

Development and Implementation of Fuzzy, Fuzzy PID and LQR Controllers for an Roll-plane Active Hydraulically Interconnected Suspension

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Abstract— A new active Hydraulically Interconnected Suspension (HIS) has been developed to compensate the limitations of conventional active suspensions such as expensive cost and high energy consumption. In this paper, the mechanism of proposed active HIS system has been briefly introduced. Fuzzy Logic Control, Fuzzy proportional-integral-derivative (PID) control and optimal linear quadratic regulator (LQR) theory have been adopted to control vehicle body's roll motion. A combination of a half-car model and the active suspension model is then derived through their mechanical-hydraulic coupling in the cylinders for the model based LQR control. Three controllers have been developed and implemented in Simulink. Two different road excitations have been used to validate the robustness of the designed controllers. The effectiveness of all these three controllers has been verified by the simulation results with considerable roll angle reductions, and the Fuzzy PID controller shows better effect and stability than other two controllers.

I. INTRODUCTION

IN terms of mechanical definition, vehicular suspension system refers to a combination of spring and damper (shock absorbers) components connecting between wheel and vehicle body to dissipate a vibration due to an external disturbance[1]. Vehicle suspension can be divided to three categories regard to passive, semi-active and active. Passive suspension is constructed basically by the combination of the spring and damper component. Whilst semi-active is inherited from passive, but with adjusting stiffness and damper can be improved vehicle performance in both riding comfort and handling performance[2]. However, compromising between riding comfort and directional stability of the vehicle in dynamic is superior to passive and semi-active which cannot provide a sufficient large amount of external energy to stabilize a motion of vehicle's body. Active suspension is referred to a suspension which can actively control a motion of the vehicle body by providing external energy. Implementing active suspension can satisfy a vehicle driving standard as riding comfort, handling performance as well ensuring safety issues. There are two concepts which are proposed to develop and design active suspension namely independent active suspension and interconnected active suspension. The concept of independent active suspension is to individually generated control forces in each actuator[2]. The advantage of this concept is easy to manufacture, implement as well as derive a control scheme. However, there are still many challenges such as high implementing cost and consuming a large

amount of energy to activate the actuators. On the other hand, interconnected suspension which is referred to interconnected pneumatic or hydraulic energy system between each actuator to produce active control force[3]. A benefit of interconnected structure is to reduce manufacturing cost and less energy consumption of active suspension as appose to independent active suspension.

The novel of Demand Dependent Active Suspension (DDAS) is proposed by Nong Zhang etc. [4] with a primary purpose of compensating the drawback of the conventional active suspension such as high implemented cost and power consumption. The DDAS system is constructed by four double directional hydraulic actuators which are hydraulically interconnected. The system is controlled by a compact manifold and a pressure control unit. The manifold plays a key role by switching between each control mode such as roll, bounce, pitch etc. by changing the direction of pressure in the manifold to each actuator[4]. While the control unit will generate a restoring force to actively tilt the vehicle against its motion. Evolving from the DDAS system, the developed roll plane active HIS is a subsection of DDAS, which is a safety-orientated, low-cost hydraulic active suspension with interconnected circuits in roll-plane. It is capable to control the roll motion of the vehicle to reduce the rollover car accident.

In last decades, many control algorithms are proposed for active suspension system such as LQR, robust H_∞ and Fuzzy Logic Controller (FLC)[5]. LQR and H_∞ are highlighted for theirs controllability by giving out an optimal and robust performance. FLC is emphasized by offering major advantage of the capability to handle the nonlinear system[6]. Control inputs are mathematically transformed by deriving FLC from linguistic statement to desired output which is basically from the knowledge and experience. Besides, developing FLC do not need a mathematical model of the system, but the dynamic range of the input of the controller is assumed by engineer's knowledge. While deriving optimal or robust controller such as LQR and H_∞ controllers, both controllers depend on the mathematical model-base. If the model is derived accurately, the performance of these controllers will be significantly improved.

The objective of this project is to design and apply different types of controllers on the implementation of the proposed roll plane active HIS. Particularly, LQR, Fuzzy and Fuzzy-PID controllers are developed preliminary in simulation for a purpose of indicating controllability. The results will be analyzed to verify the control performance of each controller. In this paper, the controller will be focused on

ground excitation which can lead to roll motion of the vehicle body. The control objective is to minimize the roll angle of the vehicle's body to reduce the risk of rollover.

