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A Numerical Two-Scale Model of Multi-granularity Linguistic Variables and It's Application to Group Decision Making

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***Abstract*—Many group decision making (GDM) problems under uncertain environments have vague and imprecise information in linguistic variable formats. Multi-granularity linguistic method with the process of symbolic computation is a typical tool to solve such problems, and can be associated with the popular computing with words domain. In the paper, we present numerical two-scale model which is the extension of symbolic computation model. Firstly, numerical two-scale representation model is proposed with two scale measurements. One is used to reflect the order of linguistic variable and the other is used to model vagueness. Secondly, we give numerical two-scale computational model in group decision making. We discuss the rules of symbolic method which directly compute with the two numerical measurements. Finally, some aggregation operators are developed. The new model has the advantage of avoiding the vague information losing without adding the calculation difficulty.**

I. INTRODUCTION

When attempting to qualify phenomena related to human perception, for example, speed of a car, linguistic terms like "fast", "very fast", "slow" may be used instead of numerical values. There may be two reasons. One is the cost of its computation is too high, so an "approximate value" may be tolerated. The other is such accuracy is unnecessary. Linguistic variable as the good method to approximate human activities is provided. Linguistic variables are often presented as labels belonging to a linguistic term set and have the relation with fuzzy subsets. Computing with words (CWW) can deal with uncertainty, imprecision problems. As decision making is a typical human mental process, it seems natural to apply the CWW methodology in order to create and enrich decision models in which the information that is provided and manipulated has a qualitative nature [1]. In recent years, CWW has been used in group decision making (GDM) or multi-expert decision making (MEDM) by many researchers [1-5]. Because of the real-world's complexity and dynamics, many financial and economical problems require advanced and sophisticated methods and tools to deal with issues like fast-learning, uncertainty etc [6].

In classical linguistic modeling [7-11], the fuzzy set or membership function associated with each linguistic label is often used to represent its semantic. They use classical fuzzy arithmetic to deal with the membership functions and obtained the outcome of a fuzzy set. Ranking functions are use to obtain a final numerical evaluation [12, 13]. But, in some practical applications, the determination of membership functions or fuzzy sets associated with linguistic labels is difficult or impossible. So these approaches based on fuzzy set or membership function have their restrictions. Linguistic symbolic computational models based on ordinal scales were researched [14-24]. Since the 2-tuple linguistic computational model [25] is proposed, many extensions to the 2-tuple linguistic model are developed. Xu [18] extend a discrete term set to a continuous term set by virtual linguistic terms. Tang and Zheng [26] model the linguistic uncertainty directly by a fuzzy relation on the set of linguistic labels. In addition, classical aggregation operators are extended to operators for linguistic information [21, 27-29].

Since decision makers with different culture and different knowledge, it is reasonable for decision makers to provide their preferences with linguistic term sets of different cardinalities. This type of information is referred to as multi-granularity linguistic information [8]. Transformation from a coarse granularity to a fine granularity is often needed in existing fusion methods. Some fusion methods [15, 30] for multi-granularity linguistic term sets are proposed to solve the GDM problem based on 2-tuple linguistic model. Previous methods can't solve multi-granularity linguistic problems effectively. Computing with words can't be manipulated like computing with real numbers. Some extended forms of classical aggregate operators can't be directly used.

To make the models more flexible, we present the concept of the numerical two-scale for ordinal linguistic term set and give suitable measurements with the purpose of taking

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account the vagueness of linguistic terms. Computational techniques for multi-granularity linguistic terms are extended for GDM.

II. PRELIMINARY

A. Linguistic Variables

The concept of linguistic variables [31-33] is defined as a quintuple $(L, T(L), U, S, M)$, in which L is the name of the variable, $T(L)$ is the term set of labels or words of L, U is the universe of discourse, S is the syntactic rule, and M is the semantic rule which associates with each linguistic label. In our model, the linguistic labels in term set $T(L)$ will be ordered. In our model, we assume the linguistic labels in a term set are ordered. We will use an ordered linguistic term set $S=\{s_0, s_1, \cdots, s_n\}$ with $s_0 < s_1 < \cdots < s_n$ to represent a vague concept. Fig. 1 is an example of the linguistic tearm set *T (Height) = {None(N), Very Low(VL), Low(L), Medium(M), High(H), Very High(VH), Perfect(P)}*.

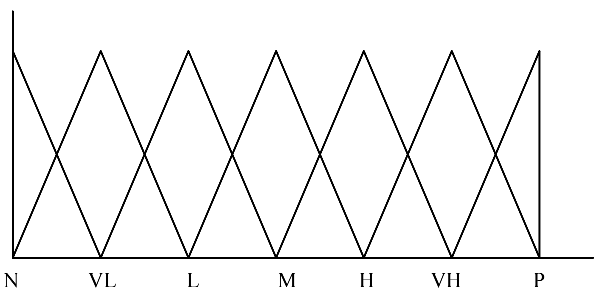

Fig.1 A set of seven terms with their semantics

B. Computational models Based on Symbolic Model

Herrera and Martinez [25] define a symbolic representation model called linguistic 2-tuples and the process of symbolic translation. Together with this representation model they presente a computational technique to deal with linguistic 2-tuples without loss of information.

