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Analysis of Fuzzy Cognitive Maps with Multi-Step Learning Algorithms in Valuation of Owner-Occupied Homes

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***Abstract*—In the paper some analysis of multi-step learning algorithms for fuzzy cognitive map (FCM) is given. FCMs, multi-step supervised learning based on gradient method and unsupervised one based on nonlinear Hebbian learning (NHL) algorithm are described. Comparative analysis of these methods to one-step algorithms, from the point of view of the speed of convergence of learning algorithm and the influence on the decision support system for the valuation of owner-occupied homes was performed. Simulation results were obtained with the use of ISEMK (Intelligent Expert System based on Cognitive Maps) software tool. The results show that the implementation of the multi-step technique gives certain possibilities to get quicker values of target FCM relations and improve the operation of the learned system.**

Keywords*—*Fuzzy Cognitive Maps, Multi-step Learning Algorithms, Gradient Method, Nonlinear Hebbian Learning, Decision Support System

I. Introduction

Fuzzy cognitive map (FCM) is a computational model, that describes problem as a set of concepts and links (relations). It is flexible, easy to use tool for modeling complex dynamic decision support systems [1]–[7]. Quite a large number of publications is devoted to the FCMs extensions, which wide review is given in [8]. Each one of them in some aspect improves the operation of traditional maps. Examples of these extensions are augmented FCMs [9], fuzzy grey cognitive maps [10], [11] or relational FCMs [12], [13].

The crucial issue connected with the modeling of decision support systems based on fuzzy cognitive maps are learning algorithms, aimed to improve reliability and performance of FCMs by unsupervised learning (Hebbian algorithm [14]–[18]) or automated generation of FCM by supervised learning (gradient method [18], [17], evolutionary computing [14], [16], [19]–[24]). The researchers constantly strive to achieve the most faithful FCM for the modeled system. An intelligent approach based on artificial immune systems is described to perform the task of classification using FCM learning [25]. The methodology of learning of FCMs using the Bacterial Evolutionary Algorithm (BEA) is proposed in [26]. Multi-step algorithms of FCM weights adaptation, which are some kind of generalization of known one-step methods of learning, is proposed in [17]. Comparative analysis of these methods to one-step algorithms, from the point of view of the influence on the work of the medical prediction system was performed [18].

In this paper, basic multi-step methods of learning are described, together with their comparative analysis to one-step methods, from the point of view of the speed of convergence of learning algorithm and the influence on the decision support system for the valuation of owner-occupied homes. Learning of FCM was based on historical data, taken from the UCI Machine Learning Repository. Simulations together with the analysis results were done with the use of ISEMK (Intelligent Expert System based on Cognitive Maps) software tool. Section II describes the structure and dynamic model of FCM. Section III presents the multi-step supervised and unsupervised learning algorithms for FCMs. Section IV describes the basic features of ISEMK software tool. Section V presents selected results of simulation research of the developed algorithms, done in ISEMK. Section VI contains a summary of the paper.

II. Fuzzy Cognitive Maps

The structure of FCM is based on a directed graph [27]:

$$< X, R > , \tag{1}$$

where $X = [X_1, ..., X_n]^T$ is the set of the concepts, $X_i(t)$ is the value of the i-th concept, $i = 1, 2, ..., n$, n is the number of concepts, $R = \{r_{j,i}\}$ is relations matrix, $r_{j,i}$ is the relation weight between the j-th concept and the i-th concept.

In this paper a nonlinear dynamic model described by the equation (2) was used [28].

$$X_i(t+1) = F\left(X_i(t) + \sum_{j \neq i} r_{j,i} \cdot X_j(t)\right) , \tag{2}$$

where t is discrete time, $t = 0, 1, 2, ..., T$, T is end time of simulation, $F(x)$ is stabilizing function, which can be chosen in the form:

$$F(x) = \frac{1}{1 + e^{-cx}} , \tag{3}$$

where $c > 0$ is a parameter.

The values of the concepts are calculated until FCM reaches one of the following states [29]:

- a fixed point attractor,
- a limit cycle,

- a chaotic attractor.