II. ROLL PLANE ACTIVE HIS

The schematic drawing of this active HIS is provided in Fig.1. Similar to DDAS, active HIS consists of four double direction hydraulic actuators which are interconnected to form two hydraulic circuits A and B. The pressures in these two circuits are then controlled by a hydraulic power unit which consists of a tank, a pressure pump, an accumulator and a servo-valve. The design drawing of this power unit is shown in Fig.2. The cylinders are mounted between wheels. This structure allows direct control of the roll motion of the vehicle by generating moment which will then tilt the vehicle against the roll motion[7].

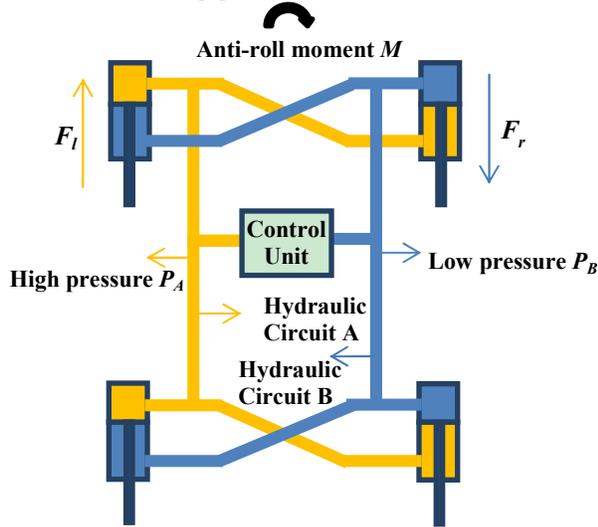


Fig.1. Schematic diagram of Active HIS

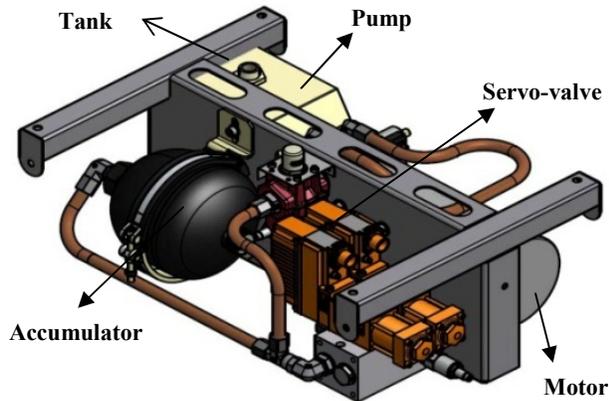


Fig.2. Designed and assembled pressure control unit for active suspension

To be specific, in the ground excited roll motion control, one side (e.g., left) of the suspension needs to extend while another side (e.g., right) need to compress in order to maintain vehicle body's leveling. In this circumstance, pressure control unit will pump oil into the circuit A to increase its pressure, while reducing the pressure (if not tank pressure) in the circuit B, as shown in Figure 1. The difference between the pressures

of two circuits will generate forces in four cylinders between wheels and vehicle chassis, which will form an anti-roll moment to the vehicle body to reduce the roll tendency of the vehicle. The control forces and moment are presented in Equation (1-3).

$$F_l = 2a(P_A - P_B) \quad (1)$$

$$F_r = 2a(P_A - P_B) \quad (2)$$

$$M = F_l l / 2 + F_r l / 2 \quad (3)$$

where, F_l and F_r are the forces effected on the vehicle body by the left and right actuators respectively; P_A and P_B are the pressures inside the two hydraulic circuits; a is the piston area; M is the anti-roll moment and l is the distance between left and right actuators.

However, active HIS is characterized as non-linear due to the complexity of the power unit. Thus, there is a challenge in both mathematical modeling and deriving suitable control algorithm for this system.

III. MODELING

In order to derive LQR and Non-linear fuzzy controller, mathematical model of linear half car need to be developed. Controllers will be tested base on this model by using Matlab Simulink. Half car model will be presented first, following by the development of pressure control unit model and finally integrating half car and pressure unit model in one.

The pressure unit model and the integrated model are obtained detailed from[8].

