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Cooperative and Hierarchical Fuzzy MPC for Building Heating Control

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***Abstract*— A combined cooperative and hierarchical control structure utilizing Fuzzy Model Predictive Control (FMPC) for building heating is introduced. The structure comprises three types of Model Predictive Controllers (MPC): For different independent zones of the building FMPCs optimize the fast acting input variable fan coils (FC) while a global linear MPC optimizes the slowly acting thermally activated building systems (TABS). Cooperation between these two groups of controllers is guaranteed by an inter-sample iteration. This cooperative structure acts as master in a hierarchical structure, where the slave is a mixed-inter MPC (MI-MPC) in the supply level. While the cooperative structure ensures user comfort in the building, the MI-MPC optimizes monetary costs of heat supply. This structure allows for decoupled and independent modeling of FMPCs, simple incorporation of the coupling input TABS, and decoupled design of the supply level control. A discussion on stability and sub-optimality of the control structure is given. A simulation of a large office building incorporating disturbances of ambient temperature, radiance, and occupancy demonstrates the performance of the proposed concept.**

***Index Terms*— fuzzy MPC; TS-fuzzy system modelling; building climate control; cooperative MPC; hierachical MPC.**

I. Introduction

The application of standard control designs to large nonlinear complex processes requires specific modeling techniques. One possible approach presented here is the division into smaller locally linear sub-processes with Fuzzy modeling. This allows for application of linear control structures, and moreover, individual local sub-processes can be added or removed without affecting the model or controller parameters of the other sub-processes. However, if there exist inputs acting simultaneously on all sub-processes, the individual local linear models are no longer decoupled. In order to actively compensate these input couplings an additional global controller is proposed. All controllers are chosen as Model Based Predictive Controllers (MPC), as it is an effective way to handle measurable or predictable disturbances, large numbers of inputs and outputs, and to cope with constraints, [4].

The resulting control scheme is a hierarchical and cooperative Fuzzy MPC (FMPC) structure which enables a modular control design. Parallel FMPCs, one for each sub-process, optimize their specific output by considering the output of the global MPC as a known disturbance. The global MPC optimizes only the coupling inputs based on a simplified global model and the known disturbances of all FMPC-inputs. The principles of such a cooperative structure are analyzed in [16] and [12] where a cooperative distributed predictive control design is shown, and in [14] sub-optimality and stability of such MPC is discussed. Those results indicate that good performance can be achieved even if global optimality is lost.

Additionally, if there exists a hierarchical split of the sub-processes in a higher (HiLe) and lower (LoLe) level, the introduced structure provides a methodology to decouple them. Comparable hierarchical structures are discussed in [13] and an example for building heating is given in [7].

The concept of cooperative and hierarchical FMPC was applied to building heating control of large modern buildings. In this case, the formulated HiLe is the building itself with several separately controllable zones coupled by one common input – the thermally activated building systems (TABS). Since TABS represents a slow dynamic heating system, one global MPC is employed for compensating this coupling between individual zones. For each zone the objective is to maximize the user's comfort by minimizing heating energy. Therefore, a separate FMPC is optimizing the zone's room temperature based on a network of local linear models (LLM). These networks are identified by a data-driven approach, as introduced and shown in [5]. On the other hand, a hierarchically decoupled LoLe MPC ensures the energy supply at minimal monetary costs [7]. All MPCs are subject to constraints.

The remainder of the paper is structured as follows: In Sec. II the data-driven system identification is introduced. The cooperative control structure is presented in Sec. III succeeded by the Sec. IV where the cooperative MPC formulation is given. The prove of concept is done for a demonstration building. The simulation results are shown in Sec. V-C before the paper ends with a Conclusion and an outlook to further research.