FCMs have the ability to learn the relations matrix R based on expert knowledge or historical data. The idea of multi-step learning algorithms for fuzzy cognitive maps is presented below.

III. Multi-Step Learning Algorithms for FCMs

The concept of multi-step pseudogradient algorithms (methods with adequate memory) was introduced in optimization of statistic objects, and then analyzed in many works about the parametric adaptation (identification) of dynamic systems [30]. Later, similar methods were introduced in the supervised learning algorithms for artificial neuron networks type MLP (e.g. the backpropagation method with the momentum is a typical two-step algorithm of supervised learning). Using a momentum term is a method of increasing the rate of learning and avoiding the danger of instability [31].

This section describes multi-step learning algorithms for FCMs, which are a form of generalization of known one-step methods. The characteristic feature of these algorithms is the estimation of a current value of the relations matrix elements on the basis of a few previous estimations. Simulation analysis of multi-step learning algorithms for fuzzy cognitive maps was made with the use of the gradient method [17], [18] and the nonlinear Hebbian learning (NHL) algorithm [15].

A. Supervised Learning Using Gradient Method

Multi-step supervised learning based on gradient method is described by the equation [17], [18]:

$$\begin{aligned} r_{j,i}(t+1) = P_{[-1,1]}(\sum_{k=0}^{m_1} \alpha_k \cdot r_{j,i}(t-k) + \\ \sum_{l=0}^{m_2} (\beta_l \cdot \eta_l(t) \cdot (Z_i(t-l) - X_i(t-l)) \cdot y_{j,i}(t-l))) \ , \end{aligned} \tag{4}$$

where $\alpha_k, \beta_l, \eta_l$ are learning parameters, which are determined using experimental trial and error method, $k = 1,...,m_1; l = 1,...,m_2$, m_1, m_2 are the number of the steps of the method, t is a time of learning, $t = 0, 1, ..., T$, T is end time of learning, $y_{j,i}(t)$ is a sensitivity function, $P_{[-1,1]}(x)$ is an operator of design for the set [-1,1] and described by an exemplary relation:

$$P_{[-1,1]}(x) = \begin{cases} 1 & \text{for } x \geq 1 \\ x & \text{for } -1 < x < 1 \\ -1 & \text{for } x \leq -1 \end{cases} . \tag{5}$$

Sensitivity function $y_{j,i}(t)$ is described as follows:

$$\begin{aligned} y_{j,i}(t+1) = (y_{j,i}(t) + X_j(t)) \cdot F'(X_i(t) \\ + \textstyle\sum_{j \neq i} r_{j,i} \cdot X_j(t)) \ , \end{aligned} \tag{6}$$

where $F'(x)$ is derivative of the stabilizing function.

Termination criterion for the gradient method (4) can be expressed by the formula:

$$J(t) = \frac{1}{n} \sum_{i=1}^{n} (Z_i(t) - X_i(t))^2 < e \ , \tag{7}$$

where $X_i(t)$ is the value of the i-th concept, $Z_i(t)$ is the reference value of the i-th concept and e is a level of error tolerance.

Learning parameters α_k, β_l, η_l have to satisfy the conditions (8)–(12) to reach the convergence of the multi-step gradient method [17], [18].

$$\sum_{k=0}^{m_1} \alpha_k = 1 \ , \tag{8}$$

$$0 < \eta_l(t) < 1 \ , \tag{9}$$

$$\eta_l(t) = \frac{1}{\lambda_l + t} \ , \tag{10}$$

$$\lambda_l > 0 \ , \tag{11}$$

$$\beta_l \geq 0 \ . \tag{12}$$

A special case of multi-step gradient method is the one-step algorithm, which modifies the relations matrix on the basis of one previous estimation, according to the formula [17], [18]:

$$\begin{aligned} r_{j,i}(t+1) = P_{[-1,1]}(r_{j,i}(t) + \\ \beta_0 \cdot \eta_0(t) \cdot (Z_i(t) - X_i(t)) \cdot y_{j,i}(t)) \ . \end{aligned} \tag{13}$$

