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New Fuzzy Model with Second Order Terms for the Design of a Predictive Control Strategy

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***Abstract*— In this paper a novel predictive control scheme based on Takagi-Sugeno model whose consequences include second order terms is proposed. Fuzzy models are used in order to approximate the non-linear behavior present on industrial dynamic systems. Quadratic approximations are used in the consequences because several systems has restricted controllable regions in the states domain. Thus, even fuzzy models may not be enough for representing the system dynamics in that regions, producing unexpected closed loop-behavior and loss of performance. The main difference between the proposed scheme and the ones reported in the literature is that iterative procedures and/or point to point approximation is not required. Reducing the computational burden of the controller. A continuous stirred tank reactor is used for testing the proposed control scheme.**

I. Introduction

DEVELOPMENT of mathematical models for real processes is an important task due to the capabilities they bring for making analysis, design, forecasting among other important issues inside engineering. But, with modeling there are several pending troubles to be addressed. For instance, the compromise between accuracy and interpretability of the models. In this way, fuzzy modeling arises as a feasible alternative to represent nonlinear systems through a set of rules and consequences. Many studies have been developed about fuzzy modeling, highlighting Takagi and Sugeno work [1]. Regarding this work, the specialized literature has been dedicated to Takagi-Sugeno models with linear consequences, with a wide use in systems identification. For example, in [2] the authors proposed a methodology for identification of fuzzy models with linear consequences; and in [3], an optimal methodology for the same models is proposed. Although these methodologies (and the others reported in the literature) have successfully demonstrated their applicability in systems identification and representation, in control theory linear models still having a restricted applicability. Such restricted applicability is because of only when the control inputs has an effect on the system state trajectories and outputs they can be used, e.g., when the system is controllable. Therefore, there are several operating points belonging to the feasible operating space where the system cannot be driven, because linear models fails in their representation of the system dynamics. For tackling this drawbacks (even present in fuzzy models with linear consequences) in [4] pointed out the use of non-linear terms in Takagi-Sugeno models for control systems design. Following the same line, in [5] the authors proposed the use of second order approximations of the system model with the purpose of increasing the controllable space. In this approach, at each time step the second order approximation of the system model was computed around the current operating point.

Regarding control applications, there are several fuzzy model based control strategies. Nowadays, the fuzzy based control strategy that highlights is the fuzzy based predictive control. In this control technique a fuzzy model is used to predict the behavior of the system several time steps ahead. Then, according to the prediction the control actions to be applied to the system are determined based on a minimization of a cost function (see [6] for details about model predictive control). This kind of strategy has increased its use in the industry lately due to its ability to manage with system constraints [7]. The wide use of fuzzy modeling in predictive control is motivated by the fact that often linear models are used in this control strategy, and they are not adequate for handling with strongly nonlinear systems that often are found in the industry. Indeed, several non-linear predictive control schemes have been reported, e.g., the ones proposed in [8], [9], [10], [11], [12], [13]. In all these approaches, iterative procedures are proposed to approximate the solution of the non-linear programming problem resulting from the model predictive control (MPC) formulation, or proposed the use of the Hamilton-Jacobi-Bellman equation to approximate the solution. In the first case, the iterative procedures are highly time consuming, making them sometimes not feasible for real time implementations. In the second case, the solution is restricted to non-linear affine systems. Therefore its applicability in real systems is highly reduced.

Since computational burden is a pending issue in both fuzzy based and non-linear model based predictive control (see e.g., [2], [14]), alternatives based on evolutionary algorithms have also been proposed. In [15] proposed the use of genetic algorithms for implementing non-linear predictive controllers, while in [16] the authors proposed a solution based on particle swarm optimization technique. Although both solutions were successfully evaluated in terms of accuracy and computational cost, they cannot assure the convergence to the optimum value of the control solutions. Hence, the stability of the closed-loop system cannot be assured. In order to tackle the drawbacks associated with the computational burden of non-linear MPC, as well as with the model representation in this work the use of second order fuzzy models as prediction models in an MPC strategy is

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proposed (here second order fuzzy models are understood as fuzzy models with second order consequences). The use of fuzzy models allowed avoiding the computation of the gradients and Hessians of the second order model at each time step (procedure required in the approach proposed in [5]). Also, the addition of the second order terms allow increasing the domain where the system is controllable. Moreover, since often the predictive control cost function is quadratic and the model is quadratic convexity properties can be derived in order to have an efficient algorithm for computing the optimal control actions to be applied to the system. The remaining of this paper is organized as follows: Section II presents a motivation for using second order approximations; Section III presents the proposed second order fuzzy model; Section IV presents the formulation of the model predictive control strategy based on second order fuzzy models; Sections V and VI gather with the simulation results and concluding remarks.

