] is proposed to be the optimization tool for minimizing finite-horizon quadratic cost. The reason why we use HTGA rather than traditional genetic-algorithms (TGA) is that Chou and his associate have shown that HTGA may obtain robust solution and better performance index [3,4]. By integrating LMI and HTGA, the controller design problem is transformed into a static-parameter constrained optimization problem represented by algebraic equations together with LMI conditions. Thus, the design problem can be solved in numerical way.

This paper is organized as follows. In section 2, the background of discrete-time T-S fuzzy system and problem formulation are provided. Then, in section 3, we first presented the main results of disturbance attenuation for nominal systems. In the next subsection, the results are extended to systems with time varying parametric uncertainties to deal with robust design. HTGA algorithm is followed to find the robust quadratic-optimal controllers of discrete-time T-S fuzzy system with time varying uncertainties.

The notation used in this paper is quite standard. $((A)+\star)=(A+A^T)$. $R^{m\times n}$ denotes $m\times n$ real matrix. $*$ denotes the symmetric terms in a block matrix. $h_i^+(t)$ denotes $h_i(t+1)$. For short hand, we also sometimes will drop time t in time functions; for example, h_i instead of $h_i(t)$.

II. PRELIMINARIES AND PROPLEM FORMULATION

Consider a class of uncertain discrete-time non-linear systems governed by the following IF-THEN rule based T-S fuzzy system:

Plant Rule i : If $f_1(t)$ is M_1^i and … and $f_s(t)$ is M_s^i,

Then

$$\begin{bmatrix} x(t+1) \\ z(t) \end{bmatrix} = \begin{bmatrix} \bar{A}_i & \bar{B}_{wi} & \bar{B}_{ui} \\ \bar{C}_{zi} & \bar{D}_{zwi} & \bar{D}_{zui} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix}, \qquad (1)$$

where M_j^i denotes an j -based fuzzy set used for the ith fuzzy rule, $f_i(t)$ is the ith premise variable, and r is the total number of fuzzy rules. $x(t)\in R^{n\times n}$ is the state vector, $w(t)\in R^{m_w}$ is the disturbance input vector, $u(t)\in R^{m_u}$ is the input vectors, and $z(t)\in R^{p}$ is the controlled output vectors. $\bar{A}_i\in R^{nxn}$, $\bar{B}_{wi}\in R^{n\times m_w}$, $\bar{B}_{ui}\in R^{n\times m_u}$, $\bar{C}_{zi}\in R^{n\times p_z}$, $\bar{D}_{zwi}\in R^{p_z\times m_w}$, $\bar{D}_{zui}\in R^{p_z\times m_u}$ are the system matrices with time varying uncertainties defined as

$$\bar{A}_i = A_i + \Delta A_i, \quad \bar{B}_{wi} = B_{wi} + \Delta B_{wi}, \quad \bar{B}_{ui} = B_{ui} + \Delta B_{ui},$$
$$\bar{C}_{zi} = C_{zi} + \Delta C_{zi}, \quad \bar{D}_{zwi} = D_{zwi} + \Delta D_{zwi}, \quad \bar{D}_{zui} = D_{zui} + \Delta D_{zui},$$

and

$$\begin{bmatrix} \Delta A_i & \Delta B_{wi} & \Delta B_{ui} \\ \Delta C_{zi} & \Delta D_{zwi} & \Delta D_{zui} \end{bmatrix} = \begin{bmatrix} H_{xi} F_{\Delta x} \left(E_{xi} \quad E_{wi} \quad E_{ui} \right) \\ H_{zi} F_{\Delta z} \left(E_{zi} \quad E_{zwi} \quad E_{zui} \right) \end{bmatrix},$$

where H_{xi}, H_{zi}, E_{xi}, E_{ui}, E_{wi}, E_{zi}, E_{zwi}, and E_{zui} are known constant matrices with appropriate dimension and $F_{\Delta x}$ and $F_{z\Delta}$ represent he unknown nonlinear time-varying function satisfying $F_{\Delta x}^T(t) F_{\Delta x}(t) \le I$ and $F_{z\Delta}^T F_{z\Delta} \le I$.

Given a pair of $x(t)$ and $u(t)$, the final outputs of fuzzy system can be inferred as:

$$\begin{bmatrix} x(t+1) \\ z(t) \end{bmatrix} = \sum_{i=1}^{r} h_i(f(t)) \begin{bmatrix} \bar{A}_i & \bar{B}_{wi} & \bar{B}_{ui} \\ \bar{C}_{zi} & \bar{D}_{zwi} & \bar{D}_{zui} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix}, \qquad (2)$$

where $f(t) = [f_1(t),\ f_2(t), \ldots, f_s(t)]$ is the premise variable vector, $\mu_i(f(t)) = \prod_{j=1}^{s} M_i^j(f_j(t))$, and

$$h_i(f(t)) = \mu_i(f(t)) \Big/ \sum_{i=1}^{r} \mu_i(f(t)).$$

In this paper, we have $\mu_i(f(t)) \ge 0$. And we have following properties

$$h_i(f(t)) \ge 0, \ i = 1, \ldots, r, \ \text{and} \ \sum_{i=1}^{r} h_i(f(t)) = 1,$$

for all $t \ge 0$. In what follows, we will use h_i instead of $h_i(f(t))$ for simplicity. Before stating our control object, we give a definite regarding to disturbance attenuation level.