In the linguistic 2-tuples definition, let $S=\{s_0, s_1, \cdots, s_g\}$ be a linguistic term set. Linguistic variables can be represented as (s_i, α), where s_i is a linguistic term and α is a numeric value representing the symbolic translation. This form can be translated to a value $\beta \in [0, g]$ which is used to represent the value of linguistic 2-tuples. The translation function Δ^{-1} and retranslation function Δ are as following[25]:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5), \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha), with \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i \end{cases}$$

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{2}$$

Function (1) and (2) have defined the functions transforming numerical values into a 2-tuples and vice versa without loss of information, therefore, any numerical aggregation operator can be easily extended for dealing with linguistic 2-tuples.

Wang and Hao [19] extend the 2-tuple fuzzy linguistic representation model to proportional 2-tuples. Proportional 2-tuples is represented as $(\alpha s_i, (1-\alpha)s_{i+1}) \in \overline{S}$. $\beta \in [0, g]$ is also used to represent the value of proportional 2-tuples. The transform function π and retransform function π^{-1} between numerical values and a proportional are given.

$$\pi(\alpha s_i, (1-\alpha)s_{i+1}) = i + (1-\alpha) \tag{3}$$

$$\pi^{-1}(x) = ((1-\beta)s_i, \beta s_{i+1}) \tag{4}$$

where $i = E(x)$, E is the integral part function, $\beta = x - i$.

The aggregation operators for proportional 2-tuples are also given [19]. These aggregation operators can deal with linguistic labels which are not symmetrically distributed. So we can use this tool to deal with some complex problems.

Xu [18] proposes the concept of virtual linguistic terms where a discrete term set is extended into a continuous term set. He called s_α the original linguistic term, where $S=\{s_\alpha | s_1, s_2, \cdots, s_t\}$. And he called s_α the virtual linguistic term, where $\overline{S}=\{s_\alpha | s_1 \le s_\alpha \le s_t, \alpha \in [1, t]\}$.

Dong, Xu and Yu [34] define the concept of numerical scale and extend the 2-tuple fuzzy linguistic representation models under the numerical scale (*NS*). They define the function $NS: S \rightarrow R$, where $NS(s_i)$ is the numerical index of s_i.

We can see some applications of the symbolic models in the literatures [6, 23, 24, 35]. Aggregation operators that operate in numerical model can be easily extended in this linguistic model such as LOWA, LWD, LWA. But the proposed symbolic models don't seem to prepare to solve multi-granularity linguistic problems.

III. A NEW VERSION OF SYMBOLIC MODEL

A. Linguistic Hierarchy of Multi-granularity Linguistic Variables

The linguistic hierarchical structure is composed of a set of layers. Each layer contains linguistic partitions with different granularity levels and fuzzy rules. It seems natural to allow decision makers to provide their preferences using different linguistic term sets, with different cardinalities and with different meanings for each label. For example, in a grading system a decision maker could choose to use a linguistic term set S_1 =(*Low, Medium, High*) and another decision maker could prefer a linguistic term set with higher granularity as S_2 = (*Very Low, Low, Medium, High, Very High)* (he might be able to better discriminate his preferences about the alternatives). A linguistic hierarchy in two levels is established (see Fig. 2).