II. Fuzzy System Model

A. General Approach

A data-driven system identification (black-box) approach for modeling nonlinear dynamic systems by LLM networks is chosen to model the individual building zones. This approach is an efficient way of modeling globally non-linear systems. Since the validity of the resulting LLMs is confined to certain regions within the so-called partition space, this model class is also named Takagi-Sugeno Fuzzy models, [17]. For each zone a separate LLM network is identified utilizing the linear model tree algorithm (LOLIMOT, [10]). In addition to the control input (fan coils – FC), LOLIMOT utilizes disturbances as further inputs: weather forecast, radiance,

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occupancy information, and the control input of the global MPC (TABS). Detailed information on thermal modeling of buildings can be found in [6] and [8]. Information on FMPC zone selection is given in [15], [22].

A single linear model for the global MPC is identified comprising the slow coupling dynamics of the TABS input. Disturbance inputs are the same as above, except for TABS, which is replaced by the sum of all FCs from the individual zones.

B. Takagi-Sugeno (TS) Models

Motivated by results of classical linear MPC theory [4], linear model structures are used for controller design. Nonlinear dynamical systems can often be represented by a nonlinear autoregressive model structure with exogeneous input (NARX), [1]. Such NARX systems may be approximated by TS models, [17]. The basic element of a fuzzy system is a set of fuzzy inference rules. In general, each inference rule consists of two elements: the IF-part, called the antecedent of a rule, and the THEN-part, called the consequent of the rule. For each rule $\mathbf{R}^j$ the following structure holds:

$$\begin{aligned}\mathbf{R}^j :& \text{ IF } \zeta_1 \text{ is } A_1^j \text{ and } \dots \zeta_m \text{is } A_m^j \\ &\text{THEN } y^j(k+1) = \sum_{i=1}^{n_y} a_i^j y(k-i+1) \\ &+ \sum_{i=1}^{n_u} b_i^j u_l(k-i-n_d+1) + c^j.\end{aligned} \tag{1}$$

Here $j = \{\text{FC}\}$, $\zeta = [\zeta_1, \dots, \zeta_m]$ is the vector of input fuzzy variables, and $A_1^j, \dots, A_m^j$ are the forgoing fuzzy sets or regions for the j-th rule $\mathbf{R}^j$ with corresponding membership functions $\mu_{A_1}^j, \dots, \mu_{A_m}^j$, with $\mu_{A_j}(\zeta_j) \mapsto [0, 1]$, for $j = 1, \dots, m$, [6], [10]. The elements of the fuzzy vector are usually a subset of the past input and outputs, [1]:

$$\begin{aligned}\zeta \in &\{y(k), \dots, y(k-n_y+1), u_l(k-n_d), \\ &\dots, u_l(k-n_u-n_d+1)\}.\end{aligned} \tag{2}$$

The overall output of the TS fuzzy model can be written as $y(k+1) = \sum_{j=1}^{r} \omega^j(\zeta) y^j(k+1)$, where r denotes the number of rules. The degree of fulfillment if the j-th rule can be computed using the product operator: $\mu^j(\zeta) = \prod_{i=1}^{m} \mu_{A_i}^j(\zeta_i)$, furthermore, the normalized degree of fulfillment can be computed as: $\omega^j(\zeta) = \frac{\mu^j(\zeta)}{\sum_{i=1}^{r} \mu^i(\zeta)}$. If all consequents of the rules have identical structure, the TS model can be expressed as a pseudo-linear model with input-dependent parameters:

$$\begin{aligned}y(k+1) = &\sum_{i=1}^{n_y} a_i(\zeta) y(k-i+1) \\ &+ \sum_{i=1}^{n_u} b_i(\zeta) u_l(k-i-n_d+1) + c(\zeta),\end{aligned} \tag{3}$$

where:

$$\begin{aligned}a_i(\zeta) = \sum_{j=1}^{r} \omega^j(\zeta) a_i^j,\;\; b_i(\zeta) = \sum_{j=1}^{r} \omega^j(\zeta) b_i^j, \\ c(\zeta) = \sum_{j=1}^{r} \omega^j(\zeta) c^j.\end{aligned} \tag{4}$$