II. MOTIVATION

Consider the ideal continuous stirred tank reactor (CSTR) described by (1). In such tank a reversible exothermic reaction $A \rightleftarrows B$ take place, with kinetic constants $k_1(t)$ for the reaction $A \rightarrow B$, and $k_2(t)$ for the reaction $A \leftarrow B$. Let $x_1(t)$, $x_2(t)$, and $x_3(t)$ denote the conversion, reactor temperature, and level of the liquid inside the reactor respectively. Let $u_1(t)$ and $u_2(t)$ be the inlet flow of reactants and the temperature of the inlet flow of cooling fluid respectively. As in [5], in this case it was assumed $x_3(t)$ constant and equals to 0.16, which means that $u_1(t)$ is constant as well and equals to 1. Accordingly, the only manipulated variable in this reactor is the temperature of the inlet flow of cooling fluid. That is $u(k) = u_2(k)$ in the remaining of this paper. Parameters used for these simulations were taken from [5].

$$\begin{aligned}
\dot{x}_1(t) &= -0.16x_1(t)x_3^{-1}(t)u_1(t) \\
&\quad + k_1(t)(1-x_1(t)) - k_2x_1(t) \\
\dot{x}_2(t) &= -0.16u_1(t)x_3^{-1}(t)u_2(t) - 0.16x_2(t)x_3^{-1}(t)u_1(t) \\
&\quad + 5(k_1(t)(1-x_1(t)) - k_2x_1(t)) \\
k_1(t) &= k_{o1}\exp\left(\frac{-E_1}{x_2(t)}\right),\ k_2(t) = k_{o2}\exp\left(\frac{-E_2}{x_2(t)}\right)
\end{aligned} \tag{1}$$

The objective of the CSTR described by (1) is to maximize the conversion without overtaking the capacity limits of the reactor. From [5] the CSTR under study has a maximum of conversion equals to 0.509, value used in the current paper as a reference for evaluating the performance of the controllers. Specifically, in order to regulate the conversion of the reactor to its maximum value a model predictive control strategy was used. This control strategy had the aim of manipulating the temperature of the coolant to provide favorable conditions inside the reactor for increasing the conversion. With this purpose, two models were used for predicting the trajectories of the conversion and the temperature of the reactor: a linear and a second order model. Figure 1 shows the results obtained through simulations for the closed-loop operation of the CSTR. It is worth to point out that for implementing the predictive controllers, at each time step k the linear and second order approximations of the system model were computed. In Figure 1 the MPC with linear prediction model is called linear MPC while the MPC with second order terms prediction model is called quadratic MPC.

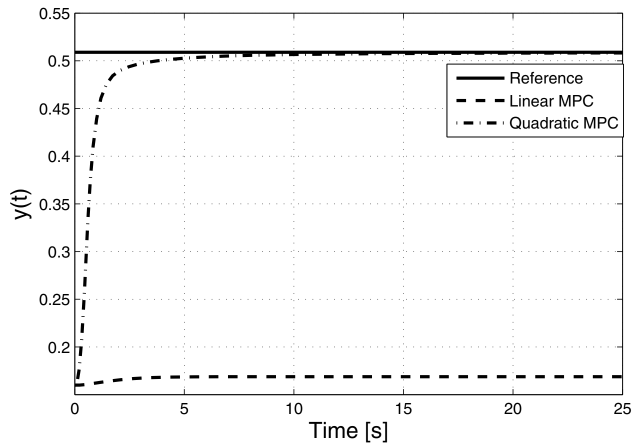

Fig. 1. Closed-loop behavior of the conversion in the CSTR described by (1). Here, the performance of predictive controllers with linear and quadratic prediction models is done.

It can be noticed that the predictive control with the second order prediction model was able to drive the system to the desired value, while the predictive controller with linear prediction model cannot be able to drive the system to the maximum conversion conditions. Mathematically, this result can be justified from the formulation of the optimization problem. In this case, for both controllers a quadratic cost function as (2) was used.

$$L(x(k), u(k)) = e^T(k)Qe(k) + \Delta u^T(k)R\Delta u(k) \tag{2}$$

In (2) $e(k) = y_{\text{ref}}(k) - \hat{y}(k)$, and $\Delta u(k) = u(k) - u_{\text{o}}(k)$, $\hat{y}(k)$ being the predicted output trajectory and $u_{\text{o}}(k)$ the input value corresponding to the operating point at time step k, and $Q, R > 0$ weighting matrices. In the system described by (1) the output is the conversion, i.e., $y(k) = x_1(k)$. Let $C = [0, 1]$ and $x(k) = [x_1(k), x_2(k)]^T$. Then $y(k) = Cx(k)$. Let $J_{\text{x}}(k)$ and $J_{\text{u}}(k)$ be the Jacobians of the CSTR model with respect to the states and the inputs respectively. Hence, the linear CSTR prediction model becomes

$$\hat{y}(k) = C\left(f(x_{\text{o}}(k), u_{\text{o}}(k)) + J_{\text{x}}(k)\Delta x(k) + J_{\text{u}}(k)\Delta u(k)\right) \tag{3}$$