B. Unsupervised Learning Using NHL Algorithm

Multi-step unsupervised learning based on the NHL algorithm can be expressed as follows [32]:

$$\begin{aligned} r_{j,i}(t+1) = P_{[-1,1]}(\sum_{k=0}^{m_1} \alpha_k \cdot r_{j,i}(t-k) + \sum_{l=0}^{m_2} (\beta_l \\ \eta_l(t) X_i(t-l) \left(X_j(t-l) - r_{j,i}(t-l) X_i(t-l) \right))) \ . \end{aligned} \tag{14}$$

Termination criterion for the NHL algorithm (14) can be described by the formula:

$$J(t) = \frac{1}{n} \sum_{i=1}^{n} (X_i(t) - X_i(t-1))^2 < e \ . \tag{15}$$

Learning parameters α_k, β_l, η_l have to satisfy the conditions (16)–(20) to obtain the convergence of the multi-step NHL algorithm [30], [32].

$$\sum_{k=0}^{m_1} \alpha_k = 1 \ , \tag{16}$$

$$0 < \eta_l(t) < 1 \ , \tag{17}$$

$$\eta_l(t) = e^{-\lambda_l \cdot t} \ , \tag{18}$$

$$\lambda_l > 0 \ , \tag{19}$$

$$\beta_l \geq 0 \ . \tag{20}$$

A special case of multi-step NHL algorithm is the one-step method, which can be described by the equation [32]:

$$\begin{aligned} r_{j,i}(t+1) = P_{[-1,1]}(r_{j,i}(t) + \\ \beta_0 \cdot \eta_0(t) \cdot X_i(t)(X_j(t) - r_{j,i}(t) X_i(t))) \ . \end{aligned} \tag{21}$$

Simulation analysis of the algorithm was done with the use of ISEMK software tool. The basic functionality of ISEMK is described below.

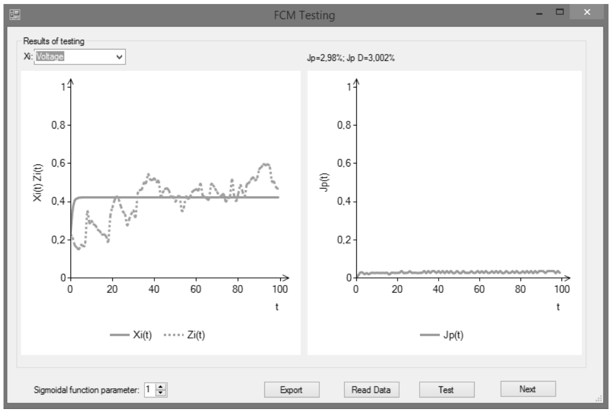

Fig. 1. Exemplary visualization of testing of learned FCM in the ISEMK system

IV. ISEMK Software Tool

ISEMK is a computer software, which is a universal tool for modeling decision support systems based on FCMs [33]. Its features are:

- the synthesis of fuzzy cognitive maps,
- the implementation of initial values of relations and concepts based on expert knowledge (using graphical user interface) and historical data (reading from .data files),
- reading and writing of FCM parameters using .xml files,
- dynamic monitoring of fuzzy cognitive maps,
- multi-step supervised and unsupervised learning, with the use of historical data (reading from .data files),
- the analysis of designed learned FCMs by testing of system operation based on historical data (reading from .data files),
- exporting data of learning and FCMs analysis to .csv files,
- proper visualizations of done research (in the form of charts) on devised computer software.

Fig. 1 shows an exemplary visualization of testing of learned FCM.