A. Half Car model

To simplify the problem of analyzing the dynamics of the vehicle body roll motion, a half-car model was utilized as shown in Fig. 2.

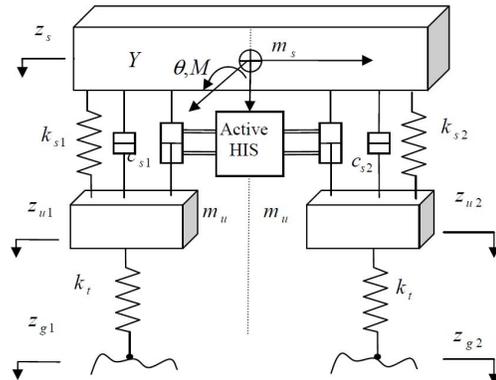


Fig. 2. Half-car model integrated with active HIS

Equation (1) and (2) express the vertical and angular motion of the sprung mass.

$$m_s \ddot{z}_s = -k_s(z_s + \theta l - z_{u1}) - k_s(z_s - \theta l - z_{u2}) \quad (1)$$

$$-c_s(\dot{z}_s + \dot{\theta} l - \dot{z}_{u1}) - c_s(\dot{z}_s - \dot{\theta} l - \dot{z}_{u2}) - f_1 - f_2$$

$$I \ddot{\theta} = -lk_s(z_s + \theta l - z_{u1}) - lc_s(\dot{z}_s + \dot{\theta} l - \dot{z}_{u1}) + \quad (2)$$

$$lk_s(z_s - \theta l - z_{u2}) + lc_s(\dot{z}_s - \dot{\theta} l - \dot{z}_{u2}) - f_1 l + f_2 l$$

The vertical motions of unsprung masses are expressed in equations (3) and (4).

$$m_u \ddot{z}_{u1} = -k_t z_{u1} + k_s (z_s + \theta l - z_{u1}) + c_s (\dot{z}_s + \dot{\theta} l - \dot{z}_{u1}) + f_1 \quad (3)$$

$$m_u \ddot{z}_{u2} = -k_t z_{u2} + k_s (z_s - \theta l - z_{u2}) + c_s (\dot{z}_s - \dot{\theta} l - \dot{z}_{u2}) + f_2 \quad (4)$$

where θ , z_s , z_{u1} , z_{u2} are angular displacement, vertical displacement of the vehicle body, left and right wheel vertical displacement, respectively.

Equations (1-4) are organized in the following state-space form

$$\dot{X}_{car} = A_{car} X_{car} + B_{input} F \quad (5)$$

Where the system state vector is defined as

$$X_{car} = [\dot{\theta} \quad \dot{z}_s \quad \dot{z}_{u1} \quad \dot{z}_{u2} \quad \theta \quad z_s \quad z_{u1} \quad z_{u2}]^T$$

And $F = [w_1 \quad w_2 \quad f_1 \quad f_2]^T$ is the input vector, where $w_{1,2}$ is the road input and $f_{1,2}$ is the actuator forces. The state space matrices of the model are provided

$$A_{car} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad B_{input} = \begin{bmatrix} 0 & 0 \\ B_{road} & B_{contr} \end{bmatrix}$$

The system coefficient matrices M, C, K, and input coefficient matrices B_{road} and B_{contr} are further shown below

$$M = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & m_s & 0 & 0 \\ 0 & 0 & m_u & 0 \\ 0 & 0 & 0 & m_u \end{bmatrix} \quad C = \begin{bmatrix} 2l^2 c_s & 0 & -l c_s & l c_s \\ 0 & 2c_s & -c_s & -c_s \\ -l c_s & -c_s & c_s & 0 \\ l c_s & -c_s & 0 & c_s \end{bmatrix}$$

$$K = \begin{bmatrix} 2l^2 k_s & 0 & -l k_s & l k_s \\ 0 & 2k_s & -k_s & -k_s \\ -l k_s & -k_s & k_s + k_t & 0 \\ l k_s & -k_s & 0 & k_s + k_t \end{bmatrix}$$

$$B_{road} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_t / m_u & 0 \\ 0 & k_t / m_u \end{bmatrix} \quad B_{contr} = \begin{bmatrix} -l / I & l / I \\ -1 / m_s & -1 / m_s \\ 1 / m_u & 0 \\ 0 & 1 / m_u \end{bmatrix}$$