with $\Delta x(k) = x(k) - x_{\text{o}}(k)$, $x_{\text{o}}(k)$ being the measured state at time step k, and $f(x_{\text{o}}(k), u_{\text{o}}(k))$ the value at $(x_{\text{o}}(k), u_{\text{o}}(k))$ of the non-linear vector function describing the CSTR dynamics. Assuming the predictive control implementation without constraints, the analytic solution of the corresponding optimization problem (when a linear prediction model is used) is given by

$$\begin{aligned}
\Delta u^*(k) = -Q(y_{\text{ref}}(k) - [C(f(x_{\text{o}}(k), u_{\text{o}}(k)) \\
+ J_{\text{x}}(k)\Delta x(k) + J_{\text{u}}(k)\Delta u(k))])CJ_{\text{u}}(k)
\end{aligned} \tag{4}$$

Notice that in (1) the trajectory of the conversion is independent of the control input $u_2(k)$, which is the decision variable in the optimization problem. Therefore, $J_{\mathrm{u}}(k)$ can be written as $J_{\mathrm{u}}(k) = [0, k]^T$ with $k \in \mathbb{R}$ a constant determined by the current operating point. Since $C = [0, 1]$ the resulting optimal control is given by $\Delta u^*(k) = 0,\ \forall k$. Hence, the control actions do not change along the simulation. If constraints over the control actions are added, the corresponding Lagrange function is

$$\mathbb{L}(u(k), \lambda) = L(x(k), u(k)) + \lambda^T g(u(k)) \tag{5}$$

where $g(u(k)) = [(\Delta u(k) - u_{\max}(k)), (u_{\min}(k) - \Delta u(k))]^T$. Thus the Kuhn-Tucker conditions for the constrained optimization problem are

$$\begin{aligned} 2R\Delta u(k) + \lambda_1 - \lambda_2 &= 0 \\ g(u(k)) &\leq 0 \\ \lambda^T g(u(k)) &= 0 \\ \lambda_1, \lambda_2 &\geq 0 \end{aligned} \tag{6}$$

According to the Kuhn-Tucker conditions, the feasible solutions for the optimization problem are

$$\Delta u^*(k) = -Q(y_{\mathrm{ref}}(k) - \hat{y}(k))CJ_{\mathrm{u}}(k) \tag{7}$$

$$\Delta u^*(k) = u_{\min} - u_{\mathrm{o}}(k) \tag{8}$$

$$\Delta u^*(k) = u_{\max} - u_{\mathrm{o}}(k) \tag{9}$$

obtaining again the trivial control action $\Delta u^*(k) = 0,\ \forall k$ as the solution of the optimization problem. This is why the predictive control strategy with linear prediction model cannot be able to drive the system to appropriate conditions for reaching the maximum conversion (here $u_{\min}$ and $u_{\max}$ denote the maximum and minimum values for the control actions). Indeed, including second order terms avoids the vanishment of the control action $\Delta u^*(k)$. Thereby, the capabilities of driving the system to the desired values is increased. These facts motivate the proposed control scheme, in which models with second order terms are used for predicting the state and/or output trajectories. As a consequence of the use of models with second order terms, an increasing on the complexity of the control strategy is expected. But as will be shown, the region where second order approximations are valid is greater than the same region for linear approximations. Thus, the complexity of the proposed control strategy is not higher than the complexity of similar strategies in which linear models are used. In sections below the proposed approach is described.

III. Second Order Fuzzy Modeling

Let start the current section introducing some notation required for the remaining of this paper. Let $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^z$ denote the state, inputs, and outputs of a dynamical system at time step k. Let $F : \mathbb{R}^{n+m} \to \mathbb{R}^n$ and $G : \mathbb{R}^{n+m} \to \mathbb{R}^z$ be C^∞ functions defining the discrete-time dynamic evolution of $x(k)$ and $y(k)$ respectively, namely,

$$\begin{aligned} x(k+1) &= F(x(k), u(k)) \\ y(k) &= G(x(k), u(k)) \end{aligned} \tag{10}$$

Let introduce the Morse Lemma. This Lemma will be used to derive the expressions for the second order approximations used in the new fuzzy models.

Lemma 1: **Morse Lemma [5]:**
Consider a function $f : \mathbb{R}^p \to \mathbb{R}$ with a non-degenerate critical point at $z = z_{\mathrm{o}}$. Then, in the neighborhood of the critical point, there is a smooth local change of coordinates to $y_1(z), \ldots, y_p(z)$ such that $y_1(z) = \ldots = y_p(z) = 0$ and $f(z)$ takes the exact form

$$f(z) = f(z_{\mathrm{o}}) + \frac{1}{2}\sum_{i,j} H_{ij}(z_{\mathrm{o}}) y_i y_j$$

with $i, j = 1, \ldots, p$ and $H_{ij}(z_{\mathrm{o}})$ the Hessian of $f(\cdot)$ with respect to the coordinates i, j evaluated at z_{o}.

