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A New Fuzzy Ratio and Its Application to the Single Input Rule Modules Connected Fuzzy Inference System

[1]Chian Haur Jong, *[1]Kai Meng Tay, [2]Chee Peng Lim

[1]Faculty of Engineering, Universiti Malaysia Sarawak, Sarawak, Malaysia
*kmtay@feng.unimas.my

[2]Centre for Intelligent Systems Research, Deakin University, Australia.

***Abstract*—The principle of ratios has been applied to many real world problems, e.g. the part-to-part and part-to-whole ratio formulations. As it is difficult for humans to provide an exact ratio in many real situations, we introduce a fuzzy ratio in this paper. We use some notions from fuzzy arithmetic to analyze fuzzy ratios captured from humans. An application of the formulated fuzzy ratio to a Single Input Rule Modules connected Fuzzy Inference System (SIRMs-FIS) is demonstrated. Instead of using a precise weight, fuzzy sets are employed to represent the relative importance of each rule module. The resulting fuzzy weights are explained as a fuzzy ratio on a weight domain. In addition, a new SIRMs-FIS model with fuzzy weights and part-to-whole fuzzy ratio is devised. A simulated example is presented to clarify the proposed SIRM-FIS model.**

Keywords: Fuzzy ratio, fuzzy arithmetic, single input rule modules connected fuzzy inference system, fuzzy weights

I. INTRODUCTION

In general, a ratio is concerned with the relationship of two magnitudes of the same kind, e.g., the size of two objects, or the height of two persons [1]. Two types of ratios are available, i.e., part-to-part and part-to-whole ratios. As an example, there are five students in a class, with two boys and three girls. Based on the principle of the part-to-part ratio, the ratio of boys to girls is 2:3. On the other hand, based on the principle of the part-to-whole ratio, the ratio of boys to all students is 2:5 and. Similarly, the ratio of girls to all students is 3:5.

In this paper, we argue that it sometimes is difficult (if not impossible) for humans to provide an exact ratio based on their experience, in some real-world scenarios. A ratio can be imprecise in nature too. As an example, one may suggest that a good micro-nutrient ratio to burn body fat is 30% protein, 15-20% fat, 50-55% carbohydrates [2]. Another example in civil engineering is that to produce an approximately 3000 psi cubic yard of concrete (27 cubic feet), a workable concrete mixture ratio is suggested to be 517 pounds of cement, 1560 pounds of sand, 1600 pounds of stone, and 32-34 gallons of water (or approximately 267.2-283.9 pounds) [3]. In the financial world, an experienced financial advisor would suggest a good debt to income ratio for an individual is near or below 30% [4].

In this paper, the idea of a fuzzy ratio is introduced. It is argued that one out of two (or both) magnitudes of the same type are known imprecisely, instead of precisely. As such, a fuzzy set is used to represent the magnitude of the same type. Here, the trapezoidal fuzzy set, which is a generalization of the triangular fuzzy set and interval set [5], is used. Some notions from fuzzy arithmetic [5-6] are employed to analyze fuzzy ratios. The validity of the proposed approach is further analyzed mathematically.

A Single Input Rule Modules connected Fuzzy Inference System (here after denoted as SIRMs-FIS) [7-8] is used to demonstrate the applicability of the proposed fuzzy ratio. SIRMs-FIS is a relatively new fuzzy reasoning model in which its final output is obtained by summarizing the product of the importance degree and inference result from single input fuzzy rule modules [7]. In our previous investigations, a new fuzzy failure mode and effect analysis (FMEA) methodology with SIRMs-FIS was proposed [9], and the use of harmony search to optimize an SIRMs-FIS model was demonstrated [10].

One of the issues of SIRMs-FIS is the difficulty to determine the exact (precise) weight for each rule module. The weight can be imprecise in some real-world applications. Therefore, an imprecise weight (i.e., a trapezoidal fuzzy set) is employed to represent the relative importance of each rule module, instead of a precise weight, is suggested. This allows uncertainty of the weights to be included. The weight domain needs to be formally defined. In this paper, these fuzzy weights are explained as a fuzzy ratio pertaining to the weight domain, which can be normalized. A simulated example is presented to clarify the proposed approach. The resulting approach is useful for tackling decision making and assessment problems, e.g., FMEA [9].

This paper is organized as follows. In Section II, trapezoidal fuzzy sets and fuzzy arithmetic are reviewed. In Section III, the idea of fuzzy ratios is introduced, with an example included. In Section IV, the new SIRMs-FIS model with fuzzy weights is explained. Finally, concluding remarks and suggestions for further work are provided in Section V.

