2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

Faults Diagnosis based on Proportional Integral Observer for TS Fuzzy Model with Unmeasurable Premise Variable

T. Youssef, H. R. Karimi, M. Chadli

***Abstract*—In this work, we focus on the synthesis of a Proportional Integral (PI) observer for the actuators and sensors faults diagnosis based on Takagi-Sugeno (TS) fuzzy model with unmeasurable premise variables. The faults estimation method is based on the assumption that these faults act as unknown inputs under polynomials form whose their *k*th derivatives are bounded. The convergence conditions of the observer as well as the faults reconstruction are established on the basis of the Lyapunov stability theory and the L_2 optimization technique, expressed as Linear Matrix Inequalities (LMI) constraints. In order to validate the proposed approach, a hydraulic system with two tanks is proposed.**

I. INTRODUCTION

The industrial systems are become more vulnerable with respect to faults due to need to continually improve the process performances and product quality. The fault is any unacceptable deviation of the nominal value of a characteristic component of the system. It is therefore essential to guarantee the safety and good operating of these processes by means of faults diagnostic systems. The diagnostic techniques of nonlinear systems, in particular the approaches based on state observers, continue to receive an increasing interest by the scientific community in the areas of fundamental and applied research, we can mention for example the works developed in [1]-[3]. These diagnostic methods are based on the analytical redundancy principle which has the ability to have two or several ways to obtain a characteristic quantity of the system. The technique of Fault Detection and Isolation (FDI) based on the analytical model is one of the most used approaches in the field of fault diagnosis. The diagnosis is a mechanism which contains besides the detecting and locating steps, the fault estimation step. The diagnosis of nonlinear systems with state observers is a direct application of state estimation, which will be discussed in this article to estimate actuator and sensor faults of dynamic systems.

In the presence of faults, the state estimation can detect any abnormal behavior of the real system from a comparison between measured and estimated of outputs signals. Through the analytical redundancy principle, the signals comparison, which is the detection step, allows to generate fault indicators also called residues. The location step comes next, which exactly determines the affected component type, actuator or sensor in a dynamic system. Finally, the fault estimation or identification step completes the diagnosis task by evaluating the fault for each time.

Initially, the FDI systems have been developed with approaches based on linear or linearized models [4], and more recently, they have been used to realize the diagnostic task [5]. It is well known that the linear model reflects the system behavior only locally, near an operating point, and not on all the operating range. Indeed, the operating point changes with the presence of fault, therefore the linear model is not representative. It is thus clear that the model-based diagnosis requires a precise knowledge of system dynamic behavior to synthesize in a reliable way the residues. In the context of diagnosis and among nonlinear representations we have the approach by fuzzy model of Takagi-Sugeno (TS), which is an interesting alternative to approximate the nonlinear behavior of various dynamical systems [6]. Through this TS fuzzy model approach, the dynamic system is represented by a set of fuzzy IF-THEN rules which describe the linear input-output relations, and the global behavior is obtained by an interpolation of linear models using nonlinear activation functions [7]. Thus, the obtained TS fuzzy model accurately represents the dynamic system on an operating wide range. The advantage of this approach is to extend the analysis and synthesis tools of linear theory for nonlinear systems. Therefore, the estimation of fault indicators is based on the synthesis of nonlinear observers using the same activation functions as the TS fuzzy model. There are two premise variables types which can intervene in the construction of activation functions, measurable for inputs and/or outputs, and unmeasurable for state variables. The latter can approximate a large class of dynamic systems by TS fuzzy models approach [8]. The works [7] and [8] address the stability analysis and stabilization of TS fuzzy models for design of observers and control laws.

In the framework of faults estimation of actuators and/or sensors and diagnosis based on the nonlinear state observer with measurable premise variables, we can mention the works developed in [9]-[16]. The unknown inputs observers are designed in [9] for TS discrete descriptor systems applied to fault diagnosis, and in [10] for disturbed TS continuous systems for detection and location of sensor fault of an automatic steering vehicle. The authors in [11] study the estimation of actuator and sensor faults by considering them as an auxiliary variable in the synthesis phase of a TS

This work was supported in part by the Aurora program. Aurora is the Franco-Norwegian Hubert Curien Partnership (PHC).

T. Youssef is with the Laboratory of Automatic Applied (LAA), M'hamed Bougara University of Boumerdès (UMBB), Boumerdès 35000, Algeria (e-mail: tewfikyoussef@umbb.dz).

H.R. Karimi is with the Department of Engineering, Faculty of Engineering and Science, University of Agder, Grimstad 4898, Norway (e-mail: hamid.r.karimi@uia.no).

M. Chadli is with the Laboratory of Modeling, Information & Systems (MIS), University of Picardie Jules Verne (UPJV), 33 rue Saint Leu, 80039 Amiens Cedex 1, France (e-mail: mchadli@u-picardie.fr).

978-1-4799-2072-3/14/$31.00 ©2014 IEEE

descriptor observer. A robust fault detection observer is designed in [16] by using $H_{\square}/H_{\infty}$ formulation for discrete-time TS fuzzy system affected by sensor faults and unknown bounded disturbances. In the case of the Proportional Integral (PI) observer synthesis, the authors in [12] propose the sensor faults estimation for disturbed TS fuzzy models. In [13], the authors deal the sensor faults diagnosis of a bioreactor described by multiple models approach. The fouling detection in a heat exchanger is studied in [14] by TS observer for systems subject to the unknown polynomial inputs. In [15], a fault tolerant control strategy is developed which compensates an actuator fault for TS systems.