V. Simulation Results

The simulations of multi-step learning algorithms for FCMs were performed on the example of the valuation of owner-occupied homes. FCM was initialized, learned and tested on the basis of historical data taken from the UCI Machine Learning Repository [34]. The map with the following concepts was analyzed:

- X_1 – CRIM: per capita crime rate by town,
- X_2 – ZN: proportion of residential land zoned for lots over 25,000 sq.ft.,
- X_3 – INDUS: proportion of non-retail business acres per town,

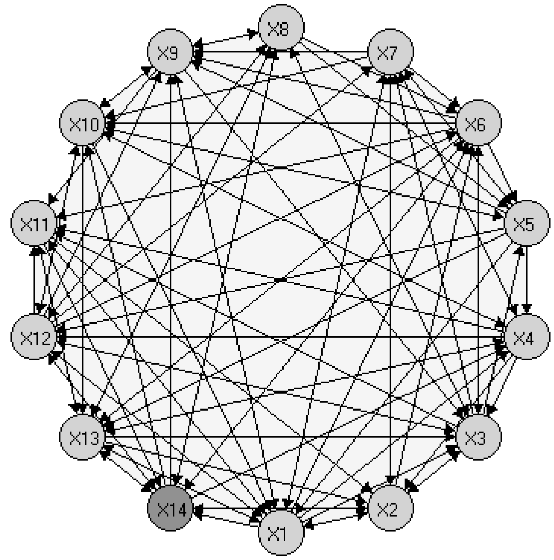

Fig. 2. Structure of the initialized map

- X_4 – CHAS: Charles River dummy variable (= 1 if tract bounds river; 0 otherwise),
- X_5 – NOX: nitric oxides concentration (parts per 10 million),
- X_6 – RM: average number of rooms per dwelling,
- X_7 – AGE: proportion of owner-occupied units built prior to 1940,
- X_8 – DIS: weighted distances to five Boston employment centers,
- X_9 – RAD: index of accessibility to radial highways,
- X_{10} – TAX: full-value property-tax rate per $10,000,
- X_{11} – PTRATIO: pupil-teacher ratio by town,
- X_{12} – B: $1000(Bk-0.63)^2$ where Bk is the proportion of blacks by town,
- X_{13} – LSTAT: % lower status of the population,
- X_{14} – MEDV: Median value of owner-occupied homes in $1000's – output concept.

The relations matrix was initialized with small random values from the interval $[-0.2, 0.2]$. The structure of the initialized map is presented in Fig. 2.

The following subsections present selected results of comparative analysis of multi-step methods to one-step algorithms, from the point of view of:

- the speed of convergence of learning algorithm and average percentage prediction errors of the map learned based on gradient method type (4),
- the speed of convergence of learning algorithm and average percentage prediction errors of the map learned based on NHL method type (14),
- the convergence of the map, learned with using gradient method type (4), to the acceptable region of concepts values.

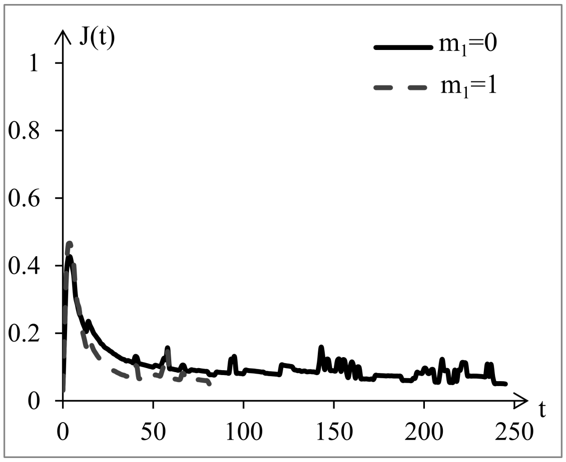

Fig. 3. Obtained values of $J(t)$ during learning

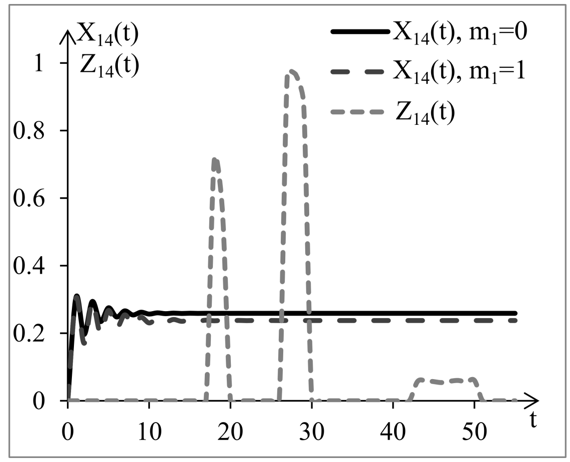

Fig. 4. Obtained values $X_{14}(t)$ and the desired values $Z_{14}(t)$ during testing