III. Structure of the Cooperative and Hierarchical Fuzzy Control

A. Theoretical Background

Cooperative distributed control is a possible way to facilitate implementation and optimization, especially if different sub-processes have different control variables but the same optimization target. Plant wide control has traditionally been implemented in a decentralized way, where each subsystem is controlled independently and network interactions are treated as disturbances to the local subsystem, [12], [16]. Each controller optimizes a plant wide cost function, e.g., the centralized controller objective. Distributed optimization algorithms are used to ensure a decrease in the plant wide objective function at each intermediate iteration. Under cooperative control, plant wide performance converges to the Pareto optimum, providing similar performance as centralized control. However, cooperative control is a form of suboptimal control for the plant wide control problem. Hence, stability is deduced from suboptimal control theory, [14]

B. Control Structure

In Fig.1 the control concept is shown. Maximization of user comfort is the goal of HiLe. All $\text{FMPC}_j, \forall j = 1, \dots, N$ optimize the same cost function by controlling different independent variables u_{FC_j}. In order to guarantee the reference indoor temperature $\vartheta_{\text{ref}}^{\text{in}}$, a global MPC with an underlying global linear building model guarantees the basic level of reference temperature with the TABS, u_{TABS}. This global linear model comprises only slow dynamics of TABS and is not able to compensate fast disturbances. The purpose of the global MPC is to set a temperature level for the whole building, not for special zones. Therefore, each FMPC is able to heat or cool their own zone in a quasi-local way with their control variable u_{FC_j}.

In this cooperative FMPC structure, for the global MPC $u_{\text{FC}_j}^{\text{fuzzy}} = z_{\text{FC}_j}^{\text{global}}$, and the other way round $u_{\text{TABS}}^{\text{global}} = z_{\text{TABS}}^{\text{fuzzy}}$, holds. Between each time step a number of iterations can be defined for calculating stationary solutions for u_j, $j = 1, \dots, N$ and u_{global}, (see III-C). This structure allows for decoupled modeling and controller design in each building zone, and simple compensation of coupling dynamics by TABS.

C. Multi Sampling Rate & Communication

The sampling time is supposed with one hour in the FMPCs, and with three hours in the global MPC ($T_s^{\text{global}} = 3\, T_s^{\text{fuzzy}}$). Hence, a multi sampling rate control results, comparable to [2] . For a correct energy balance down-sampling

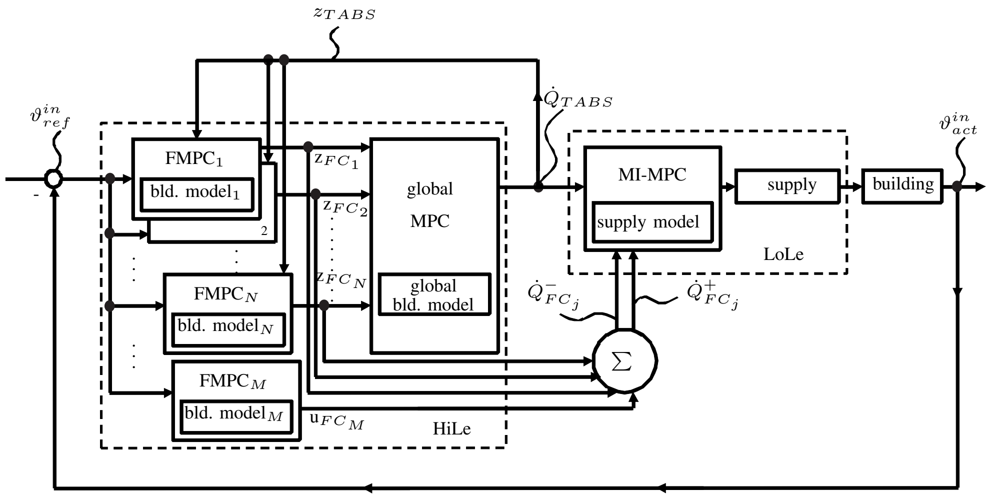

Fig. 1. Concept of hierarchical decoupled MPCs with cooperative FMPCs plus global MPC as master (HiLe) and MI-MPC as slave (LoLe)