Instead, the works developed in the framework of unmeasurable premise variables for state estimation and sensor faults based on observers, we can mention for example [17]-[20]. The unmeasurable premise variable in [17] is considered as a Lipchitz constraint on the activation functions for state estimation of a hydraulic system with three tanks represented by multiple models approach. The authors in [18] and [19] are interested to the simultaneous estimation of the state and the unknown inputs in polynomial form for TS systems by considering the unmeasurable premise variable as a disturbed system, and as a Lipchitz constraint, respectively. In [20], the robust observer synthesis with unknown input on the states and outputs is considered for continuous and discrete TS models subjected to disturbances. Most of these works use the Lyapunov stability theory and some the L_2 optimization technique to formulate the stability conditions for the state observers synthesis in term linear matrix inequalities constraints [21].

In the context of state estimation, we are particularly interested to the synthesis of a TS unknown inputs proportional integral observer with unmeasurable premise variables, because of their simultaneous estimations of states and unknown inputs. Inspired by the works developed in [18] and [19] we are addressed in this study to diagnosis of actuator and sensor faults based on PI observer of TS fuzzy model [22]. This model is subject to unknown inputs in polynomial form whose their kth derivatives are assumed bounded, and which act as faults on the dynamic system.

Our contribution in this paper is to design a proportional integral observer which deals with the faults diagnostic problem with unmeasurable premise variables by means of an additional parameter which compensates the effect due to these variables and faults which act as unknown inputs. Moreover, the proposed PI observer will allow reconstructing the time varying faults directly without using a FDI system because it allows realizing the identification stage which is an interesting approach for fault diagnosis.

The sufficient conditions are derived to design the PI observer by using the Lyapunov stability theory and L_2–gain technique to minimize the transfer of the bounded unknown input and disturbance towards the state estimation error, all formulated in terms LMI constraints.

Sections developed in this work are the following. In Section II, we show the structure of TS fuzzy model with unmeasurable premise variables. This TS model is subject to actuators and sensors faults, which can affect the dynamics and output of the system. In section III, the structure of the PI observer is presented. In section IV, the synthesis of the proposed PI observer is developed. Finally, in Section V, a simulation example is given of a hydraulic system in two tanks to show both the good estimation of states and actuator or sensor faults.

II. Structure of the TS Fuzzy Model

The considered TS fuzzy model with unmeasurable premise variables subject to faults which affect both sensor and actuator is as follow:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x)\big(A_i x(t) + B_i u(t) + E_i f_a(t)\big) \\ y(t) = Cx(t) + E f_s(t) + F w(t) \end{cases} \tag{1}$$

where $x(t) \in R^{n}$ represents the state vector, $u(t) \in R^{n_u}$ the input vector, $f_a(t) \in R^{n_{f_a}}$ and $f_s(t) \in R^{n_{f_s}}$ are the actuators and sensors faults vectors, respectively, $w(t) \in R^{n_w}$ is measurement noise vector, and $y(t) \in R^{n_y}$ represents the output vector. $A_i \in R^{n\times n}$ are the state matrices, $B_i \in R^{n\times n_u}$ are the input matrices, $C \in R^{n_y\times n}$ is the output matrix, $E_i \in R^{n\times n_{f_a}}$ and $E \in R^{n_y\times n_{f_s}}$ are the faults matrices, and $F \in R^{n_y\times n_w}$ is the disturbance matrix. The $\mu_i(x)$ represent the activation functions which depend on the state $x(t)$ of the system. These functions satisfy the convex sum property:

$$\begin{cases} \sum_{i=1}^{r} \mu_i(x) = 1, & \forall\, t \geq 0 \\ 0 \leq \mu_i(x) \leq 1, & for\ i{:}\ 1 \ldots r \end{cases} \tag{2}$$

where r is the number of linear models.

Hypothesis 1: We assume that faults $f(t)$ appear as unknown inputs in polynomial form of k-1 degree, depending on the time and their kth derivatives are bounded, denoted f_0.

The following notations are introduced:

$$\begin{cases} \dot{f}(t) = f_1(t) \\ \dot{f}_1(t) = f_2(t) \\ \quad\vdots \\ \dot{f}_{k-1}(t) = f_k(t) \\ f_k(t) \leq f_0 \end{cases} \tag{3}$$

where $f(t) = [f_a^{\,T}(t) \quad f_s^{\,T}(t)]^T$, $f(t) \in R^{n_f}$ with $n_f = n_{f_a} + n_{f_s}$.

Remark 1: The polynomial form allows considering a wide range of actuators and sensors faults affecting a dynamic system [23].

III. Structure of the PI Observer

Based on the structure of the TS fuzzy model (1) we propose the PI observer (4). This allows estimating simultaneously the states and actuators or sensors faults in presence of unmeasurable premise variables.