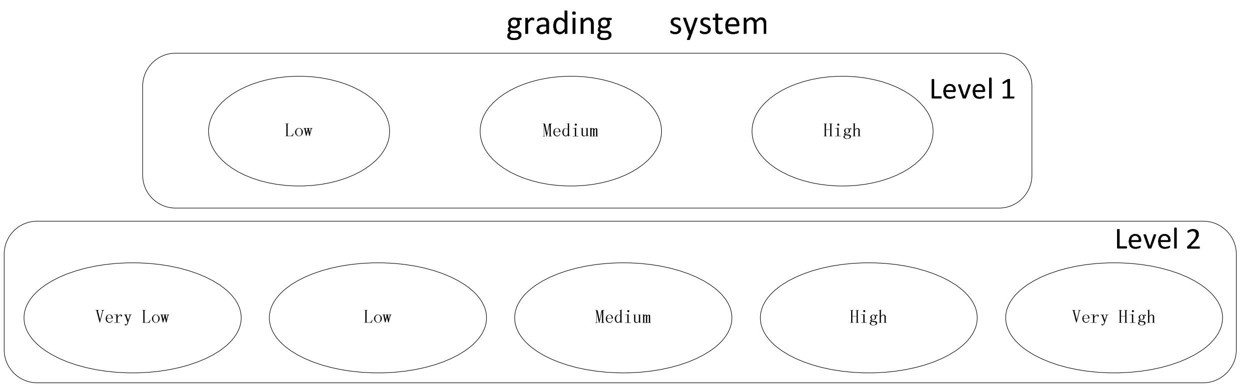

Fig2. An example of LH

In linguistic GDM analysis, most of the proposals for solving GDM problems with linguistic information are focused on the cases where the information provided by experts is represented in one level. However, in practical GDM problems, the experts maybe use linguistic terms from different levels to express their individual assessment [15, 30, 36]. **In this paper, the problem of group decision of multi-granularity linguistic term sets is the group decision whose preferences are from the different levels of linguistic hierarchy.** In horizontal direction of a linguistic hierarchy, for an ordered linguistic term set $S=\{s_0,s_1,\cdots,s_n\}$, ranking can be easily got by comparing the subscript of labels. In vertical direction of a linguistic hierarchy, experts' confidences about their preferences are reflected. In general, one expert uses labels in one level. **Different levels describe different experts' preferences. Information in vertical direction is omitted for a long time. Previous symbolic models can not reflect this information.** Information in vertical direction is very useful when we solve multi-granularity linguistic GDM problems.

When we transform the information of a label into a numerical measurement, the information should be two dimensions. The ranking information of label reflected though the functions as Δ^{-1}, CCV, NS, is the horizontal dimension. The vagueness information of label in vertical dimension is not reflected. Many papers develop linguistic decision models dealing with multi-granularity linguistic contexts and apply them to a multi-expert decision-making problem which may produce a loss of vagueness information. The vagueness information is contained in the term set, which is reflected through the granular number of the set. If we have more confidence about our preference, then we can subdivide the term set. So we choose the label in lower level whose granular number is larger. On the contrary, we choose the label in upper level.

B. A Numerical Two-Scale Representation Model

In our paper, multi-granularity linguistic variable is defined as follow:

Definition 1: Let $S^g=\{s_0^g,\cdots,s_g^g\}$ be a linguistic term set, linguistic term is represented as $s_i^g \in S^g$, where superscript g is the granule number of the term set; subscript i is the order index.

The numerical two-scale representation model is to represent a linguistic variable by two numerical measurements: order function and vague function. Selecting the $Two-Scale\ Function: S \to R$ $(Order, Vague)$ to quantify linguistic variables is the key problem. $(Order, Vague)$ should satisfy these conditions:

1) In order to normalize the values of labels in different level, we require $Order \in [0,1]$ and $Vague \in [0,1]$;

2) If the linguistic term set of A is vaguer than the set of B, then $Vague(A) > Vague(B)$.

In this paper we deal with two kinds situations, one is the linguistic terms' membership function is known, the other is unknown. If we know the membership function, it is described as triangular-shaped, symmetrical and uniformly distributed in [0,1]. So the numerical two-scale function has two types.

The numerical two-scale function to quantify linguistic terms is defined as following.

1) Linguistic term set can be described by fuzzy set or membership function

Definition 2: $(Order, Vague)$ is defined as

$$Order = \frac{\int_{S(s_i^g)} x\mu_{s_i^g}(x)dx}{\int_{S(s_i^g)} \mu_{s_i^g}(x)dx} \tag{5}$$

$$Vague = \left[\frac{\int_{S(s_i^g)} x^2\mu_{s_i^g}(x)dx}{\int_{S(s_i^g)} \mu_{s_i^g}(x)dx} - Order^2\right]^{1/2} \tag{6}$$

$x \in \Omega$ is the linguistic label description. $\mu_{s_i^g}(x)$ is a function from Ω into [0,1], and could be viewed as membership function of a fuzzy concept s_i^g.

2) Linguistic term set cannot be described by fuzzy set or membership function.

Definition 3: $(Order, Vague)$ is defined as

$$Order = \Delta^{-1}(s_i^g)/g \tag{7}$$

$$Vague = \frac{1}{g} \tag{8}$$

In order to describe more information, decision makers can use composition language to express complete opinion. We give the definition of function to transform composite linguistic variables similar in definition 3 into numerical scales.

Definition 4: Let $S^g = \{s_0^g, \cdots, s_g^g\}$ be a linguistic term set, $\overline{S}$ be the ordinal proportional 2-tuple set $(\alpha s_i^g, (1-\alpha)s_{i+1}^g) \in \overline{S}$ generated by S. $(Order, Vague)$ is defined as:

$$Order = \alpha \cdot Order(s_i^g) + (1-\alpha) \cdot Order(s_{i+1}^g) \tag{9}$$

$$Vague = \alpha \cdot Vague(s_i^g) + (1-\alpha) \cdot Vague(s_{i+1}^g) \tag{10}$$

In first place, membership functions of linguistic variables can contain more information, so it is very suitable for describe the vague information. But the models based on extension principle are very complicated. In order to compute simply, we have to discard some information in linguistic variables. We must remark that this is necessary.

On the other hand, making a depth analysis of the label indexes, we can see that the difference between two levels can be described by the subscript. So the main representation problem of multi-granularity linguistic group decision making can be solved through using order and vague function at the same time.

IV. A COMPUTATIONAL MODEL FOR NUMERICAL TWO-SCALE REPRESENTATION MODEL

A. Making the information uniform

With a view to managing the multi-granularity linguistic information, we must make it uniform [15, 30]. Here give the steps of unifying two linguistic variables $s_a^{g_1}, s_b^{g_2}$.