B. Half-pressure Control Unit Model

The pressure unit is assumed to be a linear second order system corresponding to the transfer function shown below:

$$\frac{P(s)}{I(s)} = \frac{1}{1 + (2\xi / w_n)s + s^2 / w_n^2}$$

The parameters of this transfer function are obtained via empirical method which is detailed in [8]. Fig.4 shows the experimental setup. $P(s)$ represents the measured pressure signal, and $I(s)$ is the input command signal. w_n is the natural frequency, ξ is the damping ratio. The optimized value for w_n and ξ are estimated to 68Hz and 0.57 respectively.

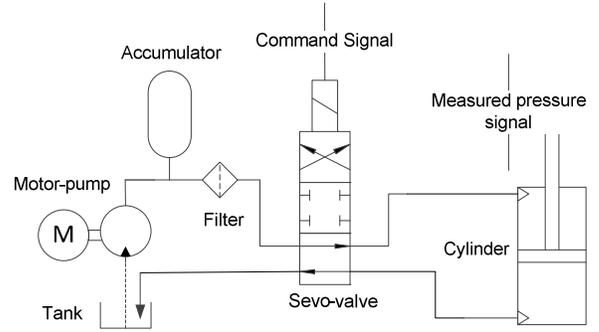


Fig. 4. Schematic drawing of the testing setup

To be consistent with the linear half-car model, it is necessary to convert the transfer function equation into state-space form.

$$\dot{X}_p = A_p X_p + B_p inp \quad Y_p = C_p X_p + D_p inp$$

where the coefficient matrixes are shown below:

$$A_p = \begin{bmatrix} -487.1 & -356.5 \\ 512 & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 16 \\ 0 \end{bmatrix},$$

$$C_p = [0 \quad 22.28], \quad D_p = [0]$$

C. Integrated Model

After integrating the vehicle model with the pressure control unit model, the integrated model is obtained, with the state vector and input vector shown below.

$$X = [\dot{\theta} \quad \dot{z}_s \quad \dot{z}_{u1} \quad \dot{z}_{u2} \quad \theta \quad z_s \quad z_{u1} \quad z_{u2} \quad x_{p1} \quad x_{p2}]^T$$

$$u = [w_1 \quad w_2 \quad p]^T$$

Then, the state space form of the integrated model is:

$$\dot{X} = A_{int} X + Bu$$

$$A_{int} = \begin{bmatrix} -M^{-1}C & -M^{-1}K & B_{contr} \begin{bmatrix} C_p a \\ -C_p a \end{bmatrix} \\ I & 0 & 0 \\ 0 & 0 & A_p \end{bmatrix},$$

$$B = \begin{bmatrix} B_{road} & 0 \\ 0 & 0 \\ 0 & B_p \end{bmatrix}$$

The value of a shown in the matrix A_{int} is the piston area with the assumption that the cylinder has equal piston area in both chambers.

IV. CONTROL DESIGN

All the designed controllers used roll angle of the vehicle body as the input signal and control force as output signal.

A. Fuzzy Controller Design

Active HIS is a complex system which contains non-linear property which mainly distributed in hydraulic power unit. This leads to difficulties in accurately modeling the system in order to develop the control scheme which requires a model-base. Alternatively, the FLC provides feasible methodology for controlling the active suspension to handle the nonlinearity effects[6]. The other advantage of Fuzzy controller is to ignore sophisticated model-base by estimating dynamic operating range of the system. In this project, deriving fuzzy controller following number of steps:

1) Defining control input and output with estimating dynamic range and mapping input and output to membership function. There are a number of points that need to be noted:

The control target is to minimize the roll angle under the ground excitation. Therefore, the displacement of roll angle (θ) of the vehicle is considered as the primary input. In order to ensure the improvement of performance of the controller, an angular acceleration ($\ddot{\theta}$) is added in as a secondary input.

An active fore (f) is the output of this controller.