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x})\left(A_i \hat{x}(t) + B_i u(t) + R_i \hat{f}(t) + K_{Pi}\big(y(t) - \hat{y}(t)\big)\right) + z_x(t) \\ \hat{y}(t) = C\hat{x}(t) + R\hat{f}(t) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}) K_{Ii}\big(y(t) - \hat{y}(t)\big) + \hat{f}_1(t) + z_f(t) \\ \dot{\hat{f}}_j(t) = \sum_{i=1}^{r} \mu_i(\hat{x}) K_{Ii}^{j}\big(y(t) - \hat{y}(t)\big) + \hat{f}_{j+1}(t) + z_{fj}(t) \quad \text{for} \quad j{:}\ 1 \ldots k-1 \end{cases} \tag{4}$$

where $R_i = [E_i \quad 0]$ and $R = [0 \quad E]$ are the faults matrices, and $K_{Pi} \in R^{n \times n_y}$ and $K_{Ii} \in R^{n_f \times n_y}$, $K_{Ii}^j \in R^{n_f \times n_y}$ represent the proportional and integral gains, respectively. The variables $z_x(t)$ and $z_f(t)$, $z_{fj}(t)$ are introduced in order to compensate the effect due to unmeasurable premise variables.

Based on *hypothesis 1*, the TS model (1) and PI observer (4) can be written in the augmented forms, respectively, as follow:

$$\begin{cases} \dot{\bar{x}}(t) = \sum_{i=1}^{r} \mu_i(x)(\bar{A}_i \bar{x}(t) + \bar{B}_i u(t)) + G f_k(t) \\ y(t) = \bar{C}\bar{x}(t) + Fw(t) \end{cases} \tag{5}$$

and

$$\begin{cases} \dot{\hat{\bar{x}}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}) \left(\bar{A}_i \hat{\bar{x}}(t) + \bar{B}_i u(t) + \bar{K}_i\big(y(t) - \hat{y}(t)\big)\right) + G\hat{f}_k(t) + z(t) \\ \hat{y}(t) = \bar{C}\hat{\bar{x}}(t) \end{cases} \tag{6}$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ f(t) \\ f_1(t) \\ \cdots \\ f_{k-1}(t) \end{bmatrix}, \hat{\bar{x}}(t) = \begin{bmatrix} \hat{x}(t) \\ \hat{f}(t) \\ \hat{f}_1(t) \\ \cdots \\ \hat{f}_{k-1}(t) \end{bmatrix}, z(t) = \begin{bmatrix} z_x(t) \\ z_f(t) \\ z_{f1}(t) \\ \cdots \\ z_{fk-1}(t) \end{bmatrix} \tag{7a}$$

with $\bar{x}(t) \in R^{(n+k \times n_f)}$, $z(t) \in R^{(n+k \times n_f)}$ and

$$\bar{e}(t) = \bar{x}(t) - \hat{\bar{x}}(t),\ \bar{e}_y(t) = y(t) - \hat{y}(t),\ \bar{E}_y(t) = \bar{e}_y - Fw \tag{7b}$$

$$\bar{A}_i = \begin{bmatrix} A_i & R_i & 0 & 0 & \cdots & 0 \\ 0 & 0 & I_{n_f} & 0 & \cdots & 0 \\ 0 & 0 & 0 & I_{n_f} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & I_{n_f} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix},\ \bar{K}_i = \begin{bmatrix} K_{Pi} \\ K_{Ii} \\ K_{Ii}^1 \\ \cdots \\ K_{Ii}^{k-1} \end{bmatrix} \tag{7c}$$

$$G = [0 \quad 0 \quad 0 \quad \cdots \quad I_{n_f}]^T,\ \bar{C} = [C \quad R \quad 0 \quad \ldots \quad 0] \tag{7d}$$

and I_{n_f} is the identity matrix.

The dynamics of the augmented state estimation error $\bar{e}(t)$ obtained on the basis of the TS model (5) and the PI observer (6) is given by the following equation:

$$\dot{\bar{e}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x})(\mathcal{A}_i \bar{e}(t) - \bar{K}_i F w) + \bar{\Delta} A \bar{x}(t) + \bar{\Delta} B u(t) + G\Delta f_k(t) - z(t) \tag{8}$$

where

$$\mathcal{A}_i = \bar{A}_i - \bar{K}_i \bar{C},\ \bar{\Delta} A = \textstyle\sum_{i=1}^{r} \bar{\mu}_i \bar{A}_i,\ \bar{\Delta} B = \sum_{i=1}^{r} \bar{\mu}_i \bar{B}_i, \tag{9.a}$$

$$\bar{\mu}_i = \mu_i(x) - \mu_i(\hat{x}),\ \Delta f_k(t) = f_k(t) - \hat{f}_k(t) \tag{9.b}$$

Remark 2: According to the convex sum property (2) of activation functions we can write $-1 < \bar{\mu}_i < 1$, note that $\sum_{i=1}^{r} \bar{\mu}_i$ are not convex sums, and the variables matrices $\bar{\Delta} A$ and $\bar{\Delta} B$ are bounded and the following conditions are satisfied:

$$\|\bar{\Delta} A\| \le \sigma_1,\ \sigma_1 = \max(\sigma_{1i}),\ \ \|\bar{\Delta} B\| \le \sigma_2,\ \ \sigma_2 = \max(\sigma_{2i}) \tag{10}$$

with $\sigma_{1i} > 0$ and $\sigma_{2i} > 0$ are the Euclidian norms of matrices $\bar{A}_i$ and $\bar{B}_i$, respectively.