II. PRELIMINARIES

A. Trapezoidal fuzzy set and its variants

A trapezoidal fuzzy set is a generalization of the triangle fuzzy set and interval set. A normal trapezoidal fuzzy set, $\widetilde{A}$, in the space of X is shown in Fig. 1. It is defined as follows.

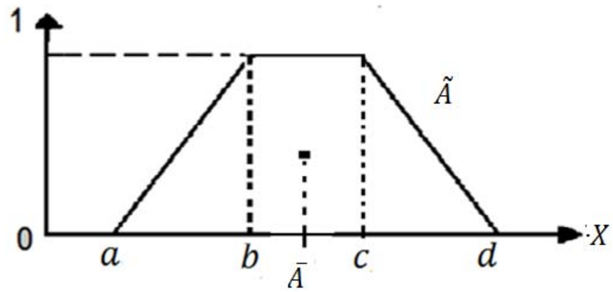

Fig. 1. A trapezoidal fuzzy set

***Definition 1*:** The trapezoidal fuzzy set, $\widetilde{A}$, in the space of X is parameterized as $\widetilde{A} = (a, b, c, d)$. $\bar{A}$ is the centroid of fuzzy set $\widetilde{A}$, which can be obtained using Eq. (1) [11].

$$\bar{A} = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d + c) - (a + b)}\right] \quad (1)$$

The trapezoidal fuzzy set is a triangular fuzzy set when $b = c$. Then, the triangular fuzzy set is parameterized as $\widetilde{A} = (a, b, d)$. The centroid of the triangular fuzzy set (i.e., $\bar{A}$) is obtained using Eq. (2) [11].

$$\bar{A} = \frac{1}{3}[a + b + d] \quad (2)$$

If $a = b$ and $c = d$, the trapezoidal fuzzy set is a rectangular fuzzy set. A rectangular fuzzy set can be interpreted as an interval set. An interval set is parameterized as $\widetilde{A} = (a, d)$. The centroid of the fuzzy set (i.e., $\bar{A}$) is obtained using Eq. (3).

$$\bar{A} = \frac{1}{2}[a + d] \quad (3)$$

B. Fuzzy arithmetic operations

Two trapezoidal fuzzy sets, i.e., $\widetilde{A}_1 = (a_1, b_1, c_1, d_1)$; $\widetilde{A}_2 = (a_2, b_2, c_2, d_2)$, are considered. The arithmetic operations of $\widetilde{A}_1$ and $\widetilde{A}_2$ [3-4] are summarized in Table I.

TABLE I. ARITHMETIC OPERATIONS FOR TRAPEZOIDAL FUZZY SETS

Arithmetic operation	Mathematic notation	Definition.
Addition	$\widetilde{A}_1 \oplus \widetilde{A}_2$	$(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
Subtraction	$\widetilde{A}_1 \ominus \widetilde{A}_2$	$(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$
Multiplication	$\widetilde{A}_1 \otimes \widetilde{A}_2$	$(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2)$
Division	$\widetilde{A}_1 \oslash \widetilde{A}_2$	$(\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2})$

Definition 2: The multiplication operation between trapezoidal fuzzy set $\widetilde{A}_1 = (a_1, b_1, c_1, d_1)$ and a precise value, i.e., k, is defined as follows:

$$k \times \widetilde{A}_1 = (k \times a_1, k \times b_1, k \times c_1, k \times d_1)$$

III. FUZZY RATIO

The following scenario of a dimension is examined.

Definition 3: Consider a dimension, R, in which the variables in R are always greater than zero. Both $\tilde{r}_1$ and $\tilde{r}_2$ are imprecise elements of R, and they are represented as fuzzy sets, i.e., $\tilde{r}_1, \tilde{r}_2 \in R$, respectively. As such, $\tilde{r}_1$ and $\tilde{r}_2$ are parameterized as follows.

$$\tilde{r}_1 = (a_1, b_1, c_1, d_1); \ \tilde{r}_2 = (a_2, b_2, c_2, d_2)$$

A. Part-to-part fuzzy ratio

The part-to-part fuzzy ratio between $\tilde{r}_1$ and $\tilde{r}_2$ is denoted as $\tilde{r}_1 : \tilde{r}_2$ or $\tilde{r}_2 : \tilde{r}_1$. Its proportion is denoted as $\tilde{q}_{\tilde{r}_1,\tilde{r}_2}$ or $\tilde{q}_{\tilde{r}_2,\tilde{r}_1}$, respectively, as shown in Eq (4).

$$\tilde{q}_{\tilde{r}_1,\tilde{r}_2} = \tilde{r}_1 \oslash \tilde{r}_2; \ \tilde{q}_{\tilde{r}_2,\tilde{r}_1} = \tilde{r}_2 \oslash \tilde{r}_1 \quad (4)$$

***Theorem 1**:* $\tilde{q}_{\tilde{r}_1,\tilde{r}_2} = \left(\tilde{q}_{\tilde{r}_2,\tilde{r}_1}\right)^{-1}$ is always true.