Definition 1: γ-disturbance attenuation: Given a constant $\gamma > 0$, the system is said to achieve γ-disturbance attenuation if $\sum_{t=0}^{\infty} z^T(t) z(t) < \gamma^2 \sum_{t=0}^{\infty} w^T(t) w(t)$ for all $w(t) \in L_2[0, \infty)$ with initial condition $x(0) = 0$. If initial condition $x(0) \ne 0$, a modified term is added to right side to become $\sum_{t=0}^{\infty} z^T(t) z(t) < \gamma^2 \sum_{t=0}^{\infty} w^T(t) w(t) + V(0)$. The γ -disturbance attenuation level also means H_∞ norm of the system from $w(t)$ to $z(t)$ is less than γ.

For control system design, we primarily consider to synthesize a feedback controller to ensure the uncertain system is robustly stable against all allowable uncertainties and also achieves adequate attenuation level in facing exogenous disturbance. However, in practical control system, only robust stability and disturbance attenuation are often not enough in controller design. The control object of minimizing quadratic-finite-horizon integral performance criterion for nominal system is also considered in many practical control applications [2,3]. Therefore, the control objective is to design a state feedback controller such that the closed loop systems are met the following goals:

(1) The closed-loop system is robustly stable for all allowable uncertainties appearing in the system matrices when $w(t) = 0$.

(2) The closed-loop system achieves γ-disturbance attenuation.

(3) The following finite-horizon-quadratic cost function is minimized:

$$J_2 = \sum_{t=0}^{t_f} (x^T(t) Q x(t) + u^T(t) R u(t)) \qquad (3)$$

where $Q > 0$ and $R > 0$ are given matrices with appropriate dimension and $t_f > 0$ is given integer.

III. MAIN RESULTS

To get more released conditions, assuming $\sum_{m=1}^{r} h_m G_m$ is invertible and following non-PDC control law is proposed

$$u(t) = \sum_{j=1}^{r} h_j F_j \left(\sum_{m=1}^{r} h_m G_m\right)^{-1} x(t), \qquad (4)$$

where F_j, G_m $(j, m = 1, \ldots, r)$ are matrices to be determined. With the controller (4), the closed-loop system becomes

$$\begin{bmatrix} x(t+1) \\ z(t) \end{bmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \begin{bmatrix} \bar{A}_{cl,ij} & \bar{B}_{wi} \\ \bar{C}_{zcl,ij} & \bar{D}_{zwi} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \qquad (5)$$

where $\bar{A}_{cl,ij} = \bar{A}_i + \bar{B}_{ui} F_j (\sum_{m=1}^{r} h_m G_m)^{-1}$ and $, \bar{C}_{zcl,ij} = \bar{C}_{zwi} + \bar{D}_{zui} F_j (\sum_{m=1}^{r} h_m G_m)^{-1}$.

A. H_∞ performance design for nominal systems

Let us consider the problem of designing a controller to reach certain disturbance attenuation level; that is H_∞ performance design. To simplify the notation and derivation, we first consider the nominal model. The following theorem gives a sufficient condition to guarantee disturbance attenuation level less than γ.

Theorem 1 [5] The system is stable and achieve γ-disturbance level if there exist $P_i > 0$, G_i, F_i, $X_{iim} = X_{iim}^T$ $(i, m = 1, \ldots, r)$ and $X_{ijm} = X_{ijm}^T$ $(i, j, m = 1, \ldots r,\ i < j)$ such that

$$M_{iim} < X_{iim} \qquad (i, m = 1, \ldots r), \qquad (6)$$

$$M_{ijm} + M_{jim} < X_{ijm} + X_{ijm}^T \quad (i, j, m = 1, \ldots r \text{ and } i < j), \quad (7)$$

$$\text{and} \quad \begin{bmatrix} X_{11m} & X_{12m} & \cdots & X_{1rm} \\ X_{21m} & X_{22m} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ X_{r1m} & X_{r2m} & \cdots & X_{rrm} \end{bmatrix} < 0 \qquad (m = 1, \ldots r), \qquad (8)$$

where

$$M_{ijm} = \left[\begin{array}{cc|cc} -P_i & 0 & \multicolumn{2}{c}{*} \\ 0 & -\gamma^2 I & & \\ \hline \tilde{\hat{A}}_{ij} & B_{wi} & -(G_m^T + G_m - P_m) & 0 \\ \tilde{\hat{C}}_{zij} & D_{zwi} & 0 & -I \end{array}\right], \quad \tilde{\hat{A}}_{ij} = A_i G_j + B_{ui} F_j \quad ,$$

$\tilde{\hat{C}}_{zij} = C_{zwi} G_j + D_{zui} F_j$.