A. Speed of convergence and average percentage prediction error for the multi-step learning with the use of gradient method

In order to analyze the influence of the steps number (m_1, m_2) on the prediction errors and speed of convergence of learning algorithm based on the gradient method, the FCM was learned with 450 records. Another 56 records were used in testing the learned FCMs. The learning process was accomplished for various learning parameters. Table I presents part of the relations matrix of the FCM learned for the following parameters $m_1 = 1$, $m_2 = 0$ ($\alpha_0 = 1.6$, $\alpha_1 = -0.6$, $\beta_0 = 10$, $\lambda_0 = 100$). Fig. 3 illustrates the results of learning of the FCM for the number of steps $m_1 = 0$, $m_2 = 0$ ($\alpha_0 = 1$, $\beta_0 = 10$, $\lambda_0 = 100$) and $m_1 = 1$, $m_2 = 0$ ($\alpha_0 = 1.6$, $\alpha_1 = -0.6$, $\beta_0 = 10$, $\lambda_0 = 100$). Fig. 4 shows the results of testing the operation of the learned map.

TABLE I. EXEMPLARY PARTIAL RELATIONS MATRIX FOR THE MAP LEARNED WITH THE TWO-STEP GRADIENT METHOD

	...X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}...
X_1	-0.6	-0.7	-0.4	0	-0.1	-0.9	-0.9	-0.6
X_2	0	-0.1	-0.7	-0.6	-0.6	-0.3	-0.3	-0.6
X_3	-0.3	-0.1	-0.2	-0.7	-0.5	0.1	-0.4	-0.1
X_4	0	-0.7	-0.1	0.1	-0.6	-0.6	-0.2	-0.7
X_5	-0.7	0	-0.9	-0.4	-0.1	-0.7	-0.2	0.1
X_6	-0.7	-0.4	0	-0.7	-0.1	0.1	-0.9	-0.6
X_7	-0.6	-0.1	-0.3	0	0.2	-0.3	-0.2	0
X_8	0	-0.1	-0.8	-0.4	0	-0.4	-0.3	-0.3
X_9	-0.4	-0.9	-0.6	-0.6	-0.1	0	-0.6	-0.3
X_{10}	-0.5	-0.6	-0.4	-0.3	-0.1	-0.6	0	-0.3
X_{11}	-0.4	-0.7	-0.4	-0.4	-0.4	-0.2	-0.5	0
X_{12}	0.1	-0.2	-0.3	-0.1	-0.2	-0.4	-0.4	-0.2
X_{13}	-0.4	-0.2	-0.3	-0.6	-0.1	-0.6	-0.2	0.2
X_{14}	-0.4	-0.8	-0.3	-0.4	-0.5	-0.5	-0.5	-0.3

The speed of convergence of learning algorithm T and the average percentage prediction errors for all concepts, described by the formula (22) and for the output concept (X_{14}), described by the formula (23) were compared.

$$J_P = \frac{1}{n_t} \sum_{t=0}^{n_t-1} \left(\frac{1}{n} \sum_{i=1}^{n} \left(Z_i(t) - X_i(t)\right)^2 \right) \cdot 100\% \ , \qquad (22)$$

where t is a time of testing, $t = 0, 1, ..., n_t-1$, n_t is the number of the test records, $X_i(t)$ is the value of the i-th concept, $Z_i(t)$ is the reference value of the i-th concept, $i = 1, 2, ..., n$, n is the number of concepts.

$$J_{P14} = \frac{1}{n_t} \sum_{t=0}^{n_t-1} \left(Z_{14}(t) - X_{14}(t)\right)^2 \cdot 100\% \ . \qquad (23)$$