Step1. Constructing S_T

We look for a S_T whose granule number (g) is the lowest common multiple (L.C.M.) of g_1, g_2. So $S_T = (s_0^g, s_1^g, \cdots, s_g^g)$

Step 2. Transforming into S_T in the form of $s_a^{g_1} = \{(\alpha(s_0^g), s_0^g), (\alpha(s_1^g), s_1^g), \cdots, (\alpha(s_g^g), s_g^g)\}$, where $\alpha(s_i^g)$ means the degree of similarity to s_i^g, and $\alpha(s_i^g) \in [0,1]$.

We define a transformation function to decide the $\alpha(s_i^g)$ for s_i^g.

$$\alpha(s_i^g) = \begin{cases} \dfrac{\frac{g}{g_1} - \left| i - \frac{a}{g_1} \cdot g \right|}{\frac{g}{g_1}}, & if\ \dfrac{\frac{g}{g_1} - \left| i - \frac{a}{g_1} \cdot g \right|}{\frac{g}{g_1}} \geq 0 \\ 0, & \dfrac{\frac{g}{g_1} - \left| i - \frac{a}{g_1} \cdot g \right|}{\frac{g}{g_1}} < 0 \end{cases} \tag{11}$$

Decision making problem with multi-granularity linguistic variables are traditional solved by fusion approach. We introduce the uniform process in order to conform to conventions of solving such problem. **The difference between our uniform process and traditional uniform process is that our process does not need to know the membership function of linguistic variables.** So if anyone want to follow fusion approach, then this subsection is useful.

But in fact our computational model does not need this process. Because the unify process need large workload. Our aim is to simplify the computational process. So in the following subsection, we introduce the new aggregation method.

B. Aggregation of Multi-granularity Linguistic Variables

We discuss the rules based on two scale numerical representation model. Suppose A and B are linguistic variables. And $k \in [0,1]$. The computational model should satisfied following rules:

1) $Order(A \oplus B) = (Order(A)/Vague(A) + Order(B)/Vague(B))$
2) $Vague(A \oplus B) = (Vague(A) + Vague(B))/2$
3) $Order(k \cdot A) = k \cdot Order(A)$
4) $Vague(k \cdot A) = Vague(A)$

so operational laws are as follows:

1) $s_a^g \oplus s_b^g = s_{(a+b)}^g$
2) $k \otimes s_a^g = s_{ka}^g$
3) $s_{a_1}^g \oplus s_{a_2}^g \oplus \cdots \oplus s_{a_n}^g = s_{\sum_{i=1}^n a_i}^g$
4) $s_a^g \oplus s_b^g = s_b^g \oplus s_a^g$
5) $s_a^{g_1} \oplus s_b^{g_2} = s^{(g_1+g_2)/2}_{\frac{a \cdot (g_1+g_2)}{2 \cdot g_1} + \frac{b \cdot (g_1+g_2)}{2 \cdot g_2}}$
6) $s_a^{g_1} \oplus s_b^{g_2} = s_b^{g_2} \oplus s_a^{g_1}$

When two linguistic variables come from a same term set, symbolic methods direct operate on the numerical value of order function. Operational laws (1)-(4) are based on the operational laws of Xu [37]. For any two labels $s_a, s_b \in S$, $S = (s_0, s_1, \cdots, s_g)$, Xu [37] defines their operational laws as follows:

$s_\alpha \oplus s_\beta = s_{\alpha+\beta}$

$\lambda \cdot s_\alpha = s_{\lambda\alpha}$, where $\lambda \in [0,1]$.

For s_a, s_b are with the same granular, we can replace $s_\alpha, s_\beta \in S$ with $s_a^g, s_b^g \in S^g$. Then we can get the Operational laws (1)-(4).

When $s_a^{g_1}, s_b^{g_2}$ are with different granular, we define

Operational law (5) as the operational rule of directly computing with the two numerical measurements

Xu [38] stated that, "in general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms only appear in operation". In our model, the linguistic terms in decision process are also virtual linguistic terms. **So the superscript g and the subscript i maybe not integers, or satisfy the propositions as we have give above.**

In the other hand, our model can get the same result as existing symbolic models if we aggregate the linguistic variables from the same tem set. So there is no conflict with the existing symbolic models.

Then we address some aggregation operators based on numerical two-scale model. In the traditional decision analysis, a weighting vector $W = (w_1, \cdots, w_p)$ is often associated with numerical values which satisfy the condition $w_i \in (0,1)$ and $\sum_{i=1}^{p} w_i = 1$. Collective preference values for the alternatives may then be obtained via a linguistic weighted aggregation operation, of the form $X = \bigoplus_{i=1}^{p} w_i \otimes x_i$. So with the aforementioned operational laws based on numerical two-scale model, we can propose some useful aggregation operators to solve most of the decision problems.