Remark 1 The nonquadratic Lyapunov function employed in the proof of the theorem is the form

$$V(x) = x^T (\sum_{m=1}^{r} h_m G_m)(\sum_{m=1}^{r} h_m P_m)^{-1} (\sum_{m=1}^{r} h_m G_m(x)) \ . \qquad (9)$$

The validity of such Lyapunov function has been shown in [1] and it can be specialized to common Lyapunov function by letting $G_m = P_m = P \ (m = 1, \ldots, r)$. Furthermore, it also can be specialized to the fuzzy dependent Lyapunov function proposed in [6,7] by letting $P_m = G_m \ (m = 1, \ldots, r)$. Hence provides more degree of freedom for obtaining the relaxed conditions.

Remark 2 Equation (6) implies $\sum_{m=1}^{r} h_m (G_m + G_m^T - P_m) > 0$. This further implies $\sum_{m=1}^{r} h_m (G_m + G_m^T) > 0$. The existence of $(\sum_{m=1}^{r} h_m G_m)^{-1}$ is followed.

B. Extensions to uncertain systems

In this subsection, we will consider a class of uncertain discrete-time T-S fuzzy model described by (2). The following theorem is an extension of Theorem 1.

Theorem 2 For given $\gamma > 0$, the uncertain system is said to be robustly stable with γ -disturbance attenuation via the controller (4) if there exist matrices $Y_{iim} \ (i, m = 1, \ldots r)$, $Y_{ijm} = Y_{jim}^T \ (i, j, m = 1, \ldots r)$, $\varepsilon_{ij}^x > 0$ and $\varepsilon_{ij}^z > 0$ such that

$$\Psi_{iim} < Y_{iim} \qquad (i, m = 1, \ldots r) \qquad (10)$$

$$\Psi_{ijm} + \Psi_{jim} < Y_{ijm} + Y_{ijm}^T \ (i, j, m = 1, \ldots r \text{ and } i < j), \quad (11)$$

$$\begin{bmatrix} Y_{11m} & Y_{12m} & \cdots & Y_{1rm} \\ Y_{21m} & Y_{22m} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ Y_{r1m} & Y_{r2m} & \cdots & Y_{rrm} \end{bmatrix} < 0 \quad (m = 1, \ldots r), \qquad (12)$$

where

$$\Psi_{ijm} = \begin{bmatrix} -P_j & * & * & * & * & * & * & * \\ 0 & -\gamma^2 I_{mw} & * & * & * & * & * & * \\ A_i G_j + B_{ui} F_j & B_{wi} & -(G_m + G_m^T - P_m) & * & * & * & * & * \\ C_{zi} G_j + D_{zui} F_j & D_{zwi} & 0 & -I_p & * & * & * & * \\ 0 & 0 & \varepsilon_{ij}^x H_x^T & 0 & -\varepsilon_{ij}^x I & * & * & * \\ E_{xi} G_j + E_{ui} F_j & E_{wi} & 0 & 0 & 0 & -\varepsilon_{ij}^x I & 0 & * \\ 0 & 0 & 0 & \varepsilon_{ij}^z H_z^T & 0 & 0 & -\varepsilon_{ij}^z I & * \\ E_{zi} G_j + E_{ui} F_j & E_{wi} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ij}^z I \end{bmatrix}.$$

C. *Finite-Horizon Optimal Controller Design*

With the results presented in previous subsections, consequently, the problem considered in this work is how to specify the matrices F_i, G_i and P_i $(i = 1, \ldots, r)$ in Theorem 2 such that the constraint of LMI-based robust H_∞ control conditions (10)-(12) for the closed-loop uncertain T-S fuzzy model based system in (2) can be satisfied, and such that the optimal performance for the nominal T-S fuzzy model-based system

$$x(t+1) = \sum_{i=1}^{r} h_i (A_i x(t) + B_{ui} u(t)) \qquad (13)$$

can be achieved by minimizing the quadratic-finite-horizon performance index function (3). Here, the design procedure can be depicted as following design steps:

Step 1: Check the LMI constraints (10)-(12).

Step 2: Minimize the quadratic-finite-horizon performance index function (3) for the nominal system (13).

Since, for the robust disturbance attenuation plus quadratic-finite-horizon optimal control problem, the LMI approach proposed in most guaranteed cost control cannot be directly applied, we integrate HTGA and LMI to solve the problem addressed in this work.

IV. CONCLUSIONS

In this paper, a systematic approach is proposed to solve the optimal finite-horizon quadratic cost subject to robust disturbance attenuation for uncertain discrete time T-S fuzzy models. We first give the relax LMI conditions for disturbance attenuation level. Then, we convert the problem into static constraint optimization problem represented by algebraic equation with LMI constraint. The HTGA is employed to search the static feedback gain to directly minimizing the finite-horizon quadratic cost for nominal system subjecting LMI constraint. Thus, the problem can be solved in efficiency way by available numerical algorithms.

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