Optimal parameters of the multi-step gradient method were chosen based on minimization of the objective function, described as follows:

$$f(J_{P14}, J_P, T) = 100 J_{P14} + 100 J_P + T \ . \qquad (24)$$

Table II presents the results of this analysis for the number of steps: $m_1 \leq 3$ and $m_2 = 0$. Obtained results show that increasing the number of steps m_1 gives certain possibilities to get quicker values of target FCM relations and consequent improves the operation of the learned FCM by reducing prediction errors. The minimum of objective function $f(J_{P14}, J_P, T) = 1657.5$ was obtained for the following parameters: $m_1 = 1$, $\alpha_0 = 1.7$, $\alpha_1 = -0.7$.

TABLE II. CHOSEN RESULTS OF ANALYSIS OF THE MULTI-STEP SUPERVISED LEARNING BASED ON GRADIENT METHOD FOR $\beta_0 = 10$, $\beta_1 = \beta_2 = \beta_3 = 0$, $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = 100$, $e = 0.05$

α_k, $k = 0, 1, ..., 3$	$J_{P14}[\%]$	$J_P[\%]$	T	$f(J_{P14}, J_P, T)$
1 0 0 0	8.652	7.988	246	1910
1.2 -0.2 0 0	8.444	7.662	207	1817.6
1.4 -0.4 0 0	8.199	7.203	189	1729.2
1.5 -0.5 0 0	8.197	7.039	163	1686.6
1.6 -0.6 0 0	7.995	8.443	82	1725.8
1.7 -0.7 0 0	7.607	8.148	82	**1657.5**
1.9 -0.9 0 0	6.173	12.552	44	1916.5
1.9 -0.7 0.4 -0.6	6.734	11.907	46	1910.1
1.5 -0.5 0.5 -0.5	8.359	7.121	176	1724
1.5 -0.6 0.1 0	8.157	7.019	163	1680.6
1.5 -0.5 0.1 -0.1	8.203	7.027	166	1689
0.6 0.4 0 0	9.252	8.227	450	2197.9

B. Speed of convergence and average percentage prediction error for the multi-step learning with using NHL method

In order to analyze the influence of the steps number (m_1, m_2) on the prediction errors (22), (23) and speed of convergence T of learning algorithm based on the NHL method, the FCM was learned with the use of two-step gradient method.