A. Average operator

$$x = \phi\{s_{a_1}^{g_1}, s_{a_2}^{g_2}, \cdots, s_{a_n}^{g_n}\} = s_a^g$$
$$g = \frac{1}{n}\sum_{i=1}^{n} g_i$$
$$a = \frac{1}{n}\sum_{i=1}^{n} a_i$$

B. Average weighted operator

$$x = \phi\{s_{a_1}^{g_1}, s_{a_2}^{g_2}, \cdots, s_{a_n}^{g_n}\} = s_a^g$$
$$g = \sum_{i=1}^{n} g_i \cdot w_i$$
$$a = g \cdot \sum_{i=1}^{n} \frac{a_i \cdot w_i}{g_i}$$

C. Linguistic ordered weighted averaging operator (LOWA)

$$x = \phi\{s_{a_1}^{g_1}, s_{a_2}^{g_2}, \cdots, s_{a_n}^{g_n}\} = s_a^g$$
$$g = \sum_{i=1}^{n} g_i^* \cdot w_i$$
$$a = g \cdot \sum_{i=1}^{n} \frac{a_i^* \cdot w_i}{g_i^*}$$

where $\{s^{*g_1}_{a_1}, s^{*g_2}_{a_2}, \cdots, s^{*g_n}_{a_n}\}$ is a permutation of $\{s_{a_1}^{g_1}, s_{a_2}^{g_2}, \cdots, s_{a_n}^{g_n}\}$, where $s^{*g_i}_{a_i}$ is the ith linguistic variable.

C. Comparison of Multi-granularity Linguistic Variables

We can obtain a rank ordering by comparison of linguistic information represented by the numerical two-scale symbolic model.

1) The first comparison way

Suppose there are two labels $s_{a_i}^{g_i}$ and $s_{a_j}^{g_j}$,

When $\frac{a_i}{g_i} > \frac{a_j}{g_j}$, $s_{a_i}^{g_i} \succ s_{a_j}^{g_j}$;

When $\frac{a_i}{g_i} < \frac{a_j}{g_j}$, $s_{a_i}^{g_i} \prec s_{a_j}^{g_j}$;

When $\frac{a_i}{g_i} = \frac{a_j}{g_j}$,

If $g_i < g_j$, then $s_{a_i}^{g_i} \prec s_{a_j}^{g_j}$;

If $g_i > g_j$, then $s_{a_i}^{g_i} \succ s_{a_j}^{g_j}$;

If $g_i = g_j$, then $s_{a_i}^{g_i} \sim s_{a_j}^{g_j}$

The rule is in accordance with the idea what decision maker doesn't like uncertainty. So when the ranking value is the same, we prefer labels with less uncertainty.

2) The second comparison way is:

Comparison of linguistic information is carried out according to the comparison of distance between two linguistic variables.

Definition 5: Distance function between two linguistic variables

$$D(s_{a_i}^{g_i}, s_{a_j}^{g_j}) = -\sum_{k=1}^{k^*}\left(\alpha_i\left(s_k^g\right) - \alpha_j\left(s_k^g\right)\right)\left(1-(k^*-k)/g\right) + \sum_{k=k^*+1}^{g}\left(\alpha_i\left(s_k^g\right) - \alpha_j\left(s_k^g\right)\right)\left(1-(k-k^*)/g\right)$$

where $k^* = \max\{k_i^*, k_j^*\}$, $\alpha_i(s_{k_i^*}^g)$ is the maximum in $\alpha_i(s_{k_i}^g)$, and $\alpha_j(s_{k_j^*}^g)$ is the maximum in $\alpha_j(s_{k_j}^g)$.

Then we give the ranking rule:

If $D(s_{a_i}^{g_i}, s_{a_j}^{g_j}) > 0$, then $s_{a_i}^{g_i} \succ s_{a_j}^{g_j}$;

If $D(s_{a_i}^{g_i}, s_{a_j}^{g_j}) < 0$, then $s_{a_i}^{g_i} \prec s_{a_j}^{g_j}$;

If $D(s_{a_i}^{g_i}, s_{a_j}^{g_j}) = 0$, then $s_{a_i}^{g_i} \sim s_{a_j}^{g_j}$.

Remark: Usually, the non-dominance choice degree defined by [39] is applied to decide the ranking of alternatives in GDM. This method can also be used to solve the problem of ranking multi-granularity linguistic term sets. Our two methods can obtain the similar ranking results to non-dominance choice degree method.

V. APPLICATION IN GDM

A. Numerical two-scale Method for GDM

A decision process of numerical two-scale method has two steps. Firstly, the collective performance evaluations of the alternatives are obtained by means of an aggregation operator. Secondly, the choice of the best alternative(s) from the collective performance evaluations is performed. Use the first comparison way to choose the best alternatives.