must be done according to $z^{\text{global}}_{\text{FC}_j}(k) = \frac{1}{3}\sum_{\nu=1}^{3} u^{\text{fuzzy}}_{\text{FC}_j} 3(k-1)+\nu$, k being the counter of the the smaller sampling time. Since the prediction horizon of the global MPC is larger than the control horizon of the FMPCs, the disturbance vector $z^{\text{global}}_{\text{FC}_j}$ has to be extrapolated with the constant last value. Each value of $u^{\text{global}}_{\text{TABS}}$ is up-sampled by 3 without filtering to define the disturbance vector $z^{\text{fuzzy}}_{\text{TABS}}$. At each T^{fuzzy}_s all demands are handed over to the LoLe controller.

D. Hierarchical Control Structure

On the right side of Fig.1 the underlying supply level (LoLe) is shown. Within the supply level, minimization of monetary costs is another important control goal. Hence, in LoLe a Mixed-Integer MPC (MI-MPC) is used for optimizing energy costs, [3]. This specific control structure, where the control variable of one MPC (i.e. HiLe) is the reference value of the underlying MPC (i.e. LoLe) is described by a decoupled hierarchical MPC structure. More details on such control structures can be found in [13], an application example is given in [7].

IV. Cooperative Fuzzy MPC Design

In Subsec.IV-A and , respectively, the cost functions of the global MPC and the FMPCs are formulated. Subsequently, in Subsec.IV-C the coordinated fuzzy MPC is formulated, and in the last Subsec.IV-D a review of suitable papers treating stability is given.

A. Constrained Cooperative Fuzzy MPC

Standard MPC formulations are well known and given in e.g. [4]. In this paper an LLM network approximates the non-linear dynamics. To avoid non-convex optimization, a set of LLMs is extracted from each TS fuzzy model, which is then utilized for MPC design, [1]. Stochastic disturbances such as weather forecast, radiance, and occupancy information, are important for HiLe modeling, [22]. As already mentioned, the global MPC and the FMPCs control the same output ϑ^{in}.

The optimization problem is formulated as follows:

$$J^\star = \min_U J(U,t) = \alpha_i \cdot \|\vartheta^{\text{in}}_{\text{act}}(U,t) - \vartheta^{\text{in}}_{\text{ref}}\|_2^2 + \beta_i \cdot \sum_i \dot{Q}_i^2$$

$$\text{s.t.} \quad \vartheta^{\text{in}}_{\min} \leq \vartheta^{\text{in}}(t) \leq \vartheta^{\text{in}}_{\max}$$
$$u_{i,\min} \leq u_i(t) \leq u_{i,\max}$$

for $U = \{u_i\}$ with $i = \text{FC}$ in case of the FMPCs and $i = \text{TABS}$ for the global MPC. Moreover, α_i and β_i are weights of the minimization criterion.

B. Fuzzy MPC formulation

Assume that the MISO TS fuzzy model can be regarded as a multi-variable linear parameter-varying system (according to equation (3)), [1],

$$y(k+1) = \sum_{i=1}^{n_y} \mathbf{F}_i(\zeta) y(k-i+1) + \sum_{i=1}^{n_u} \mathbf{H}_i(\zeta)\mathbf{u}(k-i+1) + \mathbf{c}(\zeta), \quad (5)$$

where the parameter matrices depend only on the current operating point ζ and are calculated as: $\mathbf{F}_i(\zeta) = \sum_{j=1}^{r} \mathbf{W}^j(\zeta)\mathbf{F}_i$, $i = 1,\ldots,n_y$, $\mathbf{H}_i(\zeta) = \sum_{i=1}^{r} \mathbf{W}^j(\zeta)\mathbf{H}_i^j$, $i = 1,\ldots,n_u$, and $\mathbf{c}(\zeta) = \sum_{i=1}^{r} \mathbf{W}^j(\zeta)\mathbf{c}^j$, where $\mathbf{W}^j$ is the diagonal weight matrix, which entries are normalized degrees of fulfillment of the j-th rule. n_y is the number of parameters related to the output and n_u are the number of parameters related to the inputs (control input and disturbances).