It is worth to point out that in this paper the regular perturbation technique will be used to derive the linear and quadratic approximations of the system model. Even, the resulting expressions will be used in order to obtain a fuzzy model whose consequences include second order terms. This is motivated by comparison reasons, namely, for having models and predictive controllers whose performance can be compared with the models and controllers derived in [5].

A. Second Order Approximation

Given a discrete-time dynamic model as (10), functions $F(.)$ and $G(.)$ can be interpreted as vectors whose elements are defined by nonlinear functions. For instance, consider the vector field $F(.)$, which can be expressed as the vector $F(.) = [f_1(.), \ldots, f_n(.)]^T$, where $f_i : \mathbb{R}^{n+m} \to \mathbb{R}$, $i = 1, \ldots, n$. Let $x_{\mathrm{p1}}(k)$ and $x_{\mathrm{p2}}(k)$ denote the vectors of state trajectories defined (respectively) by perturbations 1 and 2, viz., $x_{\mathrm{p1}}(k) = [x_{1,1}(k), \ldots, x_{n,1}(k)]^T$ and $x_{\mathrm{p2}}(k) = [x_{1,2}(k), \ldots, x_{n,2}(k)]^T$. Following the procedure proposed in [5], the second order perturbation approximation of the original system can be written as

$$\begin{aligned} &x_{\mathrm{p1}}(k+1) = \\ &\qquad F(x_{\mathrm{o}}(k), u_{\mathrm{o}}(k)) + J_{\mathrm{x}}(k)x_{\mathrm{p1}}(k) + J_{\mathrm{u}}(k)\Delta u(k) \end{aligned} \tag{11}$$

$$\begin{aligned} &x_{\mathrm{p2}}(k+1) = \\ &\qquad J_{\mathrm{x}}(k)x_{\mathrm{p2}}(k) + \Xi(x_{\mathrm{p1}}(k), \Delta u(k)) \end{aligned} \tag{12}$$

where the initial conditions for $x_{i,1}(k)$ and $x_{i,2}(k)$ are set to 0, and the j-th element of $\Xi(x_{i,1}(k), \Delta u(k))$ is determined by the expression

$$\begin{aligned} &\Xi_j(x_{\mathrm{p1}}(k), \Delta u(k)) = \\ &\quad \frac{1}{2}\big(x_{\mathrm{p1}}^T(k)H_{\mathrm{xx}}(k)x_{\mathrm{p1}}(k) + x_{\mathrm{p1}}^T(k)H_{\mathrm{xu}}(k)\Delta u(k) \\ &\quad +\Delta u^T(k)H_{\mathrm{ux}}(k)x_{\mathrm{p1}}(k) + \Delta u^T(k)H_{\mathrm{uu}}(k)\Delta u(k)\big) \end{aligned}$$

where $H_{\mathrm{xx}}(k)$, $H_{\mathrm{xu}}(k)$, $H_{\mathrm{ux}}(k)$, and $H_{\mathrm{uu}}(k)$ are defined as in [5]. Then, according to (11) and (12) the predicted state trajectory is defined by (13).

$$x(k+l) = x(k) + x_{\mathrm{p1}}(k+l) + x_{\mathrm{p2}}(k+l),\ l = 1, 2, 3 \ldots \tag{13}$$

In fact, following the procedure in [5] and applying the Morse Lemma the one-step-ahead predicted trajectory of each state can be computed as

$$x_i(k+1) = x_i(k) + \psi_i(k)f_i(k) + \Gamma_i(k)\delta u(k) + \psi_i(k)\eta(k)$$

where the j-th element of $\eta(k)$ is computed as

$$\begin{aligned}
\eta_i(k) &= \eta_{i,0}(k) + \eta_{i,1}(k)\Delta u(k) + \Delta u^T(k)\eta_{i,2}(k)\Delta u(k) \\
\eta_{i,0}(k) &= \frac{1}{2}\lambda^2(\psi_i(k)f_i(k))^T H_{\mathrm{xx}}(k)(\psi_i(k)f_i(k)) \\
\eta_{i,1}(k) &= \lambda(\psi_i(k)f_i(k))^T(\lambda H_{\mathrm{xx}}(k)\Gamma_i(k) + H_{\mathrm{xu}}) \\
\eta_{i,2}(k) &= \frac{1}{2}\left(\lambda^2\Gamma_i^T(k)H_{\mathrm{xx}}(k)\Gamma_i(k) + \lambda\Gamma_i^T(k)H_{\mathrm{xu}}(k)\right. \\
&\quad \left. + \lambda H_{\mathrm{ux}}(k)\Gamma_i(k) + H_{\mathrm{uu}}(k)\right)
\end{aligned}$$

with $\psi_i(k)$, $\Gamma_i(k)$, and λ defined as in [5].