Remark 3: Since the *k*th derivative of the unknown input is bounded, its estimate is also bounded and therefore their difference is bounded. The transfer of $\Delta f_k(t)$ and $w(t)$ towards augmented state estimation error $\bar{e}(t)$ is minimized by L_2–gain technique ($\|\bar{e}(t)\|_2 < \delta \|w(t)\|_2$, $\delta > 0$ with $v(t) = [\Delta f_k(t) \quad w(t)]$).

Lemma 1: *For any matrices X and Y of appropriate dimensions, the following property holds:*

$$X^T Y + Y^T X \le \alpha\, X^T X + \alpha^{-1} Y^T Y \qquad with \quad \alpha > 0$$

IV. PI Observer Synthesis

We develop in this section the synthesis method of PI observer to estimate the actuator and sensor faults. The proposed PI observer acts as a fault detection and isolation mechanism (FDI). The convergence conditions of the estimation error towards zero under LMI constraints are given in *Theorem 1*.

Theorem 1: *The dynamic error* (8) *is asymptotically stable with the L_2–gain bound δ, if there exist a matrix $Q = Q^T > 0$, matrices $\bar{X}_i$ and the positive scalars β, β_0 and $\bar{\delta}$ such as for all $i = 1, .. r$:*

$$\text{Min}\ \bar{\delta}$$

$$\begin{bmatrix} \varphi_i & QG & -\bar{X}_i F & Q \\ G^T Q & -\bar{\delta} I & 0 & 0 \\ -F^T \bar{X}_i^{\,T} & 0 & -\bar{\delta} I & 0 \\ Q & 0 & 0 & -\beta I \end{bmatrix} < 0 \tag{11.a}$$

$$\varphi_i = \bar{A}_i^{\,T} Q + Q\bar{A}_i - \bar{C}^T \bar{X}_i^{\,T} - \bar{X}_i \bar{C} + \beta_0 \sigma_1^{\,2} I + I \tag{11.b}$$

where $\delta = \sqrt{\bar{\delta}}$ and the parameters $\bar{A}_i$, $\bar{C}$, and σ_1 are given in (7c), (7d) *and* (10), *respectively.*

The gains of the PI observer (4) are defined by:

$$\bar{K}_i = Q^{-1} \bar{X}_i \tag{11.c}$$

and

$$\begin{cases} z = 0 & if \quad |\bar{E}_y| < \varepsilon \\ z = \eta_1 \sigma_1^{\,2} \dfrac{\hat{\bar{x}}^T \hat{\bar{x}}}{2\bar{E}_y^{\,T} \bar{E}_y} Q^{-1} \bar{C}^T \bar{E}_y + \eta_2\, \sigma_2^2 \dfrac{u^T u}{2\bar{E}_y^{\,T} \bar{E}_y} Q^{-1} \bar{C}^T \bar{E}_y & if \quad |\bar{E}_y| \ge \varepsilon \end{cases} \tag{11.d}$$

with variables $\hat{\bar{x}}$, z, $\bar{E}_y$ and the parameter σ_2 are given in (7a), (7b), (10), *respectively, and $\eta_1 = \left(\frac{\beta_0}{\alpha}\right)$, $\eta_2 = \left(\frac{\beta\beta_0}{\beta(1+\alpha) - \beta_0}\right)$. α and $\varepsilon > 0$ is a small scalar arbitrarily fixed.*

Proof: The Lyapunov quadratic function considered has the form $V(t) = \bar{e}^T(t) Q \bar{e}(t)$ where $Q = Q^T > 0$. The conditions (11) guarantee the asymptotic stability of the dynamic error (8). The proof is given in appendix. □

The resolution of the constraint (11.c) provides the PI observer $\bar{K}_i$ gains for the estimation of states and of actuators and sensors faults. In the next section, a simulation example is given of a hydraulic system with two tanks in order to validate the proposed approach.

V. Simulation Example

The hydraulic system (Fig. 1) is composed by two tanks of same section S which communicate by means of a cylindrical tube and a tank which supplies water both the two tanks through two pumps P_1 and P_2 of flows u_1 and u_2, respectively. The water is recovered by the leak valves of tanks, thus the hydraulic system operates in a closed circuit [11].

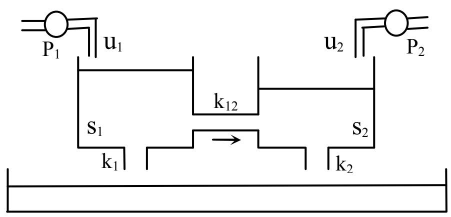

Fig. 1. Technological scheme of hydraulic system with two tanks.

The mass conservation properties lead to the following dynamic equations:

$$\begin{cases} S_1 \dfrac{d}{dt} x_1(t) = U_1 u_1(t) - q_{12} - q_1 \\ S_2 \dfrac{d}{dt} x_2(t) = U_2 u_2(t) + q_{12} - q_2 \end{cases} \tag{12}$$

where $x_1(t)$ and $x_2(t)$ represent the levels of tanks 1 and 2, respectively, $u_1(t)$ and $u_2(t)$ are the water supply flows delivered by pumps 1 and 2, respectively.