Proof:

$$\frac{1}{\tilde{q}_{\tilde{r}_2,\tilde{r}_1}} = \frac{1}{\tilde{r}_2 \oslash \tilde{r}_1}$$

$$\frac{1}{\tilde{q}_{\tilde{r}_2,\tilde{r}_1}} = \frac{1}{\left(\frac{a_2}{d_1}, \frac{b_2}{c_1}, \frac{c_2}{b_1}, \frac{d_2}{a_1}\right)}$$

Using the division operation in Table 1,

$$\frac{1}{\tilde{q}_{\tilde{r}_2,\tilde{r}_1}} = \left(\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2}\right)$$

$$\frac{1}{\tilde{q}_{\tilde{r}_2,\tilde{r}_1}} = \tilde{r}_1 \oslash \tilde{r}_2 = \tilde{q}_{\tilde{r}_1,\tilde{r}_2}$$

B. Part-to-whole fuzzy ratio

The part-to-whole fuzzy ratio between $\tilde{r}_1$ and $\tilde{r}_2$ on R is denoted as $\tilde{r}_i : \tilde{r}_w$, where $i = 1,2$ and $\tilde{r}_w = \tilde{r}_1 \oplus \tilde{r}_2$. Here, a scale and move transformation method is used to normalize $\tilde{r}_i:\tilde{r}_w$ into a fuzzy ratio in the form of $\tilde{q}_{\tilde{r}_i}:\tilde{q}_w$. The property of the part-to-whole fuzzy ratio is summarized as follows.

i. $\tilde{r}_w = \tilde{r}_1 \oplus \tilde{r}_2$

ii. $\tilde{q}_{\tilde{r}_i} = s \times \tilde{r}_i$.

iii. $\tilde{q}_w = \tilde{q}_{\tilde{r}_1} \oplus \tilde{q}_{\tilde{r}_2} = s \times \tilde{r}_w$

Note that $\tilde{q}_{\tilde{r}_i}$ is re-sized with a scale factor, s, which is obtained from Eq. (5). Besides that, $\bar{r}_i$ is the centroid of $\tilde{r}_i$, and $\bar{r}_i$ is obtained using Eq. (6). By expanding Eq. (5) using Eq. (6), Eq. (7) is obtained.

$$s = (\bar{r}_1 + \bar{r}_2)^{-1} \tag{5}$$

$$\bar{r}_i = \frac{1}{3}\left[a_i + b_i + c_i + d_i - \frac{d_i c_i - a_i b_i}{(d_i + c_i) - (a_i + b_i)}\right] \tag{6}$$

$$s = 3\left(\sum_{i=1}^{n=2}\left[a_i + b_i + c_i + d_i - \frac{d_i c_i - a_i b_i}{(d_i + c_i) - (a_i + b_i)}\right]\right)^{-1} \tag{7}$$

Theorem 2: $\frac{\tilde{r}_1}{\tilde{r}_2} = \frac{\tilde{q}_{\tilde{r}_1}}{\tilde{q}_{\tilde{r}_2}}$ is always true.

Proof:

$$\frac{\tilde{q}_{\tilde{r}_1}}{\tilde{q}_{\tilde{r}_2}} = \frac{s \times \tilde{r}_1}{s \times \tilde{r}_2} = \frac{\tilde{r}_1}{\tilde{r}_2}$$

C. An example

An example from [2] is considered. It is suggested that a good micro-nutrient ratio in a daily meal to burn body fat is 30% protein, 15-20% fat, 50-55% carbohydrates [2]. This ratio is written as $(30,30,30,30):(15,15,20,20):(50,50,55,55)$. The centroids for $(30,30,30,30)$, $(15,15,20,20)$ and $(50,50,55,55)$ are 30, 17.5 and 52.5, respectively, and $s = 30 + 17.5 + 52.5 = 100$. After scaling, the ratio is $(0.30,0.30,0.30,0.30):(0.15,0.15,0.20,0.20):(0.50,0.50,0.55,0.55)$.