B. Takagi-Sugeno Fuzzy Models

Takagi and Sugeno in [1] described a type of fuzzy models suitable for the approximation of non-linear systems. The premises are based on fuzzy sets, and the consequences are often linear models for representing different operating points of the system. Let M_{TS} denotes the number of rules of the Takagi-Sugeno model. Let $z(k) = [x_{\mathrm{pf}}^T(k), x^T(k)]^T$ denotes the vector of model input variables, where $x_{\mathrm{pf}}(k),\ x(k) \in z(k)$ denote respectively the vector of premises and the consequences at time step k. Let $\beta_j(x_{\mathrm{pf}}(k))$ be the normalized membership function of the j-th rule. Such function assigns a value between zero and one to each model depending on the fulfillment of each rule. The normalized activation degree satisfies $\beta_j(x_{\mathrm{pf}}(k)) > 0,\ j = 1, \ldots, M$, and $\sum_{j=1}^{M} \beta_j(x_{\mathrm{pf}}(k)) = 1$. Let θ_j denotes the vector of parameters of the linear model associated with the j-th rule. Then, at time step k the output of the TS model is given by (14) (see [17] for notation details and how to compute $\beta_j(x_{\mathrm{pf}}(k))$).

$$\hat{y}(k) = \sum_{j=1}^{M_{\mathrm{TS}}} \beta_j(x_{\mathrm{pf}}(k))\theta_j x(k) \tag{14}$$

At time step k, let $y(k)$ denotes the measured outputs of the system to be modeled. The procedure for identifying the structure and parameters of the fuzzy model is shown in Figure 2. This procedure is based on the methodology proposed in [3]. In such procedure, the first step is to make a data selection for training, testing, and validation, the second step is to select the relevant variables, the third step is to optimize the structure by e.g. sensitivity analysis, the fourth step is to perform a parameter identification procedure, and the fifth step is to validate the resulting model.

C. Second Order Takagi-Sugeno Fuzzy Models

Although Takagi-Sugeno fuzzy models have been widely used as universal approximators, in control field they have the same pending issues as the linear models (see Section I of the current paper for details). Namely, they may have consequences where the input effect is not reflected into

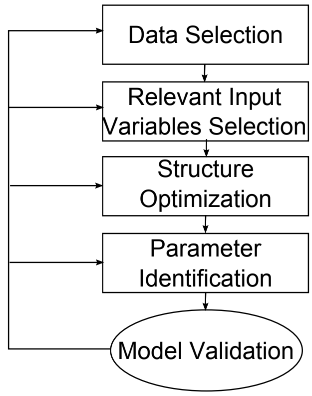

Fig. 2. Methodology used for Takagi-Sugeno model identification.

the state trajectory. Hence, if the premises are inside the domain of such consequence the controller may not drive the system to the desired operating conditions. An improvement respect to the often used Takagi-Sugeno fuzzy models is the use of consequences with second order terms instead of consequences with linear terms.

Consider a dynamic system whose dynamic evolution can be represented as (10). Let $\bar{y}(k)$ denotes the output of the fuzzy model. Based on [1] a general implication R for (10) is defined as

$$\begin{aligned}
&\text{R: If } x_{\mathrm{r1}}(k) \text{ is } A_1 \text{ and } u_{\mathrm{r1}}(k) \text{ is } B_1 \ldots \\
&\quad \ldots \text{ and } x_{\mathrm{r}M}(k) \text{ is } A_M \text{ and } u_{\mathrm{r}N}(k) \text{ is } B_N \\
&\quad \text{Then } \bar{y}(k) = \bar{\Gamma}(x_{\mathrm{r}}(k), u_{\mathrm{r}}(k))
\end{aligned} \tag{15}$$

with $\bar{\Gamma}(\cdot)$ a non-linear function, $x_{\mathrm{r}}(k)$ and $u_{\mathrm{r}}(k)$ being the premises associated with the states and the control inputs. In (15) $A_m,\ m = 1, \ldots, M$ and $B_n,\ n = 1, \ldots, N$ denote the membership function of the fuzzy sets in the premises, with M and N the number of states and control inputs respectively. From (13) the trajectory of the states of the dynamic system can be expressed as

$$x(k+1) = x(k) + x_{\mathrm{p1}}(k+1) + x_{\mathrm{p2}}(k+1)$$

which corresponds to a second order approximation of $F(\cdot)$. Thus, assuming second order consequences for the Takagi-Sugeno model yields

$$\bar{\Gamma}(x_{\mathrm{r}}(k), u_{\mathrm{r}}(k)) = C(x(k) + x_{\mathrm{p1}}(k+1) + x_{\mathrm{p2}}(k+1))$$

where C is an observation matrix. Therefore, the Takagi-Sugeno fuzzy model representing the dynamic evolution of the states of (10) is