The flows are expressed according to Torricelli's law by:

$$\begin{aligned} q_{12} &= k_{12} sign(x_1 - x_2)\sqrt{|x_1 - x_2|} \\ q_1 &= k_1 \sqrt{x_1} \\ q_2 &= k_2 \sqrt{x_2} \end{aligned} \tag{13}$$

where the constants values used in simulation are $S = S_1 = S_2 = 0.09$, $U_1 = U_2 = 10$, $k_{12} = 5.2\ 10^{-4}$, $k_1 = 2.2\ 10^{-3}$, $k_2 = 2.5\ 10^{-3}$ and $sign()$ is the sign function.

The analytical knowledge of nonlinear model (12) of the hydraulic system allows us to obtain, with the use of the sectors nonlinearity approach detailed in [7], the following TS fuzzy model:

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{4} \mu_i(x)\big(A_i x(t) + B_i u(t)\big) \\ y(t) = Cx(t) \end{cases} \tag{14}$$

with

$$A_1 = \begin{bmatrix} -0.0389 & 0 \\ 0 & -0.0500 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.0389 & 0 \\ 0 & -0.0111 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.0111 & 0 \\ 0 & -0.0500 \end{bmatrix},\ A_4 = \begin{bmatrix} -0.0111 & 0 \\ 0 & -0.0111 \end{bmatrix}$$

$$B_i = \frac{1}{0.09}\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\ (i = 1, \dots 4),\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Hypothesis 2: We assume that hydraulic system output is the state $x_1(t)$ and the state $x_2(t)$ is unmeasurable.

The TS fuzzy model (14) can take the form (1), i.e. it is subject to actuator and sensor faults with the unmeasurable premise variable $x_2(t)$ and measurement noise on the output. The considered measurement noise $w(t)$ is a centered noise represented by Fig. 2, and the inputs $u(t)$ is given by Fig. 3.

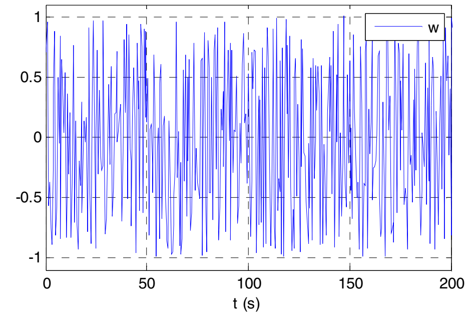

Fig. 2. Centered measurement noise.

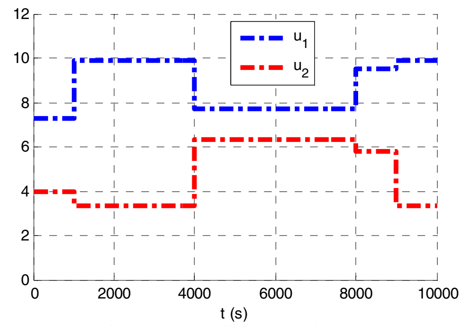

Fig. 3. Hydraulic system inputs.

In the following paragraphs we present the simulation results of simultaneous estimation of states, actuator and sensor faults.

A. Actuator Fault Diagnosis

In the case of an actuator fault, the TS model of the system with unmeasurable premise variables and measurement noise can be put under the following form:

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{r} \mu_i(x)\big(A_i x(t) + B_i(u(t) + \Delta u(t))\big) \\ y(t) = Cx(t) + Fw(t) \end{cases} \tag{15}$$

The TS model (15) can take the form (1) with the following notations:

$$B_i\ \Delta u(t) = E_i f_a(t)$$

where $f_a(t)$ is the actuator fault and

$$E_i = \begin{bmatrix} -0.022 \\ 0 \end{bmatrix}\ (i = 1, \dots 4),\quad E = 0,\quad F = 10^{-3}$$

Solving of LMIs constraints (11) of the *theorem 1* lead to the PI observer gains given in Table I. The simulation results are carried out with $\varepsilon = 10^{-3}$, $f_{0a} = 1.81\ 10^{-4}$ and initial conditions $x_0 = [0.6 \quad 0.1]$, $\hat{x}_0 = [0.5 \quad 0.2]$. The considered actuator fault has the fourth derivative bounded, and appears between t = 2000 s and t = 3000 s, as given by Fig. 4.

Actuator fault and its estimate are given in Fig 4 where the states estimation are given by Fig 5.

TABLE I

$\mu = 3,$		$\beta = 1.22\ 10^{8},$		$\beta_0 = 5.03\ 10^{-4}$
i	1	2	3	4
K_{pi}	$\begin{bmatrix} 7.74 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 7.74 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 7.79 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 7.79 \\ 0 \end{bmatrix}$
K_{Ii}	-1320.56	-1320.56	-1320.83	-1320.83
K_{Ii}^{1}	-2940.42	-2940.42	-2941.38	-2941.38
K_{Ii}^{2}	-4035.21	-4035.21	-4037.48	-4037.48
K_{Ii}^{3}	-2892.85	-2892.85	-2894.84	-2894.84

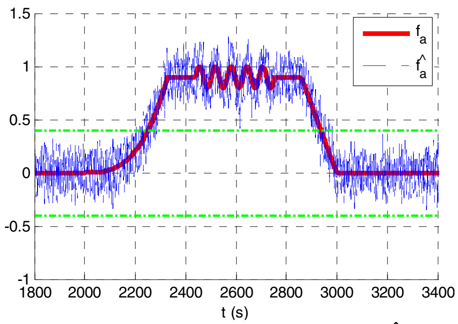

Fig. 4. Actuator fault $f_a(t)$ and its estimate $\hat{f}_a(t)$.

