2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

Dynamic Output Feedback Controller Design for T-S Fuzzy Plants with Actuator Saturation Using Linear Fractional Transformation

Yang Liu, Xiaojun Ban, Fen Wu, and H. K. Lam

***Abstract*— In this paper, a systematic synthesis method for Takagi-Sugeno fuzzy dynamic output feedback controller is proposed for T-S fuzzy plants with actuator saturation. By using the deadzone function, both the T-S fuzzy plant with actuator saturation and the T-S fuzzy dynamic output feedback controller are transformed into the form of linear fractional transformation (LFT). Within the framework of LFT, the issue of stability as well as H_∞ performance is cast as a convex optimization problem which can be approached by solving a set of linear matrix inequalities. A numerical example is presented to illustrate the effectiveness of the proposed method.**

I. Introduction

SINCE the concept of fuzzy sets is introduced into the control community in 1974 by Dr. Mamdani, fuzzy control has attracted a considerable amount of interest from both control theorists and control engineers. Some substantial research progresses have been made in both theory and applications [1, 2]. The T-S fuzzy system proposed in 1985 is a landmark in the history of fuzzy control theory [3]. This kind of fuzzy system can be regarded as a fuzzy blending of many local linear systems and can be effectively utilized to approximate nonlinear plants encountered in control engineering. Within the framework of T-S fuzzy models, numerous fuzzy control issues, such as stability analysis [4], systematic design, robust stability analysis and design [5–7], hybrid control system analysis and design [8], model reference tracking problems [9], time-delay system analysis [10], adaptive systems based on this fuzzy model [11] have been well studied [12].

On the other hand, saturation nonlinearity is widely encountered in control engineering and can significantly deteriorate the performance of a closed-loop system or even render a stable system unstable. Therefore it has attracted a lot of attention from the control community (see, for example, [13–16] and the references therein).

Because T-S fuzzy systems can be effectively used to approximate nonlinear plants, it is also significant to study the T-S fuzzy systems with input saturation nonlinearity. To this end, some research has been performed. In [17], the low-gain design approach is used to constrain the magnitude of the control output. Based on the method of dealing with actuator saturation in [13], a less conservative approach based on convex hull representations has been proposed in [18] to cope with the saturation nonlinearity in nonlinear systems. In [19], the saturation function is formulated into a specific nonlinear saturation sector and a new design approach is proposed which requires less number of LMIs. The aforementioned results concerns with the state feedback controllers. A fuzzy observer-based controller for nonlinear distributed parameter systems with control constraints is investigated in [20]. Moreover, a dynamic output feedback controller is considered in [21], in which both the amplitude saturation and the rate limitation are taken into consideration.

In this research, for the T-S fuzzy plant with actuator saturation, a T-S fuzzy dynamic output feedback controller with the same antecedents as in the T-S fuzzy plants is proposed to guarantee the stability of the closed-loop T-S fuzzy system as well as providing disturbance/error attenuation measured in L_2 norm. Distinct from the existing approaches, we are going to address the issue of stability and performance of the T-S fuzzy system with saturation nonlinearity from the perspective of gain-scheduling control. More specifically, by using the deadzone function both the T-S fuzzy plants and the T-S fuzzy controller are transformed into the form of linear fractional transformation (LFT). Within the framework of LFT, the synthesis problem is cast as a convex optimization problem in terms of linear matrix inequalities (LMIs) and can be solved efficiently by numerical algorithms. The inverted pendulum system is utilized to demonstrate the proposed saturation control approach.

The notations used in this paper are rather standard. $\mathbf{R}$ stands for the set of real numbers and $\mathbf{R}_+$ for the non-negative real numbers. $\mathbf{Z}_+$ is the set of non-negative integers. $\mathbf{R}^{m\times n}$ is the set of real $m\times n$ matrices. We use $\mathbf{S}^n$ to denote real, symmetric $n\times n$ matrices, and $\mathbf{S}^n_+$ for positive-definite matrices. A block-diagonal matrix with matrices $X_1, X_2, \cdots, X_p$ on its main diagonal is denoted as $\mathrm{diag}\{X_1, X_2, \cdots, X_p\}$. In large symmetric matrix expressions, terms denoted as $\star$ will be induced by symmetry. For two integers k_1, k_2, $k_1 < k_2$, we denote $\mathbf{I}[k_1, k_2] = \{k_1, k_1+1, \cdots, k_2\}$. $\mathrm{Co}\,\mathcal{S}$ means the convex hull of a set $\mathcal{S}$. The space of square integrable functions is denoted by $\mathcal{L}_2$, that is, for any $x \in \mathcal{L}_2$

$$\|x\|_2 := \left(\int_0^\infty x^T(t)x(t)dt\right)^{\frac{1}{2}} < \infty.$$

Yang Liu, Xiaojun Ban are with the Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, P. R. China (email: banxiaojun@hit.edu.cn). Fen Wu is with the Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC 27695, USA (email: fwu@ncsu.edu). H. K. Lam is with the Department of Informatics, King's College London, Strand, London, WC2R 2LS, United Kingdom (email: hak-keung.lam@kcl.ac.uk).