The formulation of the FMPC depends on linear models, which are obtained by interpolating the parameters of the local models in the TS model, see system (4) and equation (3); [17]. The following problem is formulated for one single FMPC and can be extended to all FMPCs.

The goal is to locally represent a TS fuzzy model by a linear state-space model, on the basis of [4]

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}_k\mathbf{x}(k) + \mathbf{B}_k\mathbf{u}(k) + \mathbf{E}_k\mathbf{z}(k) \\ y(k) &= \mathbf{C}_k\mathbf{x}(k)\end{aligned} \tag{6}$$

in which system matrices $\mathbf{A}_k, \mathbf{B}_k, \mathbf{C}_k$ and $\mathbf{E}_k$ implicitly depend on the operating point ζ and are computed from matrices $\mathbf{F}_i$ and $\mathbf{H}_i$, see e.g. [4]. In order to predict the trajectory of the controlled output, system (5) can be used. The linear extracted state-space models can be augmented to provide offset free control. Let $\boldsymbol{\xi}(k)$ denote the augmented state vector at time step k, then future process outputs are computed from the following matrix equality

$$\hat{\mathbf{Y}} = \mathbf{F}(k)\boldsymbol{\xi}(k) + \Phi_u(k)\Delta\mathbf{U} + \Phi_z(k)\Delta\mathbf{Z}, \tag{7}$$

where the matrices $\mathbf{F}$, Φ_u and Φ_z are computed as described in [4]. They have to be calculated at each time step and are then used in the following quadratic program for determining optimal future control sequences:

$$\begin{aligned}J^\star = \min_{\Delta U_j} J(U_j) =& (\mathbf{Y}_r - \hat{\mathbf{Y}}_j)^T\mathbf{Q}_j(\mathbf{Y}_r - \hat{\mathbf{Y}}_j) \\ &+ \Delta\mathbf{U}_j^T\mathbf{R}_j\Delta\mathbf{U}_j\end{aligned} \tag{8}$$

$$\begin{aligned}\text{s.t.}\quad Y_{\min,j} &\le \hat{Y}_j \le Y_{\max,j}, \\ U_{\min,j} &\le U_j \le U_{\max,j}, \\ \Delta U_{\min,j} &\le \Delta U_j \le \Delta U_{\max,j}.\end{aligned}$$

$\mathbf{Y}_r$ is the reference trajectory for all FMPCs, $U_j = \{u_{i,j}\}$, $i = \text{FC}$, j specify the optimizing FMPC $\forall j = 1,\ldots,M+N$, $\mathbf{Q}_j$ and $\mathbf{R}_j$ are positive semi-definite weighting matrices which allow for tuning, depending on the considered FMPC. The approach can be summarized in the following steps:

1) Use the obtained linear model 6 at the current operating point $\zeta(k)$ and compute the control signal $\mathbf{u}(k)$ for the whole control horizon.
2) Simulate the TS fuzzy model over the prediction horizon.
3) Freeze the TS fuzzy model along each point in the predicted operating point trajectory ζ(k+i) and obtain to parameters of (6), for $i = 1,\ldots,N_p$.
4) Use calculated (6) of step before, $i = 1,\ldots,N_p$ to construct MPC matrices $\mathbf{F}$, Φ_u and Φ_z and compute the new control sequence $\mathbf{u}(k)$.

Steps 3 and 4 are repeated until $\mathbf{u}$ converges, [1], [5], [17].

C. Formulation of Full Optimization Problem

The cooperative problem for FMPCs and coupled global MPC is iteratively solved within each control time step.
As previously defined, FMPCs are enumerated by $j = 1,\ldots,N$, the global MPC is denoted by $N+1$, sampling instances are enumerated by $k = 1,\ldots,M$, and iterations of the cooperative problem solution are denoted by $i = 1,\ldots,P$.