A threshold of 0.4 can be used as decision logic to show the presence or not of an actuator fault.

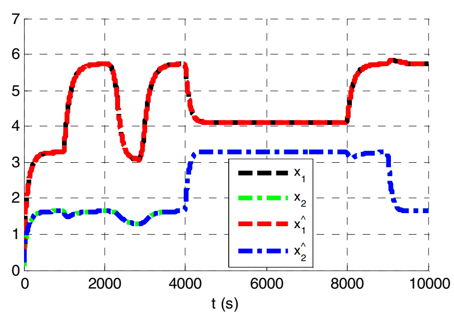

Fig. 5. States $x_1(t)$, $x_2(t)$ and their estimate $\hat{x}_1(t)$, $\hat{x}_2(t)$.

B. *Sensor Fault Diagnosis*

In this context, the TS model of the system with unmeasurable premise variables and measurement noise can be put under the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x)\big(A_i x(t) + B_i u(t)\big) \\ y(t) = Cx(t) + E f_s(t) + Fw(t) \end{cases} \tag{16}$$

The TS model (16) is subjected to a sensor fault and can take the form (1) with $f_s(t)$ $E_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $(i = 1, \dots 4)$, $E = -1$, $F = 10^{-3}$.

The resolution of LMIs constraints (11) leads to PI observer gains shown in Table II.

TABLE II

$\mu = 2,$		$\beta = 6.92\ 10^{5},$		$\beta_0 = 2.64\ 10^{-2}$
i	1	2	3	4
K_{pi}	$\begin{bmatrix} 0.004 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.004 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.001 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.001 \\ 0 \end{bmatrix}$
K_{Ii}	-11.394	-11.394	-11.377	-11.377
K_{Ii}^{1}	-69.402	-69.402	-69.104	-69.104
K_{Ii}^{2}	-247.513	-247.513	-245.645	-245.645
K_{Ii}^{3}	-459.817	-459.817	-453.450	-453.450

The simulation results are carried out with an attenuation gain of $\delta = 0.6$ with $f_{0s} = 1.81\ 10^{-4}$, $\varepsilon = 10^{-3}$ and initial conditions $x_0 = [0.6 \quad 0.1]$, $\hat{x}_0 = [0.5 \quad 0.2]$. The considered sensor fault has the fourth derivative bounded, and appears between t = 5000 s and t = 6000 s (see Fig. 6).

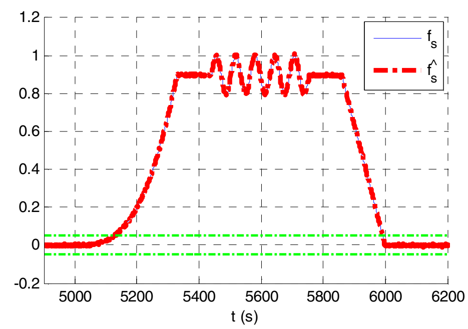

Fig. 6. Sensor fault $f_s(t)$ and its estimate $\hat{f}_s(t)$.

Fig. 6 and Fig. 7 gives the sensor fault and states and their estimate respectively. The simulation results give good simultaneous estimation of states and sensor fault.

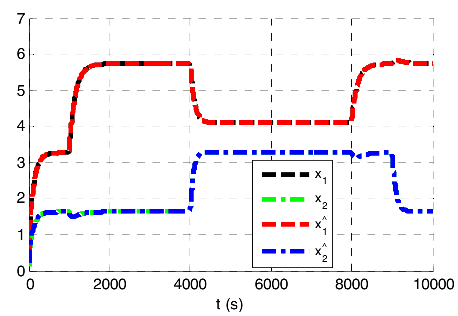

Fig. 7. States $x_1(t)$, $x_2(t)$ and their estimate $\hat{x}_1(t)$, $\hat{x}_2(t)$.

VI. CONCLUSION

This paper proposes a simultaneous estimation of both state and sensor/actuator faults for TS fuzzy model based on a proportional integral observer. The considered TS fuzzy model is subject to disturbances and unmeasurable premise variables. To take into account a wide range of faults, polynomial form with their bound kth derivatives are considered. Based on Lyapunov stability theory and L_2–gain technique, sufficient synthesis conditions of the proposed PI observer are developed in LMIs formulation. A simulation example of hydraulic system with two tanks is provided to illustrate the effectiveness and feasibility of the proposed approach. Indeed, the obtained results show the good simultaneous estimation of states and actuator or sensor faults.