$$\begin{aligned}
&\text{R1: If } x_{\mathrm{r1}}(k) \text{ is } A_1^1 \text{ and } u_{\mathrm{r1}}(k) \text{ is } B_1^1 \ \ldots \\
&\quad \ldots \text{ and } x_{\mathrm{r}M}(k) \text{ is } A_M^1 \text{ and } u_{\mathrm{r}M}(k) \text{ is } B_N^1 \\
&\quad \text{Then } \bar{y}^1(k) = C(x(k) + x_{\mathrm{p1}}^1(k+1) + x_{\mathrm{p2}}^1(k+1)) \\
&\quad \vdots \\
&\text{Rq: If } x_{\mathrm{r1}}(k) \text{ is } A_1^q \text{ and } u_{\mathrm{r1}}(k) \text{ is } B_1^q \ \ldots \\
&\quad \ldots \text{ and } x_{\mathrm{r}M}(k) \text{ is } A_M^q \text{ and } u_{\mathrm{r}M}(k) \text{ is } B_N^q \\
&\quad \text{Then } \bar{y}^q(k) = C(x(k) + x_{\mathrm{p1}}^q(k+1) + x_{\mathrm{p2}}^q(k+1))
\end{aligned} \tag{16}$$

q being the maximum number of rules defining the fuzzy model. As in the case of the original Takagi-Sugeno model, in this case the output is computed as the weighted sum of the outputs of each rule. It is worth to point out that the proposed fuzzy model with second order consequences does not require computing the Hessians and Jacobians of $F(\cdot)$ at each time step. Instead, in the fuzzy identification methodology the parameters of both are identified from input output data at different operating points. Therefore, the computational burden associated with the solution of the second order model is reduced. This fact increases the possibility of using second order Takagi-Sugeno models in real time implementations of predictive control strategies. Next section presents the formulation of the model predictive control with second order fuzzy models as prediction model.

IV. Predictive Control Strategy Design

As in Section III, consider a non-linear system whose dynamic behavior can be described by the discrete-time model (10). The general idea of non-linear model predictive control (NMPC) is to determine the sequence of control actions for the system under control by solving an optimization problem considering the predicted trajectories given by the non-linear discrete time model (10). Commonly, a quadratic cost function (that may be interpreted as the total energy of the system) is used to measure the performance of the system

$$\begin{aligned} L(\widetilde{x}(k), \widetilde{u}(k)) = & \sum_{l=1}^{N_\mathrm{p}} \left[e^T(k+l|k) Q e(k+l|k) \right] \\ & + \sum_{l=0}^{N_\mathrm{u}} \left[u^T(k+l) R u(k+l) \right] \end{aligned} \tag{17}$$

where the superscript T denotes the transpose operation, $e(k+l|k)$ denotes the predicted value of e at time step $k+l$ given the conditions at time step k, $u(k+l)$ denotes the control input u at time step $k+l$, $\widetilde{x}(k) = [x^T(k+1|k), \ldots, x^T(k+N_\mathrm{p}|k)]^T$, $\widetilde{u}(k) = [u^T(k), \ldots, u^T(k+N_\mathrm{u}), \ldots, u^T(k+N_\mathrm{p})]^T$, where $x(k|k) = x(k)$, and $u(k+l) = u(k+N_\mathrm{u})$, for $l = N_\mathrm{u}, \ldots, N_\mathrm{p}$; Q and R are diagonal matrices with positive diagonal elements, and N_u, N_p are the control and prediction horizon respectively, with $N_\mathrm{u} \leq N_\mathrm{p}$.

Let $\mathbb{Y} \subset \mathbb{R}^n$ and $\mathbb{U} \subset \mathbb{R}^m$ denote the feasible sets for the outputs and inputs of the system, i.e., $y(k) \in \mathbb{Y},\ u(k) \in \mathbb{U}$ (these sets are determined by the physical and operational constraints of the system). Then, the NMPC problem can be formulated as the non-linear optimization problem:

$$\begin{aligned} & \min_{\widetilde{u}(k)} L(\widetilde{x}(k), \widetilde{u}(k)) \\ & \textbf{s.t:} \\ & x(k+l+1) = F(x(k+l), u(k+l)) \\ & \hat{y}(k+l) = G(x(k+l), u(k+l)) \\ & \hat{y}(k+l) \in \mathbb{Y}; \qquad u(k+l) \in \mathbb{U} \end{aligned} \tag{18}$$

This optimization problem corresponds to the centralized formulation of the NMPC problem. Although widely studied, the solution of (18) is hard to compute in real time. Therefore, approximations of $F(\cdot)$ and $G(\cdot)$ are often used. The most widely used approximation is the linear. However, those approximations cannot be enough for representing the system dynamics. This is why in this paper the second order fuzzy model derived in Section III is used to predict the system output. Then, from [1] the output of the fuzzy model can be computed as

$$\hat{y}(k+l) = \sum_{j=1}^{q} \beta_j(x_\mathrm{r}(k), u_\mathrm{r}(k)) \bar{y}_j(k+l) \tag{19}$$