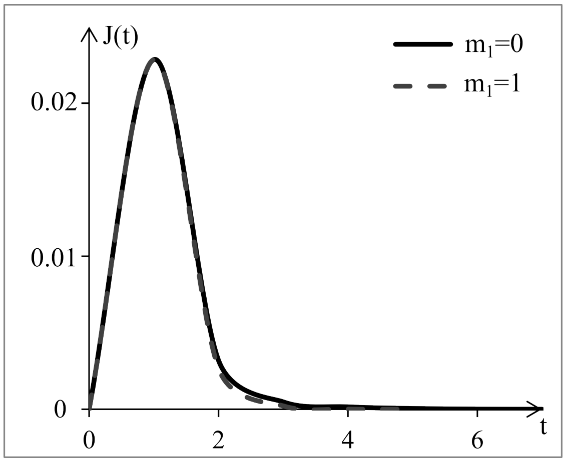

Fig. 5. Obtained values of $J(t)$ during learning using first record

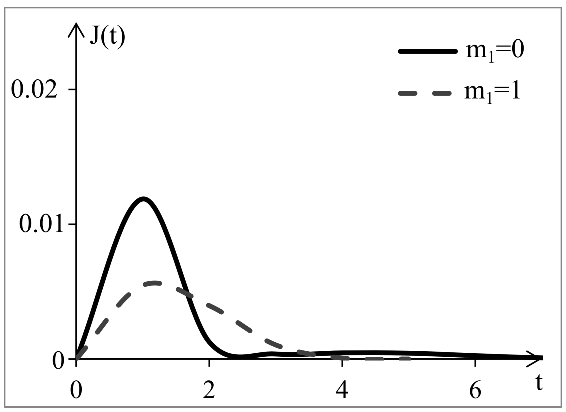

Fig. 6. Obtained values of $J(t)$ during learning using third record

Values of the concepts were initialized with 3 first records from the set of testing records for the gradient method and then the map was learned based on the NHL algorithm. Another 53 records were used in testing the operation of the learned map. Optimal parameters of the multi-step NHL method were chosen based on minimization of the objective function (24). Fig. 5 and fig. 6 illustrate the results of learning of the FCM for the number of steps $m_1 = 0$, $m_2 = 0$ ($\alpha_0 = 1$, $\beta_0 = 0.1$, $\lambda_0 = 1$) and $m_1 = 1$, $m_2 = 0$ ($\alpha_0 = 1.9$, $\alpha_1 = -0.9$, $\beta_0 = 0.1$, $\lambda_0 = 1$). Fig. 7 shows the results of testing the operation of the learned map.

Table III presents the results of the analysis for the number of steps: $m_1 \leq 3$ and $m_2 = 0$. Obtained results show that increasing the number of steps m_1 gives certain possibilities to get quicker values of target FCM relations and consequent improves the operation of the learned FCM by reducing prediction errors. The minimum of objective function $f(J_{P14}, J_P, T) = 1322.9$ was obtained for the following parameters: $m_1 = 1$, $m_2 = 0$, $\alpha_0 = 1.9$, $\alpha_1 = -0.9$ and $m_1 = 3$, $m_2 = 0$, $\alpha_0 = 1.9$, $\alpha_1 = -1$, $\alpha_2 = 0.2$, $\alpha_3 = -0.1$.

TABLE III. CHOSEN RESULTS OF ANALYSIS OF THE MULTI-STEP UNSUPERVISED LEARNING BASED ON NHL ALGORITHM FOR $\beta_0 = 0.1$, $\beta_1 = 0$, $\beta_2 = 0$, $\beta_3 = 0$, $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = 1$, $e = 0.00001$

$\alpha_l, l = 0, 1, 2, 3$	$J_{P14}[\%]$	$J_P[\%]$	T	$f(J_{P14}, J_P, T)$
1 0 0 0	7.541	6.553	30	1439.4
1.2 -0.2 0 0	7.509	6.487	31	1430.6
1.5 -0.5 0 0	7.413	6.306	23	1394.9
1.7 -0.7 0 0	7.295	6.118	19	1360.3
1.9 -0.9 0 0	7.127	5.932	17	**1322.9**
1.9 -1 0.2 -0.1	7.127	5.932	17	**1322.9**

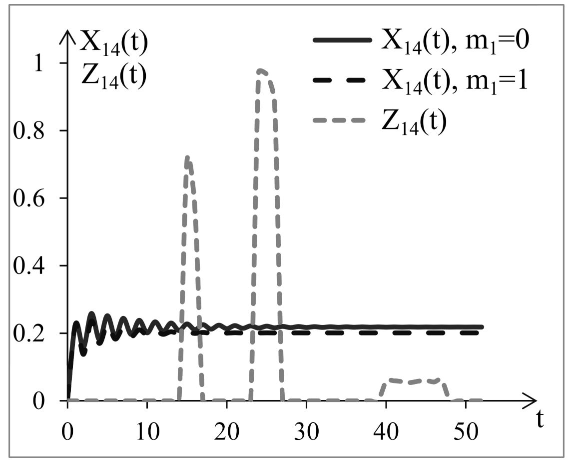

Fig. 7. Obtained values $X_{14}(t)$ and the desired values $Z_{14}(t)$ during testing