The cooperative FMPC cost function can be defined by using similar notation as eq.(8):

$$\begin{aligned}J^\star_{j,k} = \min_{\Delta U_j} J(U_j) =& (\mathbf{Y}_r - \hat{\mathbf{Y}}_j)^T_{k,i}\mathbf{Q}_j(\mathbf{Y}_r - \hat{\mathbf{Y}}_j)_{k,i} \\ &+ (\Delta\mathbf{U}_j^T\mathbf{R}_j\Delta\mathbf{U}_j)_{k,i}\end{aligned} \tag{9}$$

$$\begin{aligned}\text{s.t.}\quad Y_{\min,j} &\le \hat{Y}_j \le Y_{\max,j}, \\ U_{\min,j} &\le U_j \le U_{\max,j}, \\ \Delta U_{\min,j} &\le \Delta U_j \le \Delta U_{\max,j}.\end{aligned}$$

The following assumption is straightforward:

Assumption 1: All disturbances $z_{l,i}$, $l = 1,\ldots,n_z$, where n_z is the number of disturbances, stay constant during all iterations i at time step k.
Hence $z = z_{l,k}$ holds, instead of using $z_{\text{TABS},i}$ and $z_{\text{FC},i}$.

According to Subsec.IV-A the recursive state-space update for the i-th iteration at time step k is given by the following set of recursive equations:

$$\begin{aligned}\mathbf{x}_i^j(k+1) &= \mathbf{A}_k^j\mathbf{x}_i^j(k) + \mathbf{B}_k^j\mathbf{u}_i^j(k) + \mathbf{E}_k^j\mathbf{z}_i^j(k) \\ y_i^j(k) &= \mathbf{C}_k^j\mathbf{x}_i^j(k) \\ \mathbf{z}_i^j(k) &= \left(z^{N+1}_{\text{TABS},i-1}, z^j_{2,k}, \ldots, z^j_{n_z,k}\right)^T\end{aligned} \tag{10}$$

$$\begin{aligned}\mathbf{x}_i^{N+1}(k+1) &= \mathbf{A}_k^{N+1}\mathbf{x}_i^{N+1}(k) + \mathbf{B}_k^{N+1}\mathbf{u}_i^{N+1}(k) \\ &\quad + \mathbf{E}_k^{N+1}\mathbf{z}_i^{N+1}(k) \\ y_i^{N+1}(k) &= \mathbf{C}_k^{N+1}\mathbf{x}_i^{N+1}(k) \\ \mathbf{z}_i^{N+1}(k) &= \left(z^j_{\text{FC},i}, z^{N+1}_{2,k}, \ldots, z^{N+1}_{n_z,k}\right)^T\end{aligned} \tag{11}$$

Note that for $i = 1$ the initial value for $z^{N+1}_{\text{TABS},i-1}$ in the FMPCs state-space update (10) should be chosen constant and at the current operating point.

D. Stability of Fuzzy MPC

Stability analysis and stabilization for TS-fuzzy systems are mainly based on Lyapunov criteria. A so-called fuzzy Lyapunov-function is able to guarantee such stability, [18]. In [18], the fuzzy Lyapunov function is defined by fuzzy blending of quadratic Lyapunov-functions. Another way of stability analysis for fuzzy model based control has been applied by [9], where asymptotic stability can be shown for open-loop bounded-input-bounded-output stable systems. More recently, [20] introduced an approach to guarantee both stability and transient performance of the closed-loop system using piecewise quadratic Lyapunov funtions (PLFs) in order to reduce the conservatism those controllers based in common Lyapunov functions. Furthermore, in [21] a three-step methodology is proposed to obtain a robust input-to-state stable constraint FMPC system taking model-reality discrepancy and unknown disturbances into account. [11] shows that asymptotic stability in the sense of Lyapunov is guaranteed if the derivative of the Lyapunov function is negative-definite for all fuzzy rules. For further stability analysis for the cooperative FMPC structure presented in Sec. IV two steps will be necessary. Firstly, stability of one FMPC has to be guaranteed as proposed by [20] using PLFs and secondly the analysis must be extended to the cooperation of two FMPCs, as generally introduced for linear MPCs in [19].