There are three type of input and output membership functions that are used in this controller namely trimf, pimf and gaussmf corresponding to roll acceleration, angle and active control force, which are abbreviated as follow:

$$\mu_{A_i}(x_i) = \text{trimf}(x_i, a_i, b_i, c_i)$$

$$\mu_{B_j}(x_j) = \text{pimf}(x_j, a_j, b_j, c_j, d_j)$$

$$\mu_{C_k}(x_k) = \text{gaussmf}(x_k, c_k, \sigma)$$

2) Applying fuzzy theory to derive the controller. In general, the design of FLC has three basic stages which are fuzzification, inference engine, and defuzzification.

Fuzzification stage converts the crisp input values, namely the roll angle of the vehicle body, into fuzzy values under an association of membership function[9]. The range of the vehicle body roll angle is adopted as from -6 degree to 6 degree and divided as negative very large, negative large, negative medium, negative small, zero, positive small, positive medium, positive large and positive very large . The Fig. 5 below shows the graph result of mapping inputs membership functions and estimating dynamic range of inputs. Similarly, the range of the angular acceleration of the vehicle body is from $-1/^\circ/s^2$ to $1^\circ/s^2$ and divided as negative, zero and positive.

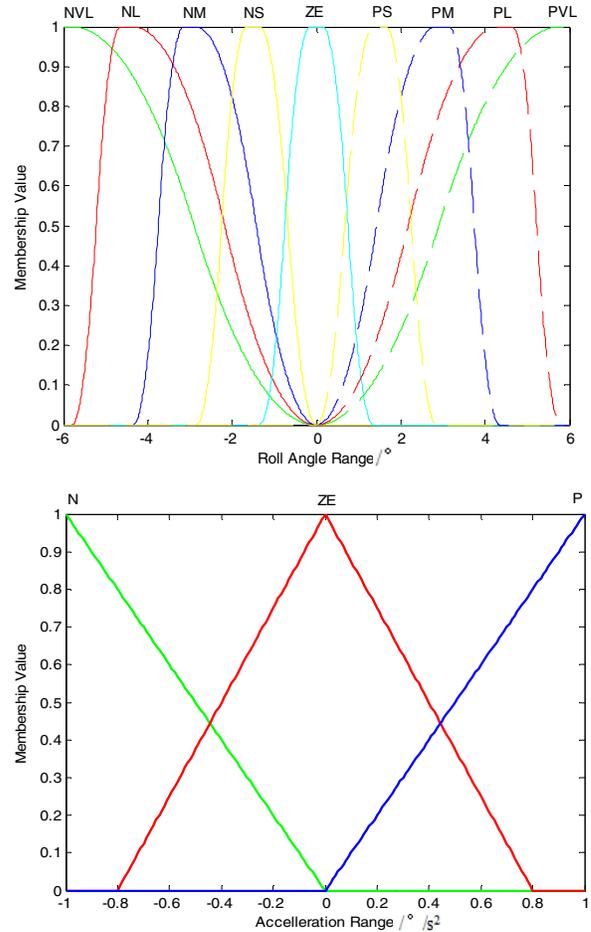


Fig. 5. Mapping of membership function of FLC inputs

Inference engine is a state in which the fuzzy value is calculated with the association of the defined rules base system and/or database build in to give out the fuzzy output . In this work, Mamdani method is used to construct the rule base inference expressing in the following formula.

Rule i: IF $x_i = A_i$ AND $x_j = B_j$ THEN $x_k = C_k$

In this controller, there are 27 rules which indicated on Table I as follow: denoting as NVL: negative very large, NL: negative large, NM: negative medium, NS: negative small, ZE: zero, PVL: positive very large, PL: positive large, PM: positive medium, PS: positive small, P: positive, N: negative.

TABLE I
FUZZY RULE BASE OF CONTROLLER

θ	$\ddot{\theta}$	f	θ	$\ddot{\theta}$	f	θ	$\ddot{\theta}$	f
NVL	N	NVL	NVL	ZE	NVL	NVL	P	NL
NL	N	NNL	NL	ZE	NL	NL	P	NM
NM	N	NL	NM	ZE	NM	NM	P	NS
NS	N	NM	NS	ZE	NS	NS	P	NS
ZE	N	ZE	ZE	ZE	ZE	ZE	P	ZE
PS	N	PM	PS	ZE	PS	PS	P	PS
PM	N	PL	PM	ZE	PM	PM	P	PS
PL	N	PVL	PL	ZE	PL	PL	P	PM
PVL	N	PVL	PVL	ZE	PVL	PVL	P	PL