B. A Numerical Example

Take the group decision example in paper [15] for example. These experts use different linguistic term sets to provide their preferences over the alternative set. The preferences of four experts come from four term sets:

$S_1 = S_4 = \left(s_0^8, s_1^8, \cdots, s_8^8\right), S_2 = \left(s_0^6, s_1^6, \cdots, s_6^6\right), S_3 = \left(s_0^4, s_1^4, \cdots, s_4^4\right)$. The preferences of four experts presented by the definition 1 are in Table.1：

TABLE 1. The preferences of experts

The preferences of experts		ALTERNATIVES			
		x_1	x_2	x_3	x_4
Experts	p_1	s_4^8	s_6^8	s_3^8	s_5^8
	p_2	s_3^6	s_4^6	s_3^6	s_5^6
	p_3	s_2^4	s_3^4	s_2^4	s_1^4
	p_4	s_4^8	s_5^8	s_3^8	s_5^8

Using the numerical two-scale method

$r_1 = \phi\{s_4^8, s_3^6, s_2^4, s_4^8\} = s_4^8$

$r_2 = \phi\{s_6^8, s_4^6, s_3^4, s_5^8\} = s_{4.5}^6$

$r_3 = \phi\{s_3^8, s_3^6, s_2^4, s_3^8\} = s_{2.5}^5$

$r_4 = \phi\{s_5^8, s_5^6, s_1^4, s_5^8\} = s_5^7$

where ϕ is the LOWA operator, $W = (0,0,0.5,0.5)$.

In the operator, $\{s^{*g_1}_{a_1}, s^{*g_2}_{a_2}, \cdots, s^{*g_n}_{a_n}\}$ is a permutation of $\{s^{g_1}_{a_1}, s^{g_2}_{a_2}, \cdots, s^{g_n}_{a_n}\}$, where the comparison of linguistic information represented by numerical two-scale model is carried out according to the comparison of distance between two linguistic variables.

Finally we also get $r_2 \succ r_4 \succ r_1 \succ r_3$.

C. Comparative Analysis

According to fusion methods [15, 30], the decision steps are concluded as following three steps. The only difference in the two papers is the transformation function. In our paper, we use the transformation function of equation (11).

1) Making the information uniform

Linguistic variables are transformed in the form of

$$s_a^{g_1} = \left\{\left(\alpha\left(s_0^g\right), s_0^g\right), \left(\alpha\left(s_1^g\right), s_1^g\right), \cdots, \left(\alpha\left(s_g^g\right), s_g^g\right)\right\} \tag{12}$$

Where $S_T = \left(s_0^g, s_1^g, \cdots, s_g^g\right)$.

2) Computing the collective performance values.

The collective performance value of an alternative is obtained by means of the aggregation of these uniform representations. This collective performance value, denoted r^i, is defined on S_T as

$$r^i = \left\{\left(\alpha_i\left(s_0^g\right), s_0^g\right), \left(\alpha_i\left(s_1^g\right), s_1^g\right), \cdots, \left(\alpha_i\left(s_g^g\right), s_g^g\right)\right\}$$

$$\alpha^i(s_j^g) = f\left(\alpha_1(s_j^g), \alpha_2(s_j^g), \cdots, \alpha_n(s_j^g)\right), \quad j = 0,1,\cdots g$$

where f is an aggregation operator.

3) Choosing the best alternatives

Use the second comparison way to choose the best alternatives.

Then we use the traditional fusion approach to solve the same numerical example, we get:

Step1. Constructing S_T

Obtain the granular of S_T, which is $LCM(8,6,4,8) = 24$. So we construct $S_{BLTS} = \left(s_0^{24}, s_1^{24}, \cdots, s_{24}^{24}\right)$.

Step2. Transforming to S_T

For example

$$s_4^8 = \left\{\left(0.33, s_{10}^{24}\right), \left(0.67, s_{11}^{24}\right), \left(1, s_{12}^{24}\right), \left(0.67, s_{13}^{24}\right), \left(0.33, s_{14}^{24}\right)\right\}$$

$$s_4^6 = \left\{\left(0.25, s_{13}^{24}\right), \left(0.5, s_{14}^{24}\right), \left(0.75, s_{15}^{24}\right), \left(1, s_{16}^{24}\right), \left(0.75, s_{17}^{24}\right), \left(0.5, s_{18}^{24}\right), \left(0.25, s_{19}^{24}\right)\right\}$$

$$s_2^4 = \left\{\left(0.1667, s_8^{24}\right), \left(0.333, s_9^{24}\right), \left(0.667, s_{10}^{24}\right), \left(0.8333, s_{11}^{24}\right), \left(1, s_{12}^{24}\right), \left(0.833, s_{13}^{24}\right), \left(0.667, s_{14}^{24}\right), \left(0.333, s_{15}^{24}\right), \left(0.1667, s_{16}^{24}\right)\right\}$$

Step3. Aggregating preferences

We take the OWA operator with weighting vector $W = (0,0,0.5,0.5)$.