C. Convergence of the map learned with the gradient method to the acceptable region of concepts values

In order to analyze the influence of the steps number (m_1, m_2) on the convergence of the decision support system to acceptable region of concepts values for proper operation, the FCM was initialized with small random values from the interval $[-0.2, 0.2]$ and learned with 450 records. Another 56 records were used in testing the learned maps. The learning process was accomplished for various learning parameters and $e = 0.01$. Fig.8 illustrates the results of testing the map learned with the number of steps $m_1 = 0$ ($\alpha_0 = 1$, $\beta_0 = 30$, $\lambda_0 = 100$) and $m_1 = 5$ ($\alpha_0 = 0.3$, $\alpha_1 = \alpha_2 = 0.2$, $\alpha_3 = \alpha_4 = \alpha_5 = 0.1$, $\beta_0 = 30$, $\lambda_0 = 100$). The FCM learned with the multi-step method reached the fixed point attractor quicker than the map learned with the one-step algorithm. Fig.9 shows the results of testing the map learned with the number of steps $m_1 = 0$ ($\alpha_0 = 1$, $\beta_0 = 50$, $\lambda_0 = 100$) and $m_1 = 5$ ($\alpha_0 = 0.3$, $\alpha_1 = \alpha_2 = 0.2$, $\alpha_3 = \alpha_4 = \alpha_5 = 0.1$, $\beta_0 = 50$, $\lambda_0 = 100$). The FCM learned with the multi-step method converged to the acceptable region of concepts values, while the map learned with the one-step algorithm did not converge.

VI. CONCLUSION

This paper contains the description of FCMs and multi-step supervised learning algorithms based on gradient method as well as unsupervised one based on NHL method. Simulation analysis of the algorithms was done with the use of ISEMK software tool. Learning and testing of FCMs was based on historical data. Chosen results of the analysis, showing the influence of the implementation of the multi-step technique on the speed of convergence of the learning algorithm and functioning of the decision support system for the valuation of owner-occupied homes, were presented. It could be stated that the implementation of the multi-step technique gives certain possibilities to get quicker values of target FCM relations.

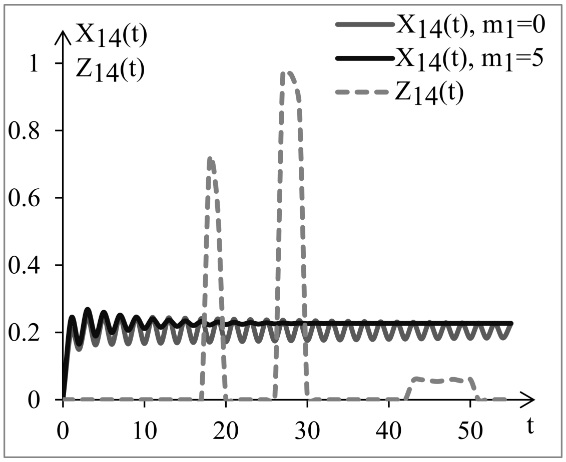

Fig. 8. Obtained values $X_{14}(t)$ and the desired values $Z_{14}(t)$ during testing

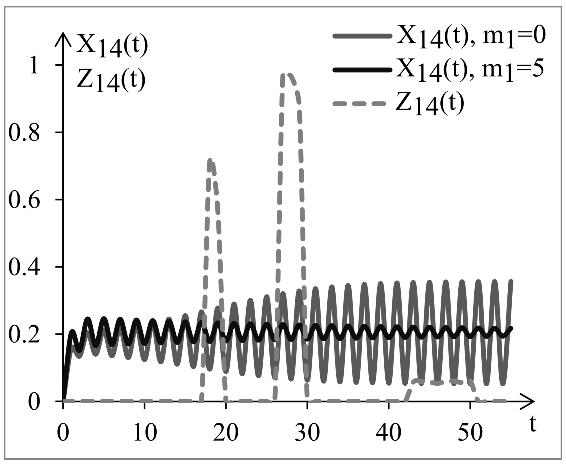

Fig. 9. Obtained values $X_{14}(t)$ and the desired values $Z_{14}(t)$ during testing

The advantage of the application of multi-step algorithms is also the improvement of the functioning of the learned system by reducing the average percentage prediction errors and obtaining convergence to the acceptable region of concepts values. There are plans of further analysis of multi-step learning algorithms in prediction of time-series. In addition, it is planned to implement evolutionary based algorithms for FCMs learning.

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