For example:

is a foundation of this method. Depending on the inputs of fuzzy logic, suitable parameters for PID will be outputted. In other word, PID parameters are regulated by the output of the fuzzy logic. Therefore, Fuzzy-PID results in better performance in comparison to conventional PID controller. The procedure to derive the controller will be revealed below:

Derive the conventional PID controller, the parameters are derived by using Ziegler-Nichols rule

Estimate interval change of PID parameters

K_p : [700 - 900]

K_i : [5 - 15]

K_d : [55 - 65]

Estimate interval change of output error and rate of change of error

$e(t)$: [-6 - 6]

de/dt : [-5 - 5]

Applying fuzzy logic with considering

$e(t)$ and de/dt as the fuzzy input

K_p , K_i , K_d as the fuzzy output

The procedure of deriving the fuzzy logic for this controller is similar to previous fuzzy controller and will not be repeated here.

The inference engine of the controller including rule base mapping system between input and output are shown in the tables below. Also, the controller uses centroid of gravity as the defuzzification method.

TABLE II
RULE BASE OF FUZZY -PID CONTROLLER

		de/dt		
		N	ZE	P
e	N	L	MD	S
	ZE	L	MD	S
	P	MD	MD	MD

		de/dt		
		N	ZE	P
e	N	S	S	MD
	ZE	S	MD	L
	P	MD	L	L

		de/dt		
		N	ZE	P
e	N	L	L	MD
	ZE	L	MD	S
	P	MD	S	S

There are nine rules for each output value with notations N (negative), ZE (zeros), and P (positive) referring to the change value of both error, and error rate. While L (large), MD (medium), and S (small) referring to the values of K_p , K_i , K_d . For example:

Rule 1 of K_p : IF $e_1 = N$ AND $de/dt_1 = N$ THEN $K_p = L$

Rule 3 of K_i : IF $e_3 = N$ AND $de/dt_3 = P$ THEN $K_i = S$

Rule 3 of K_d : IF $e_3 = N$ AND $de/dt_3 = P$ THEN $K_i = MD$

Under the mapping of rule base, the 3-D output surfaces of K_p , K_i , K_d are shown in Fig.9 below:

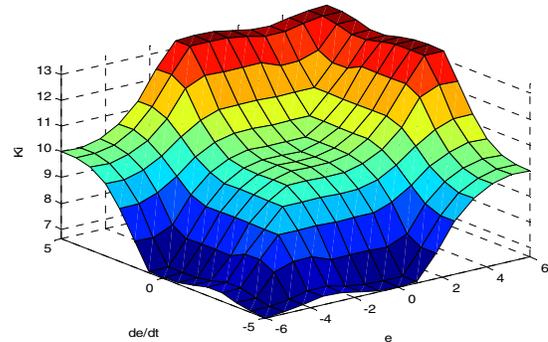
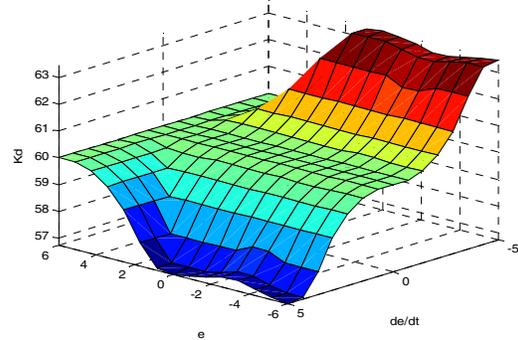
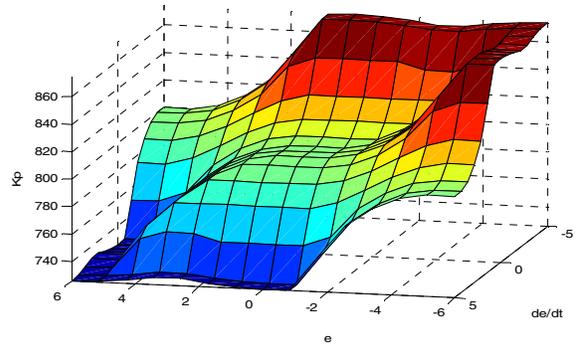


Fig.9. 3-D output surface of K_p , K_i , K_d

V. SIMULATION RESULTS AND DISCUSSION

The feasibilities of controllers are verified through simulation in Matlab. Two kinds of ground excitation namely fishhook, slalom have been tested to indicate the controllability. The results below show graphically of the ground input signal, compared results between LQR, Fuzzy and Fuzzy-PID regarding to roll angle and suspension relative displacement.