where $\beta_j(x_\mathrm{r}(k), u_\mathrm{r}(k))$ denotes the degree of activation of the j-th rule, and $\bar{y}_j(k)$ the local second order approach derived in Section III. Notice that in (19) $\beta_j(x_\mathrm{r}(k), u_\mathrm{r}(k))$ is independent of the time counter l. Thus, it is assumed that the premises remains constant along the prediction horizon, and only are updated at each time step k. That assumption allows reducing the computational burden associated with the fuzzy predictive controller. However, some loss of performance is expected, which might not be significantly because of approximations based on nonlinear functions has wider ranges of validity than the approximations done with linear functions. Moreover, it is also expected that the control actions do not move the system far away from the current operating point during the computation of the control actions over the prediction horizon. Therefore, $\hat{y}(k+l)$ has a matrix representation more suitable for NMPC implementation purposes. In this sense, the predicted output can be expressed as the linear relationship

$$\hat{y}(k+l) = \mathcal{B} y_\mathrm{rules}(k+l) \tag{20}$$

where $y_\mathrm{rules}(k+l) = [\bar{y}_1^T(k+l), \ldots, \bar{y}_q^T(k+l)]^T$ and $\mathcal{B} = [\beta_1(x_\mathrm{r}(k), u_\mathrm{r}(k)), \ldots, \beta_q(x_\mathrm{r}(k), u_\mathrm{r}(k))]$. Thus, the NMPC optimization problem (18) becomes

$$\begin{aligned} & \min_{\widetilde{u}(k)} L(\widetilde{x}(k), \widetilde{u}(k)) \\ & \textbf{s.t:} \\ & \hat{y}(k+l) = \mathcal{B} y_\mathrm{rules}(k+l) \\ & \mathcal{B} y_\mathrm{rules}(k+l) \in \mathbb{Y}; \qquad u(k+l) \in \mathbb{U} \end{aligned} \tag{21}$$

It is worth to point out that since the elements of $y_\mathrm{rules}(k+l)$ are the second order approximations of the dynamic behavior of the system (21), they can be considered convex with respect to $\widetilde{u}(k)$ in almost all the feasible space. Indeed, if the system is operating at a point where the model has non-zero gradient, then $\hat{y}(k+l) = \mathcal{B} y_\mathrm{rules}(k+l)$ tends to be a linear approach of the system dynamics, i.e., the proposed fuzzy model becomes an original Takagi-Suggeno model. But, if the system is operating at points where the model has zero gradient, then by the Morse Lemma $\hat{y}(k+l) = \mathcal{B} y_\mathrm{rules}(k+l)$ tends to be a quadratic approach of the system dynamics. In both cases, the resulting cost function is convex with respect to $\widetilde{u}(k)$. There exists an alternative set of regions where the contribution of both the gradient and the Hessian is similar. In those cases an in depth analysis must be done. Here a detailed analysis of the convexity of (21) is not included because is beyond of the scope of the current work. In the next section, the proposed model predictive control technique will be applied to the stirred tank reactor presented in Section II.

V. CASE STUDY

In order to evaluate the performance of the proposed second-order fuzzy predictive control scheme, the reactor described in Section II was used. As in [5] a one step ahead prediction horizon was considered. The selection of such prediction horizon obeys to comparison purposes. The idea is to compare the performance of the fuzzy predictive controller designed in this paper with the predictive controllers proposed in [5], whose performance was presented in Figure 1. It is worth to notice that the selection of the prediction horizon does not restricts the use of the proposed second order fuzzy predictive control. Indeed, the formulation presented in Sections III and IV is general and can be applied for any prediction horizon. Since the fuzzy predictive control strategy requires a prediction model, an identification procedure was carried out. Figure 3 presents the signal used for the identification procedure and the system response. In this Figure is evident that several steady state was reached during the experiment as well as several operating modes were exited, which is desirable in any identification process.

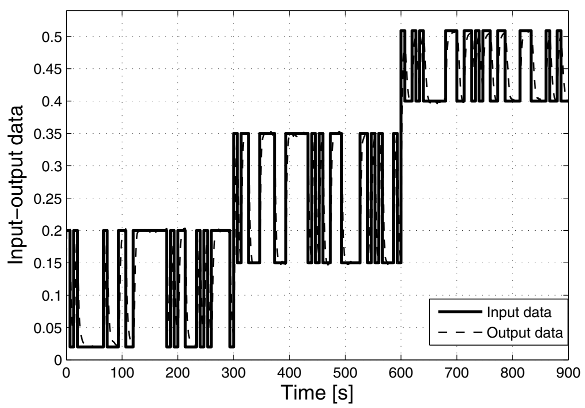

Fig. 3. Input-output data used for the system identification procedure. In this case slower dynamics has a time constant about 5s.