It is also assumed that energy provided by proteins, fats and carbohydrates are 4 calories per gram, 9 calories per gram, and 4 calories per gram, respectively [2]. For someone who needs 2000 calories daily, the proportions of protein, fat, and carbohydrates in a daily meal are as follows.

- $2000 \div 4 \times (0.30,0.30,0.30,0.30) = (150,150,150,150)$ grams of protein
- $2000 \div 9 \times (0.15,0.15,0.20,0.20) = (33.3,33.3,44.4,44.4)$ grams of fats
- $2000 \div 4 \times (0.50,0.50,0.55,0.55) = (250,250,275,275)$ grams of carbohydrates

IV. A New SIRMs-FIS Model with Fuzzy Weights

A. Proposed formulation

A zero-order SIRMs-FIS, i.e., $y = f(\bar{x})$, where $\bar{x} = (x_1, x_2 \dots x_n)$; is considered. The definitions of the input and output spaces of the zero-order SIRMs-FIS model are as follows.

Definition 4: Consider an input space, $X_1 \times X_2 \times X_3 \times \dots \times X_n$, and an output space, Y. Variables $\bar{x}$ and y are the elements of $X_1 \times X_2 \times X_3 \times \dots \times X_n$ and Y, respectively, i.e., $x_i \in X_i$, $i = 1,2,3,\dots,n$, and $y \in Y$.

The zero-order SIRMs-FIS model consists n fuzzy rule modules. Each rule module consists of m_i fuzzy rules, as follows.

$$SIRM\text{-}i: \quad \left\{Rule_i^{j_i}: if\ x_i\ is\ A_i^{j_i}\ then\ y_i = c_i^{j_i}\right\}_{j_i=1}^{m_i}$$

where $SIRM\text{-}i$ is the i-th rule module, x_i is the sole variable in the antecedent of the fuzzy rules. $Rule_i^{j_i}$ is the j-th rule in $SIRM\text{-}i$, where $1 \le j_i \le m_i$. $A_i^{j_i}$ is a fuzzy set in domain X_i (denoted as $\mu_i^j(x_i)$), and $c_i^{j_i}$ is a numerical output or fuzzy singleton. A fuzzy rule, $Rule_i^{j_i}$, can also be viewed as a mapping from $A_I^{j_i}$ to $c_i^{j_i}$, i.e., $Rule_i^{j_i}: A_i^{j_i} \to c_i^{j_i}$.

The output of $SIRM\text{-}i$, i.e., y_i, is obtained using Eq. (8). The outputs from the rule modules are aggregated using Eq. (9), where w_i is a numerical value or a precise weight that reflects the relative importance of the i-th rule module.

$$y_i = \frac{\Sigma_{j_i=1}^{m_i}\left[\mu_i^j(x_i) \times c_i^j\right]}{\Sigma_{j_i=1}^{m_i}\left[\mu_i^j(x_i)\right]} \tag{8}$$

$$y = \Sigma_{i=1}^{n}[w_i y_i] \tag{9}$$

B. The Proposed SIRMs-FIS Model

As explained earlier, the traditional zero-order SIRMs-FIS model adopts a precise weight to reflect the relative importance of a rule module to the inference output. In this paper, imprecise weights, represented by a trapezoidal fuzzy set, are used instead. An SIRMs-FIS model with imprecise weights is illustrated in Fig. 2. The imprecise weights provided by human experts are denoted as $\tilde{r}_i$. The domain containing $\tilde{r}_i$ is defined in Definition 5. The relationships among $\tilde{r}_i$ are explained as ratios. Note that $\tilde{r}_i$ should be scaled to $\widetilde{w}_i$. The domain containing w_i and/or $\widetilde{w}_i$ is defined in Definition 6.

Definition 5: A weight space, i.e., R, is considered. The imprecise weight variables in R are denoted as $\tilde{r}_i$, and are represented as fuzzy sets, i.e., $\tilde{r}_i \in R$ and $\tilde{r}_i = (a_i, b_i, c_i, d_i)$, $i = 1,2,3,\dots,n$.

Definition 6: A weight space, i.e., W, is considered. The precise weight variables in W are denoted as w_i, i.e., $w_i \in W$ where. $i = 1,2,3,\dots,n$. The imprecise weight variables in space W are denoted as $\widetilde{w}_i$, i.e., $\widetilde{w}_i \in W$, in which $\widetilde{w}_i$ is a fuzzy set.