V. Simulation Results: Office Building

A. Building Model

The demonstration building is the 27.000 m^2 University building in the center of Salzburg, Austria, which has five floors above ground containing several large and numerous smaller meeting rooms, offices and lecture rooms. For this study, the third floor comprising about 400 office rooms is considered which is split into two zones. For the simulation study the zone north-east (NE) and the zone south-west (SW) constitute the individual zones. The energy input for this floor is supplied by fast FC inputs and by a common slow TABS input. While the energy input from the FCs can be controlled for each zone separately, the common input via the activated concrete core (TABS) constitutes the coupling. Hence, for each zone an FMPC is formulated according to Sec. IV-A, whereas a global MPC computes the optimal input for TABS.

The considered period for simulation was March 2012. Measurements for this period have been obtained from records of the building management system. These data have been used for model identification introduced in Sec. II. Common system disturbances are the outside temperature and the occupancy profile. For the SW zone the radiance is an additional relevant disturbance input. The historic outside temperature and the radiance was provided by the ZAMG[1] Austria.

B. Design of Cooperative MPC

In this work only simulations results of the HiLe, the cooperative MPC, are shown. Results of the hierachical structure can be found in [7]. This cooperative structure is built up of two FMPCs and one classical linear MPC. The FMPCs are for zone NE and zone SW, while the classical MPC controls the basic temperature level over the whole floor, manipulated by TABS. In Fig. 2 mentioned topology of control zones is shown.
The underlying TS-fuzzy models for the FMPCs are built with LOLIMOT. From there parameters can be calculated and used to fit an ARX model for building the TS-fuzzy models as formulated in Sec. II by equations (3) and (4). For each non-linear model a net of 3 LLMs is obtained, each with 2 partitioning variables, ambient temperature ϑ^{amb} and supply heat.

C. Simulation Results

Results of a closed-loop simulation with the proposed control scheme are shown in Fig. 3. 13 days of the considered period are plotted. Constraints are shown in green dashed lines. The first plot shows the reference trajectory and the indoor temperature ϑ^{in} for the global MPC. The second

[1] Zentralanstalt für Meteorologie und Geodynamik - The central institute for meteorology and geodynamics

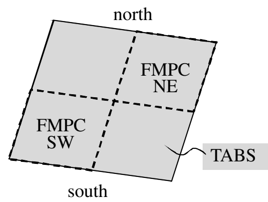

Fig. 2. Scheme of cooperative MPCs demonstrated on one exemplary level (3rd floor) of the demonstration building.

plot depicts the corresponding control variable $u_{\mathrm{global}} = \{\mathrm{TABS}\}$ of the global MPC. Here, the large dead time of TABS, approximately 36 hours, can be perfectly seen. In plot 3 and plot 4 the results of the two FMPCs can be seen, where the red lines are associated with NE and the black ones with SW. The $\vartheta_j^{\mathrm{in}}$ of the FMPCs (plot 3) are the actual indoor temperatures of the respective zones. Both excellent output tracking and disturbance rejection can be observed.

In the fourth plot the different zone dynamics become apparent, as the individual control inputs $\mathrm{FMPC_{NE}}$ and $\mathrm{FMPC_{SW}}$ show different behavior. Note that input constraints become active during the set-point changes for $\mathrm{FMPC_{NE}}$. The fifth plot shows the ambient temperature until hour 210, where an (artificial) step in the disturbance occurs. All three MPCs slightly reduce the demanded supply temperatures for FCs as well as for the TABS.