Once obtained the input-output data, the fuzzy modeling was done, following the procedure presented in Figure 2. Since the linear model presented in Section II did not represent the behavior of the system, two fuzzy models were identified for performance comparison: one with linear consequences and one with second order consequences. Both were used as prediction models of 1 inside a model predictive control strategy. Figure 4 presents the results of the identification procedure for the fuzzy model with second order terms. In that Figure, the validation data is compared with the output of the fuzzy model. A root mean square error of 0.0051 in the output of the fuzzy model was obtained after the training procedure. It is worth to remark that compared with Takagi-Sugeno models with linear consequences, including the second order terms allowed decreasing the number of rules required to represent the system behavior. The reduction of the number of rules comes form the fact that approximations done with nonlinear functions (e.g., with models including second order terms) have a wider range of validity than the approximations done with linear functions. For instance, the model with second order terms has only two rules while the model with linear consequences has seventh rules. Hence a trade off between complexity and identificability of the model is achieved. That is, although the model with second order terms has more parameters to be identified, there are significantly less rules than in the models with linear consequences. Thus, the identificability of the model whose consequences have second order terms is not an issue.

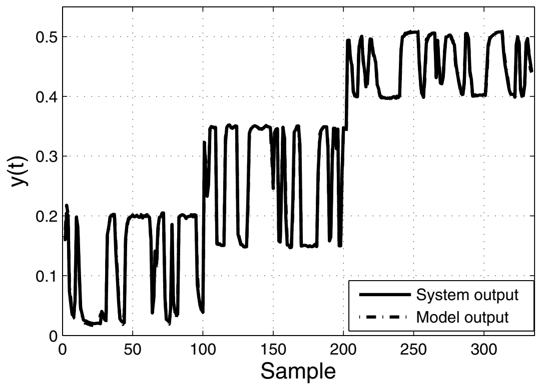

Fig. 4. Comparison of the real output of the system and the output of the fuzzy model after the training procedure.

Figure 5 presents the simulation results obtained for the experiment introduced in Section II. In this Figure is clear that although linear fuzzy model improves the representation of the system behavior, which is reflected in the enhancement of the closed-loop performance of the system, linear approximations are not enough for capturing all the interactions among the states and inputs. This fact is evident in the permanent deviation of the conversion respect to its desired value. Such deviation is not present in the closed-loop behavior of the system when the prediction is performed using the second order fuzzy model.

Finally, with the aim of determining the improvement achieved with the second order fuzzy model respect to the conventional second order approximation, a uniformly distributed signal in the range of -5 and $+5$ was added in the manipulated variable. Figure 6 shows the obtained results. In this Figure, after 30s the predictive control using conventional second order approximation model cannot maintain the conversion at its desired level. This does not happens when the control actions were computed using the proposed second order fuzzy model as a prediction model. In this way, at least for disturbances in the manipulated variable the proposed model provides an increasing in the system robustness.

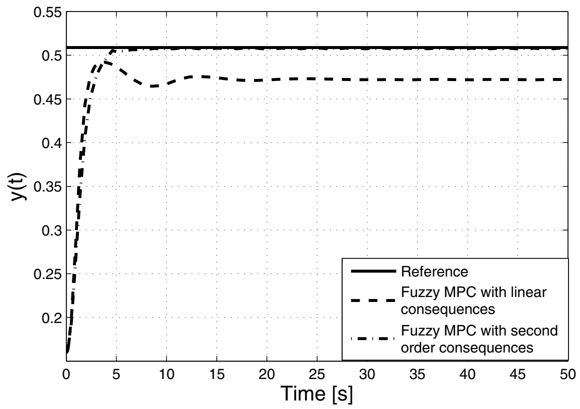

Fig. 5. Simulation results. Here the performance of predictive control with both linear and quadratic fuzzy prediction models is compared.

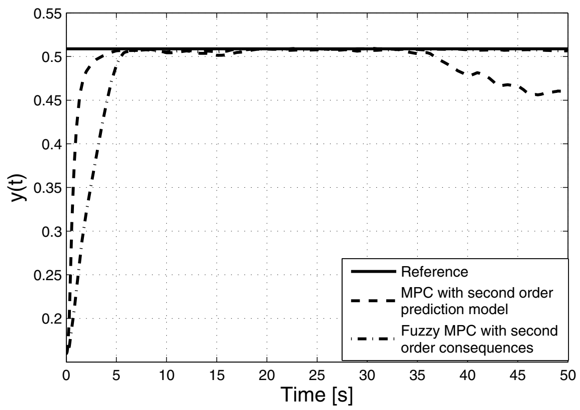

Fig. 6. Simulation results. Here the performance comparison between an MPC with conventional second order approximation model with an MPC with the proposed second order fuzzy prediction model is presented.

VI. Concluding Remarks

In this paper a novel approach based on second order fuzzy models for model predictive control was proposed. This model selection is motivated by the fact that often linear models result very restrictive form the control point of view, even in the neighborhood of the equilibrium point where they were computed. This implies a shrinking of the domain where the system can be controlled. In this paper, adding second order term allows increasing the controllable space of the system. Moreover, using second order fuzzy models enhance the robustness of the system and reduces the computational burden of the second order model predictive controller. Namely, with the second order fuzzy model is not required computing the gradients and the Hessians at each time step. This is the main difference between the proposed predictive control scheme with the approaches already reported in the literature.

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