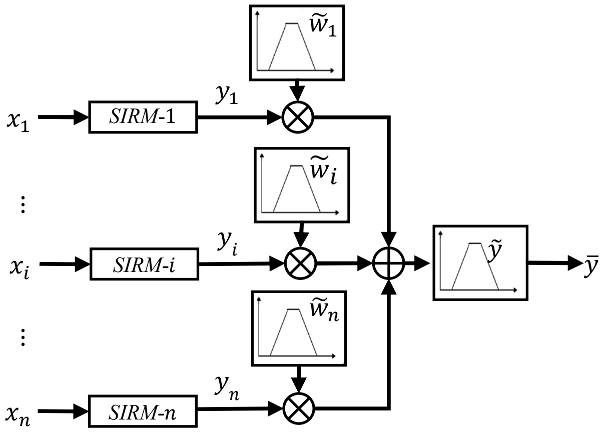

Fig. 2 An SIRMs-FIS model with imprecise weights

Similarly, the output of $SIRM\text{-}i$, i.e., y_i, is obtained using Eq. (8). In this paper, the outputs of all rule modules, i.e., $y_1, y_2 \ldots$, and y_n, reside in the Y domain. The final inference result is obtained by a weighted sum of all rule modules, as in Eq. (10).

$$\tilde{y} = (y_1 \times \widetilde{w}_1) \oplus (y_2 \times \widetilde{w}_2) \oplus \ldots \oplus (y_n \times \widetilde{w}_n) \tag{10}$$

where

$$\widetilde{w}_i = s \otimes \tilde{r}_i \tag{11}$$

and

$$s = [\Sigma_{i=1}^{n}(\bar{r}_i)]^{-1} \tag{12}$$

Both $\tilde{r}_i$ and $\widetilde{w}_i$ are fuzzy sets used to reflect the relative importance of the i-th rule module to the final inference result, i.e., $\tilde{y}$. $\tilde{r}_i$ is scaled to $\widetilde{w}_i$ with a scale factor (i.e., s) as in Eq. (14). $\tilde{r}_i : \tilde{r}_{i'}$ and $\widetilde{w}_i : \widetilde{w}_{i'}$ are always in proportion, where $i, i' \in [1, n]$ and $i \neq i'$. Eq. (12) can be expanded to Eq. (13) and Eq. (14).

$$\tilde{y} = (y_1 \otimes s \otimes \tilde{r}_1) \oplus (y_2 \otimes s \otimes \tilde{r}_2) \oplus \ldots \oplus (y_n \otimes s \otimes \tilde{r}_i)$$

$$\tilde{y} = \left(s \sum_{i=1}^{n} y_i a_i, s \sum_{i=1}^{n} y_i b_i, s \sum_{i=1}^{n} y_i c_i, s \sum_{i=1}^{n} y_i d_i\right) \tag{13}$$

$$\bar{y} = \frac{s}{3} \sum_{i=1}^{n} y_i \left[a_i + b_i + c_i + d_i - \frac{d_i c_i - a_i b_i}{(d_i + c_i) - (a_i + b_i)}\right] \tag{14}$$

C. A Simulated Example

A two-input SIRMs-FIS model, i.e., $y = f(x_1 \in X_1, x_2 \in X_2)$, with two rule modules is considered, as shown in Fig. 3. The two rule modules are $SIRM\text{-}1$ and $SIRM\text{-}2$, in which x_1 and x_2 are the sole variables at the antecedents. The fuzzy sets at X_1 and X_2 are depicted in Fig. 4 and Fig. 5, respectively.

$$SIRM\text{-}1: \begin{Bmatrix} Rule_1^1: & if\ x_1\ is\ A_1^1\ then\ y_1^1 = 1 \\ Rule_1^2: & if\ x_1\ is\ A_1^2\ then\ y_1^2 = 10 \\ Rule_1^3: & if\ x_1\ is\ A_1^3\ then\ y_1^3 = 20 \end{Bmatrix}$$

$$SIRM\text{-}2: \begin{Bmatrix} Rule_2^1: & if\ x_2\ is\ A_2^1\ then\ y_2^1 = 1 \\ Rule_2^2: & if\ x_2\ is\ A_2^2\ then\ y_2^2 = 3 \\ Rule_2^3: & if\ x_2\ is\ A_2^3\ then\ y_2^3 = 14 \\ Rule_2^4: & if\ x_2\ is\ A_2^4\ then\ y_2^4 = 20 \end{Bmatrix}$$