In order to evaluate the influence of the inter-sample iteration, the control scheme (10) and (11) was run with different numbers of iterations P. Tab. I lists the gained Mean Squared Error (MSE) of the Δu_j as formulated in (12). The result shows that with only four iterations the MSE is reduced to almost the same level as for the asymptotic value (approximated by $P = 64$). Note that in the MSE of the control error ($\vartheta_{\mathrm{ref}}^{\mathrm{in}} - \vartheta_{\mathrm{act}}^{\mathrm{in}}$) did not show any significant changes for variation of P.

$$MSE_j = \frac{1}{M-1}\sum_{n=2}^{M}(u_{j,n} - u_{j,n-1})^2 \qquad (12)$$

for $j = 1,..,N,N+1$ MPCs and M simulation steps

TABLE I
Comparison of the MSE of the FMPC control variable for different number of iterations P

P	$\mathrm{MSE_{global}}$	$\mathrm{MSE_{FMPC_{NE}}}$	$\mathrm{MSE_{FMPC_{SW}}}$
0	0.603	1.56	0.269
4	0.467	1.47	0.270
64	0.461	1.46	0.270

VI. Conclusion

A cooperative and hierarchical constrained MPC control structure for building heating control has been presented. The cooperative part consists of several FMPCs, each of them dedicated to control one building zone by optimizing the fast acting input variable FC. The dynamics of the individual zones are only coupled by another input, the slowly acting

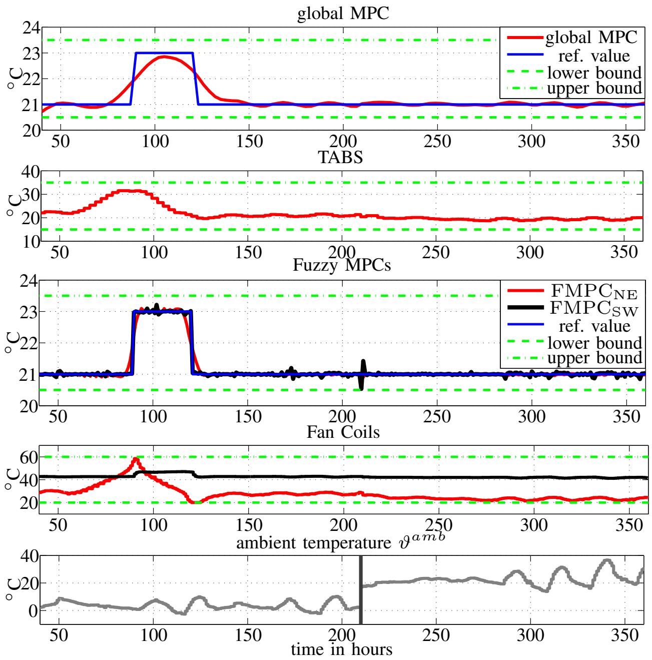

Fig. 3. Cooperative Fuzzy MPCs with 4 iterations between each time step; reference step and disturbance step ϑ^{amb}

TABS. For optimizing this input a global linear MPC is designed based on a simple linear model of the overall building. Cooperation between FMPCs and global MPC is guaranteed by an inter-sample iteration, where the control variables of the FMPCs are the disturbances to the global MPC and vice versa. The results of the simulation study given in Tab. I clearly show that this iteration converges fast and asymptotically to the stationary end value. Therefore, it is successfully demonstrated, that a Fuzzy MPC structure can be effectively integrated in a cooperative controller scheme.

The hierarchical part is given by the cooperative structure as master and a supply level MI-MPC as slave. Since the master feeds the overall heating demand of the building to the slave, the MI-MPC is completely decoupled from the building dynamics and focuses on the optimization of the monetary costs of the heat supply.

This structure ensures a straightforward and simple controller design, since all MPCs can be designed based on independent and decoupled models. Only the simple global linear MPC design has to consider the coupling effect of the slow TABS input. Furthermore, if zones change in their thermal behavior, are added or removed, only the affected FMPC and the global MPC have to be re-designed, all other parts of the scheme can be adopted unchanged. This property will be preserved for similar processes.

Although a review on specific literature has been included, a formal proof of stability and convergence of the proposed scheme is still to be completed. This is a topic of current research as well as the extension of the simulation to both heating and cooling operation.

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