Table III indicates the simulation results by comparing result of roll angle and relative displacement of the three proposed controller. Through the simulation results, there are several points of note which are:

1) Control ability will be justified by indicating the percentage of reduction of roll angle when comparing with passive system.

2) Relative displacement of both left and right side is similar, hence one is omitted. It will be evaluated via the level of stability.

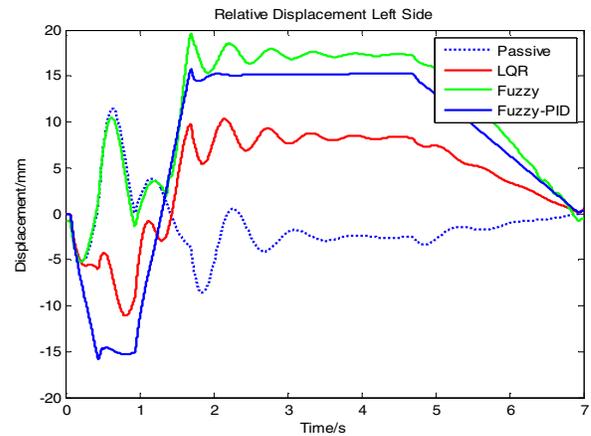
TABLE III

THE COMPARISON OF SIMULATION RESULTS

Signal	Roll angle			Displacement		
	LQR	Fuzzy	F-PID	LQR	Fuzzy	F-PID
Fishhook	50%	70%	60%	Unstable	Unstable	Stable
Slalom	40%	80%	70%	Unstable	Unstable	Stable

Fig.10 shows the results of comparison of roll angle and suspension deflection, namely relative displacement between vehicle body and road wheels, under fishhook ground input. From the graph, the roll angle is significantly reduced under active control. Notice that fuzzy-PID produces the best performance with approximately 60% reduction. Although LQR and fuzzy have sufficiently decreased the roll angle, there is unstable state of roll angle after the control has been implemented which can be seen through the simulation results.

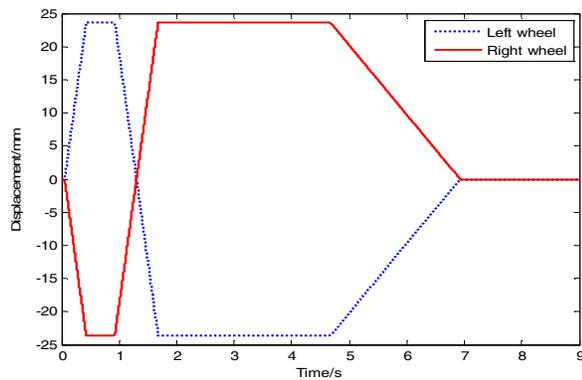
For the relative displacement of suspension, the result indicates that under active control, the relative displacement values are significantly larger than ones in passive system. This is due to the controller generating control force to tilt the sprung mass against its roll motion.



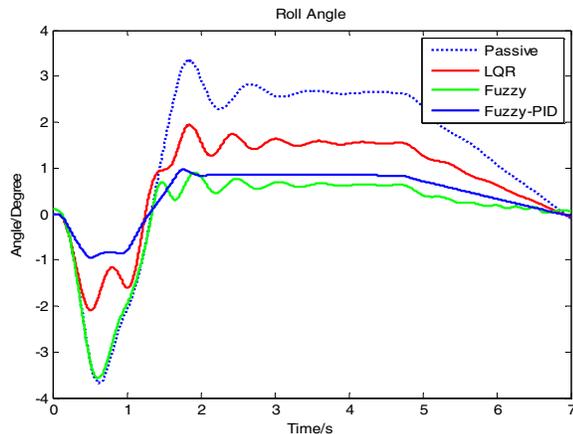
Relative Displacement at Left Side
Fig.10. Simulation results of fishhook ground input

With slalom ground excitation input, the three controllers produce optimized results, which can be found in Fig.11. Roll angle is significantly decreased when applying fuzzy, fuzzy PID and LQR which results in 80%, 70% and 40% reduction respectively. The result of fuzzy controller indicates good performance. However, the results exhibited an oscillation with the fuzzy controller but not presented in LQR and fuzzy-PID control. Furthermore, fuzzy-PID captures control target which is reducing the roll angle by producing high stability and performance.