APPENDIX

The time-derivative of Lyapunov quadratic function $V(t) = \bar{e}^T(t)Q\bar{e}(t)$ where $Q = Q^T > 0$ leads:

$$\dot{V} = \sum_{i=1}^{r} \mu_i(\hat{x}) \left(\bar{e}^T \left(\bar{\mathcal{A}}_i^{\,T} Q + Q\bar{\mathcal{A}}_i \right) \bar{e} - \bar{e}^T Q \bar{K}_i F w - w^T F^T \bar{K}_i^{\,T} Q \bar{e} \right)$$
$$+ \bar{x}^T \bar{\Delta} A^T Q \bar{e} + \bar{e}^T Q \bar{\Delta} A \bar{x} + u^T \bar{\Delta} B^T Q \bar{e}$$
$$+ \bar{e}^T Q \bar{\Delta} B u + \Delta f_k^{\,T} G^T Q \bar{e} + \bar{e}^T Q G \Delta f_k - 2\bar{e}^T Q z \quad (A.1)$$

Then, taking into account (7b) and using *Lemma 1*, we obtain:

$$\dot{V} \le \sum_{i=1}^{r} \mu_i(\hat{x}) \left(\bar{e}^T \left(\bar{\mathcal{A}}_i^{\,T} Q + Q\bar{\mathcal{A}}_i \right) \bar{e} - \bar{e}^T Q \bar{K}_i F w - w^T F^T \bar{K}_i^{\,T} Q \bar{e} \right)$$
$$+ \alpha_1 \sigma_1^{\,2} (\bar{e}^T \bar{e} + \hat{\bar{x}}^T \hat{\bar{x}} + \alpha^{-1} \hat{\bar{x}}^T \hat{\bar{x}} + \alpha\, \bar{e}^T \bar{e})$$
$$+ \alpha_1^{-1} \bar{e}^T Q^2 \bar{e} + u^T \bar{\Delta} B^T Q \bar{e} + \bar{e}^T Q \bar{\Delta} B u$$
$$+ \Delta f_k^{\,T} G^T Q \bar{e} + \bar{e}^T Q G \Delta f_k - 2\bar{e}^T Q z \quad (A.2)$$

using again *Lemma 1*:

$$\dot{V} \le \sum_{i=1}^{r} \mu_i(\hat{x}) \Big(\bar{e}^T \left(\bar{\mathcal{A}}_i^{\,T} Q + Q\bar{\mathcal{A}}_i + \beta_0 \sigma_1^{\,2} I + \beta^{-1} Q^2 \right) \bar{e} - \bar{e}^T Q \bar{K}_i F w$$
$$- w^T F^T \bar{K}_i^{\,T} Q \bar{e} \Big) + \Delta f_k^{\,T} G^T Q \bar{e} + \bar{e}^T Q G \Delta f_k$$
$$+ \eta_1 \sigma_1^{\,2} \hat{\bar{x}}^T \hat{\bar{x}} + \eta_2 \sigma_2^{\,2} u^T u - 2\bar{e}^T Q z \quad (A.3)$$

with

$\beta_0 = \alpha_1(1+\alpha)$, $\beta^{-1} = (\alpha_1^{-1} + \alpha_2^{-1})$,

$\eta_1 = \alpha_1(1+\alpha^{-1}) = \left(\frac{\beta_0}{\alpha}\right)$, $\eta_2 = \alpha_2 = \left(\frac{\beta\beta_0}{\beta(1+\alpha)-\beta_0}\right)$.

By substituting the expression of z (11.d) in equation (A.3), we get:

$$2\bar{e}^T Q z = 2\,\bar{e}^T Q \eta_1 \sigma_1^{\,2} \frac{\hat{\bar{x}}^T \hat{\bar{x}}}{2\bar{E}_y^{\,T} \bar{E}_y} Q^{-1} \bar{C}^T \bar{E}_y$$
$$+ 2\,\bar{e}^T Q\, \eta_2\, \sigma_2^2 \frac{u^T u}{2\,\bar{E}_y^{\,T} \bar{E}_y} Q^{-1} \bar{C}^T \bar{E}_y \quad (A.4)$$
$$= \eta_1 \sigma_1^{\,2} \hat{\bar{x}}^T \hat{\bar{x}} + \eta_2 \sigma_2^{\,2} u^T u$$

with $\bar{C}\bar{e} = \bar{E}_y(t)$ et $\bar{E}_y^{\,T} = \bar{e}^T \bar{C}^T$.

The system (8) is stable and satisfied the L_2–gain technique ($\|\bar{e}(t)\|_2 < \delta \|w(t)\|_2$, $\delta > 0$ with $v(t) = [\Delta f_k(t) \quad w(t)]$) if the following condition is respected:

$$\dot{V} + \bar{e}^T \bar{e} - \delta^2 \Delta f_k^{\,T} \Delta f_k - \delta^2 w^T w < 0 \quad (A.5)$$

Taking into account (A.4), the condition (A.5) is satisfied if the following matrices inequalities are satisfied:

$$\begin{bmatrix} \varphi_i + \beta^{-1} Q^2 & QG & -Q\bar{K}_i F \\ G^T Q & -\delta^2 I & 0 \\ -F^T \bar{K}_i^{\,T} Q & 0 & -\delta^2 I \end{bmatrix} < 0 \quad (A.8)$$

with $\varphi_i = \bar{\mathcal{A}}_i^{\,T} Q + Q\bar{\mathcal{A}}_i + \beta_0 \sigma_1^{\,2} I + I \quad (i = 1, \dots r)$.