Globe preferences of alternatives are:

$$r_1 = \left\{\left(0.33, s_{10}^{24}\right), \left(0.67, s_{11}^{24}\right), \left(1, s_{12}^{24}\right), \left(0.67, s_{13}^{24}\right), \left(0.33, s_{14}^{24}\right)\right\},$$

$$r_2 = \left\{\begin{array}{l}\left(0.0833, s_{13}^{24}\right), \left(0.1667, s_{14}^{24}\right), \left(0.25, s_{15}^{24}\right), \left(0.5, s_{16}^{24}\right), \left(0.5, s_{17}^{24}\right), \\ \left(0.25, s_{18}^{24}\right), \left(0.125, s_{19}^{24}\right)\end{array}\right\}$$

$$r_3 = \left\{\left(0.0833, s_7^{24}\right), \left(0.1667, s_8^{24}\right), \left(0.375, s_9^{24}\right), \left(0.5833, s_{10}^{24}\right), \left(0.333, s_{11}^{24}\right)\right\}$$

$$r_4 = \left\{\left(0.29, s_{17}^{24}\right)\right\}$$

Step4. Ranking

Compare the globe preferences of alternatives:

$D(r_1, r_2) = -1.099$

$D(r_1, r_3) = 2.498$

For example $D(r_3, r_4) = -0.0291$

$D(r_2, r_4) = 0.665$

$D(r_1, r_4) = -2.665$

Finally we get $r_2 \succ r_4 \succ r_1 \succ r_3$.

Remark: The result of the two methods is the same as the result in Chen and Ben-Arieh [30] paper. While in Herrera, Herrera-Viedma and Martinez [15] paper the alternatives x_2 and x_4 are non-dominated with degree 1. So our result has more information. We can conclude the method is effective.

VI. CONCLUSION

Extension model based on linguistic 2-tuple representation is proposed in this paper. The model makes a bridge between manipulation of perceptions and manipulation of measurements. In the linguistic hierarchy structure of multi-granularity linguistic variables, two measurements are given. So transformation between multi-granularity linguistic variables becomes easier. The vagueness information losing problem can be solved. Another important innovation is that the fuzzy sets or membership functions of linguistic variables need not be known. This change conforms to reality.

Some computing techniques based on the extension model are given for decision making. The aggregation operators for linguistic labels directly compute on numerical measurements. In terms of future research, the proposed approach can be extended to multiple attribute GDM problems where decision-makers' preferences are in the form of uncertain linguistic variables or non-homogeneous information.

However, our research is based on the premise that labels are symmetrical and uniformly distributed. If we want to apply for unbalanced linguistic variables, new research is needed.

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References

[1] F. Herrera et al., “Computing with words in decision making: foundations, trends and prospects,” Fuzzy Optimization and Decision Making, vol. 8, no. 4, pp. 337-364, Dec, 2009.

[2] Y.-P. Jiang, Z.-P. Fan, and J. Ma, “A method for group decision making with multi-granularity linguistic assessment information,” Information Sciences, vol. 178, no. 4, pp. 1098-1109, 2008.

[3] M. Espinilla, J. Liu, and L. Martínez, “An extended hierarchical linguistic model for decision-making problems,” Computational Intelligence, vol. 27, no. 3, pp. 489-512, 2011.

[4] L. Martinez, and F. Herrera, “An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges,” Information Sciences, vol. 207, no. 0, pp. 1-18, 2012.

[5] R. M. Rodríguez, and L. Martínez, “An analysis of symbolic linguistic computing models in decision making,” International Journal of General Systems, vol. 42, no. 1, pp. 121-136, 2013/01/01, 2012.

[6] Z. Zhang, and C. Guo, “A method for multi-granularity uncertain linguistic group decision making with incomplete weight information,” Knowledge-Based Systems, vol. 26, no. 0, pp. 111-119, 2012.

[7] W. Pedrycz, and A. V. Vasilakos, “Linguistic models and linguistic modeling,” IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics, vol. 29, no. 6, pp. 745-757, Dec, 1999.

[8] O. Cordon, and F. Herrera, “A proposal for improving the accuracy of linguistic modeling,” IEEE Transactions on Fuzzy Systems, vol. 8, no. 3, pp. 335-344, 2000.

[9] J. Lawry, “An alternative approach to computing with words,” International Journal of Uncertainty,Fuzziness and Knowledge-based Systems(Suppl), vol. 9, pp. 3-16, 2001.

[10] O. Cordón, F. Herrera, and I. Zwir, “A hierarchical knowledge-based environment for linguistic modeling: models and iterative methodology,” Fuzzy Sets and Systems, vol. 138, no. 2, pp. 307-341, 2003.

[11] J. A. Roubos, and R. Babuska, “Comments on the benchmarks in "A proposal for improving the accuracy of linguistic modeling" and related articles,” IEEE Transactions on Fuzzy Systems, vol. 11, no. 6, pp. 861-865, Dec, 2003.

[12] G. Fu, “A fuzzy optimization method for multicriteria decision making: An application to reservoir flood control operation,” Expert Systems with Applications, vol. 34, no. 1, pp. 145-149, 2008.

[13] K. Anagnostopoulos, H. Doukas, and J. Psarras, “A linguistic multicriteria analysis system combining fuzzy sets theory, ideal and anti-ideal points for location site selection,” Expert Systems with Applications, vol. 35, no. 4, pp. 2041-2048, 2008.