Fig. 3 Rule modules for the example

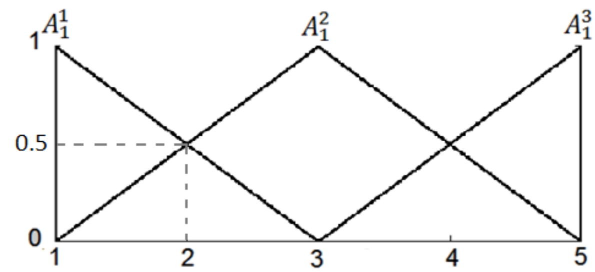

Fig. 4 Fuzzy set for $SIRM$- 1

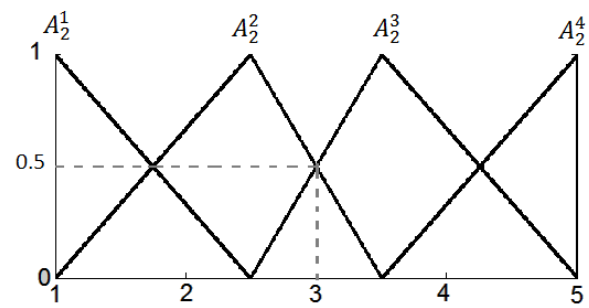

Fig. 5 Fuzzy set for $SIRM$- 2

It is assumed that $\tilde{r}_1 = (1.0, 1.2, 1.8, 3.0)$ and $\tilde{r}_2 = (2.0, 3.2, 3.8, 4.0)$. Using Eq. (1), the centroids of $\tilde{r}_1$ and $\tilde{r}_2$ are 1.79 and 3.21, respectively. Using Eq. (12), the scale factor is determined, i.e., $s = 0.2$. Let $x_1 = 2$ and $x_2 = 3$, the inference results for $SIRM\text{-}1$ and $SIRM\text{-}2$ are $y_1 = 0.5 \times 1 + 0.5 \times 10 = 5.5$ and $y_2 = 0.5 \times 3 + 0.5 \times 14 = 8.5$, respectively. Eq. (13) and Eq. (14) are used to produce $\tilde{y} = (4.50, 6.76, 8.44, 10.1)$ and $\bar{y} = 7.42$. A plot of $\bar{y}$ versus x_1 and x_2 is depicted in Fig. 6.

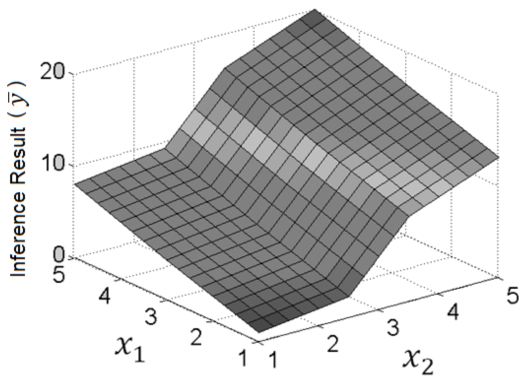

Fig. 6 The inference result from the proposed SIRMs-FIS model

D. Remarks

The contributions of this paper are two folds, (i) exploration of the general concept of the fuzzy ratio and fuzzy weight, and (ii) introduction of a new SIRMs-FIS model with fuzzy weights. Fuzzy weights for the new SIRMs-FIS model are interpreted as a ratio of the importance of rule modules to the inference result.

To the best of our knowledge, even though a number of investigations on SIRMs-FIS have been conducted, a proper definition of the weight domain for SIRMs-FIS is not available. As such, a definition of the weight domain is provided (i.e., Definition 6) in this paper. If an fuzzy ordering exists among $\widetilde{w}_i$, fuzzy ratio is possible. Besides that, to the best of our knowledge, little attention is given to the definition of y_i. In this paper, we interpret the y_i domain as Y. Such definition is useful for practical application of SIRMs-FIS in assessment and decision making problems, e.g., FMEA [7].

V. CONCLUSIONS

In this paper, the idea of a new fuzzy ratio is introduced. The proposed fuzzy ratio has been analyzed mathematically. The usefulness of the fuzzy ratio for modeling of SIRMs-FIS with imprecise weights has also been illustrated and discussed.

For further work, it is useful to examine the properties of the fuzzy ratio. Besides that, the applicability of SIRMs-FIS with imprecise weights to real-world problems (e.g., FMEA [9] and education assessment [12]) will be investigated.

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