The Schur complement to the condition (A.8) with the variables change $\bar{\delta} = \delta^2$, $\bar{\mathcal{A}}_i = \bar{A}_i - \bar{K}_i \bar{C}$ and $\bar{X}_i = Q\bar{K}_i$ leads to linear matrix inequalities (11.a).

REFERENCES

[1] J. Chen, and R. J Patton, *Robust model-based fault diagnosis for dynamic systems*. Kluwer Academic Publishers, Norwell, MA, USA, 1999.

[2] Ron J. Patton, Paul M. Frank, and R. N. Clark, *Issues of Fault Diagnosis for Dynamic Systems*. London: Springer-Verlag, 2000.

[3] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and fault-tolerant control*. 2nd ed. Germany: Springer, 2006.

[4] R. Patton, P. Frank, and R. Clark, *Fault Diagnosis in Dynamic Systems*: *Theory and Application*. Prentice Hall international, 1989.

[5] S. X. Ding, *Model-Based Fault Diagnosis Techniques Design Schemes*, *Algorithms and Tools*. Springer-Verlag, 2008.

[6] T. Takagi, and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control," *IEEE Transactions on Systems*, Man and Cybernetics, vol. SMC–15, no. 1, pp. 116–132, 1985.

[7] K. Tanaka, and H. O. Wang, *Fuzzy Control Systems Design and Analysis : A LMI Approach*. John Wiley and Sons, 2001.

[8] J. Yoneyama, "H∞ filtering for fuzzy systems with immeasurable premise variables: an uncertain system approach," *Fuzzy Sets and Systems*, vol. 160, no. 12, pp. 1738–1748, 2009.

[9] B. Marx, D. Koenig, and J. Ragot, "Design of observers for Takagi-Sugeno descriptor systems with unknown inputs and application to fault diagnosis," *IET Control Theory and Applications*, vol. 1, no. 5, pp. 1487–1495, 2007.

[10] A. Akhenak, M. Chadli, J. Ragot, and D. Maquin, "Design of observers for Takagi-Sugeno fuzzy models for Fault Detection and Isolation," *7th IFAC Symposium on Fault Detection, Super-vision and Safety of Technical Processes, SAFE-PROCESS'09*, Barcelona, Spain, 2009.

[11] M. Bouattour, M. Chadli, A. El Hajjaji, and M. Chaabane, "State and Faults Estimation for T-S Models and Application to Fault Diagnosis," *7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, *SAFEPROCESS'09*, 2009.

[12] A. Kheder, K. Ben Othman, D. Maquin, and M. Benrejeb, "State and sensor faults estimation via a proportional integral observer," *6th Multi-Conf on Systems, Signals and Devices*, *SSD'2009*, Tunisia, 2009.

[13] R. Orjuela, B. Marx, J. Ragot, and D. Maquin, "Diagnosis of nonlinear systems by multiple models approach," *6th International Conference Francophone of Automatic*, *CIFA'2010*, France, 2010.

[14] S. Delrot, T. M. Guerra, M. Dambrine, and F. Delmotte, "Fouling detection in a heat exchanger by observer of Takagi–Sugeno type for systems with unknown polynomial inputs," *Engineering Applications of Artificial Intelligence*, September 13, 2012.

[15] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "Observer based actuator fault tolerant control for nonlinear Takagi-Sugeno systems: an LMI approach," *18th Mediterranean Conference on Control and Automation*, *MED'10*, Athens, Greece, June 23–25, 2010.

[16] M. Chadli, A. Abdo, Steven X Ding. H−/H∞ fault detection filter design for discrete-time Takagi–Sugeno fuzzy system. Automatica, Volume 49, Issue 7, pp. 1996–2005, July 2013.

[17] A. M. Nagy, B. Marx, G. Mourot, G. Schutz, and J. Ragot, "State estimation of the three tank system using a multiple model," *48th IEEE Conference on Decision and Control*, *CDC'09*, pp. 7795–7800, Shanghai, China, December 16–18, 2009.

[18] Z. Lendek, J. Lauber, T. M. Guerra, R. Babuska, and B. De Schutter, "Adaptive observers for TS fuzzy systems with unknown polynomial inputs," *Fuzzy Sets and Systems*, vol. 161, no. 15, pp. 2043–2065, 2010.

[19] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "Simultaneous state and unknown inputs estimation with PI and PMI observers for Takagi-Sugeno model with unmeasurable premise variables," *17th Mediterranean Conference on Control and Automation*, Thessaloniki, Greece, June 24–26, 2009.

[20] M. Chadli, and H. R. Karimi, "Robust Observer Design for Unknown Inputs Takagi-Sugeno Models," *IEEE Trans. on Fuzzy Systems*, vol. 21, no. 1, pp. 158–164, 2013.

[21] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. SIAM, Studies In Applied Mathematics, 1994.

[22] T.Youssef, M. Chadli, H. R. Karimi, and M. Zelmat, "Design of unknown inputs proportional integral observers for fuzzy models," *Neurocomputing*, vol. 123, pp. 156–165, 2013.

[23] Z. Gao, S. X. Ding, and Y. Ma, "Robust fault estimation approach and its application in vehicle lateral dynamic systems," *Optimal Control Applications and Methods*, vol. 28, pp. 143–156, 2007.