[14] F. Herrera, and E. Herrera-Viedma, “Linguistic decision analysis: steps for solving decision problems under linguistic information,” Fuzzy Sets and Systems, vol. 115, no. 1, pp. 67-82, 2000.

[15] F. Herrera, E. Herrera-Viedma, and L. Martinez, “A fusion approach for managing multi-granularity linguistic term sets in decision making,” Fuzzy Sets and Systems, vol. 114, no. 1, pp. 43-58, 2000.

[16] F. Herrera, and L. Martinez, “A model based on linguistic 2-tuples for dealing with multigranular hierarchical linguistic contexts in multi-expert decision-making,” IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol. 31, no. 2, pp. 227-234, 2001.

[17] E. Herrera-Viedma et al., “A model of fuzzy linguistic IRS based on multi-granular linguistic information,” International Journal of Approximate Reasoning, vol. 34, no. 2–3, pp. 221-239, 2003.

[18] Z. S. Xu, “A method based on linguistic aggregation operators for group decision making with linguistic preference relations,” Information Sciences, vol. 166, no. 1–4, pp. 19-30, 2004.

[19] J.-H. Wang, and J. Hao, “A new version of 2-tuple fuzzy linguistic representation model for computing with words,” IEEE Transactions on Fuzzy Systems, vol. 14, no. 3, pp. 435-445, 2006.

[20] E. Herrera-Viedma et al., “A fuzzy linguistic model to evaluate the quality of Web sites that store XML documents,” International Journal of Approximate Reasoning, vol. 46, no. 1, pp. 226-253, 2007.

[21] R. R. Yager, “Aggregation of ordinal information,” Fuzzy Optimization and Decision Making, vol. 6, no. 3, pp. 199-219, Sep, 2007.

[22] Z. P. Fan et al., “An approach to group decision-making with uncertain preference ordinals,” Computers and Industrial Engineering, vol. 58, no. 1, pp. 51-57, Feb, 2010.

[23] Y. Xu, and H. Wang, “Approaches based on 2-tuple linguistic power aggregation operators for multiple attribute group decision making under linguistic environment,” Applied Soft Computing, vol. 11, no. 5, pp. 3988-3997, 2011.

[24] L. Mart??nez, and F. Herrera, “An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges,” Information Sciences, vol. 207, no. 0, pp. 1-18, 2012.

[25] F. Herrera, and L. Martinez, “A 2-tuple fuzzy linguistic representation model for computing with words,” IEEE Transactions on Fuzzy Systems, vol. 8, no. 6, pp. 746-752, 2000.

[26] Y. Tang, and J. Zheng, “Linguistic modelling based on semantic similarity relation among linguistic labels,” Fuzzy Sets and Systems, vol. 157, no. 12, pp. 1662-1673, 2006.

[27] Z. S. Xu, “Eowa and eowg operators for aggregating linguistic labels based on linguistic preference relations,” International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems, vol. 12, no. 6, pp. 791–810, 2004.

[28] Z. S. Xu, “Induced uncertain linguistic OWA operators applied to group decision making,” Information Fusion, vol. 7, no. 2, pp. 231-238, 2006.

[29] R. R. Yager, “Centered OWA operators,” Soft Computing, vol. 11, no. 7, pp. 631-639, May, 2007.

[30] Z. Chen, and D. Ben-Arieh, “On the fusion of multi-granularity linguistic label sets in group decision making,” Computers and Industrial Engineering, vol. 51, no. 3, pp. 526-541, 2006.

[31] L. A. Zadeh, “The concept of a linguistic variable and its application to approximate reasoning—I,” Information Sciences, vol. 8, no. 3, pp. 199-249, 1975.

[32] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—II," Information Sciences, vol. 8, no. 4, pp. 301-357, 1975.

[33] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning-III," Information Sciences, vol. 9, no. 1, pp. 43-80, 1975.

[34] Y. Dong, Y. Xu, and S. Yu, "Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model," IEEE Transactions on Fuzzy Systems, vol. 17, no. 6, pp. 1366-1378, 2009.

[35] L. Martinez, D. Ruan, and F. Herrera, "Computing with words in decision support systems: an overview on models and applications," International Journal of Computational Intelligence Systems, vol. 3, no. 4, pp. 382-395, Oct, 2010.

[36] S.-L. Chang, R.-C. Wang, and S.-Y. Wang, "Applying a direct multi-granularity linguistic and strategy-oriented aggregation approach on the assessment of supply performance," European Journal of Operational Research, vol. 177, no. 2, pp. 1013-1025, 2007.

[37] Z. S. Xu, "Deviation measures of linguistic preference relations in group decision making," Omega, vol. 33, no. 3, pp. 249-254, 2005.

[38] Z. S. Xu, "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment," Information Sciences, vol. 168, no. 1–4, pp. 171-184, 2004.

[39] S. A. Orlovsky, "Decision-making with a fuzzy preference relation," Fuzzy Sets and Systems, vol. 1, no. 3, pp. 155-167, 1978.