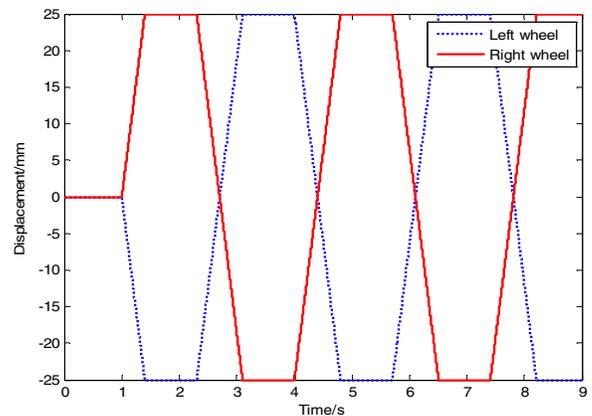
In the graph of relative displacement, the range of LQR controller is much less than the results in other two controllers, which means LQR provides less active force control than fuzzy, and fuzzy-PID. This factor can be considered as energy consumption advantage of LQR controller. But when compared with fuzzy-PID, the active control force of LQR is oscillating and lead to unstable.



Ground input signal



Roll angle



Ground input signal

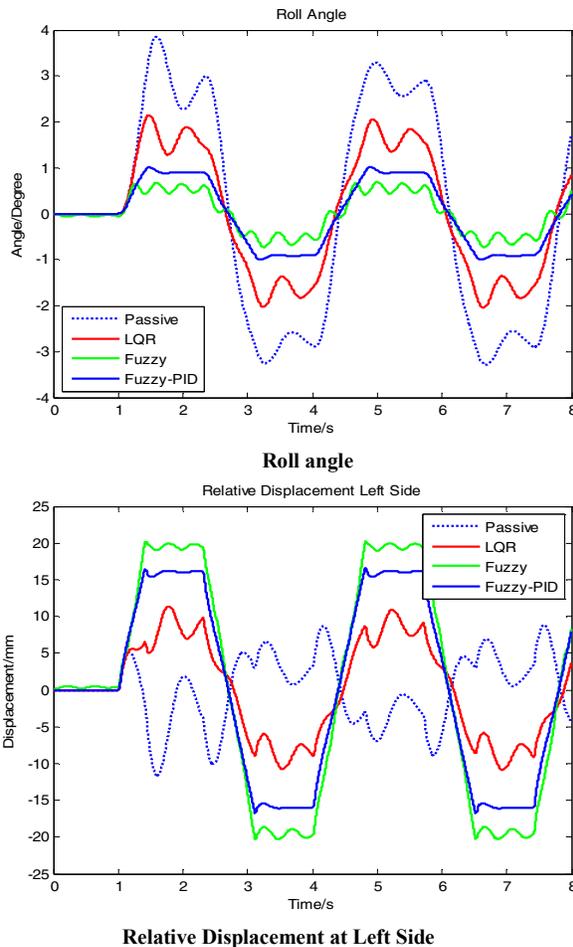


Fig.11. Simulation results of slalom ground input

VI. CONCLUSION

The primary concern and target of carrying out this topic was to increase the level of safety of the driver as well as passengers in a car. Focusing on SUV vehicle which has lower roll stability limit, half car model was developed along with various control systems. Three controllers namely LQR, Fuzzy and Fuzzy-PID controllers were developed and then implemented in simulation. The results were recorded and discussed. The simulation results of all the controllers show expected results with considerable reduction in roll angle and thus the improved vehicle safety in terms of rollover.

The fuzzy controller shows a relatively lower level of control ability in roll angle than the other two controllers and also shows instability in the suspension deflection. Although the LQR controller consumes less energy when it is working, it also exhibited instability. The designed fuzzy-PID control shows the best control ability and stability in the results, so it is more suitable to be adopted to apply in the proposed active HIS than the other two controllers.

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