X. Ban's work is supported by the National Natural Science Foundation of China (NSFC) under Grant No. 61273095, NSFC under Grant No. 61304006 and the Fundamental Research Funds for the Central Universities (Grant No. HIT. NSRIF. 2013036). F. Wu's work is supported in part by the NSF Grant CMMI-0800044.

II. Problem Formulation

Consider a T-S fuzzy model which represents the dynamics of a nonlinear plant subject to actuator saturation. The ith rule can be described by the following linguistic rule.

Rule i :

IF z_1 is M_i^1, and, $\cdots$, z_r is M_i^r,

$$\text{THEN} \quad \begin{cases} \dot{x} &= A_i x + B_{1,i} d + B_{2,i} \text{sat}(u) \\ e &= C_{1,i} x + D_{11,i} d + D_{12,i} \text{sat}(u) \\ y &= C_{2,i} x + D_{21,i} d \end{cases}, \tag{1}$$

where $i \in \mathbf{I}[1,m]$, m is a positive integer which denotes the number of fuzzy rules. $z_j, j \in \mathbf{I}[1,r]$ are premise or antecedent variables. They could be arbitrary measurable variables. M_i^j are fuzzy or linguistic terms which can be quantified by membership functions, including triangular, tripezoid, and Gaussian-shaped membership functions and so on. $x \in \mathbf{R}^n$ is the plant state; $u \in \mathbf{R}^{n_u}$ is the control input; $d \in \mathbf{R}^{n_d}$ is the exogenous input, possibly including disturbance, measurement noise or reference signals; $y \in \mathbf{R}^{n_y}$ is the measurement output and $e \in \mathbf{R}^{n_e}$ is the performance output. $\text{sat}(\cdot)$ is a vector saturation function with the saturation levels given by a vector $\bar{u} \in \mathbf{R}^{n_u}$, $\bar{u}_i > 0, i \in \mathbf{I}[1, n_u]$. More specifically, $\text{sat}(u_i) = \text{sgn}(u_i) \min\{\bar{u}_i, |u_i|\}$. It is assumed that $(A_i, B_{2,i})$ is stabilizable and $(C_{2,i}, A_i)$ is detectable.

In this research, for disturbance attenuation, we are mainly concerned with a class of energy-bounded disturbances

$$\mathcal{W}_s = \left\{ d: \ \mathbf{R}_+ \to \mathbf{R}^{n_d}, \int_0^\infty d^T(\tau) d(\tau) d\tau \le s^2, \ d \in \mathcal{L}_2 \right\}$$

in which s is a given positive scalar.

By using the "product operation" to quantify the linguistic term "AND", and the "product inference mechanism" and "center-average" defuzzification method, the above T-S fuzzy system can be formulated as the following analytic formula.

$$\begin{cases} \dot{x} &= \sum_{i=1}^m g_i(z)[A_i x + B_{1,i} d + B_{2,i} \text{sat}(u)] \\ e &= \sum_{i=1}^m g_i(z)[C_{1,i} x + D_{11,i} d + D_{12,i} \text{sat}(u)] \\ y &= \sum_{i=1}^m g_i(z)(C_{2,i} x + D_{21,i} d) \end{cases} \tag{2}$$

where $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_r \end{bmatrix}^T$, and

$$g_i(z) = \frac{w_i(z)}{\sum_{i=1}^m w_i(z)} \tag{3}$$

$$w_i(z) = \prod_{j=1}^r M_i^j(z_j) \tag{4}$$

$M_i^j(z_j), j \in \mathbf{I}[1,r], i \in \mathbf{I}[1,m]$ is the degree of membership of z_j in M_i^j. Since the degree of membership is confined into the closed interval $[0,1]$, the following two properties hold.

$$0 \le w_i(z) \le 1 \tag{5}$$

$$\sum_{i=1}^m g_i(z) = 1 \tag{6}$$

By introducing a nominal model, equation (2) can be rewritten as

$$\begin{cases} \dot{x} &= A_0 x + B_{1,0} d + B_{2,0} \text{sat}(u) \\ & \quad + \sum_{i=1}^m g_i(z)[A_{\delta i} x + B_{1,\delta i} d + B_{2,\delta i} \text{sat}(u)] \\ e &= C_{1,0} x + D_{11,0} d + D_{12,0} \text{sat}(u) \\ & \quad + \sum_{i=1}^m g_i(z)[C_{1,\delta i} x + D_{11,\delta i} d + D_{12,\delta i} \text{sat}(u)] \\ y &= C_{2,0} x + D_{21,0} d \\ & \quad + \sum_{i=1}^m g_i(z)[C_{2,\delta i} x + D_{21,\delta i} d] \end{cases}$$

with

$$\begin{aligned} A_0 + A_{\delta i} &= A_i, & B_{1,0} + B_{1,\delta i} &= B_{1,i}, \\ B_{2,0} + B_{2,\delta i} &= B_{2,i}, & C_{1,0} + C_{1,\delta i} &= C_{1,i}, \\ C_{2,0} + C_{2,\delta i} &= C_{2,i}, & D_{11,0} + D_{11,\delta i} &= D_{11,i}, \\ D_{12,0} + D_{12,\delta i} &= D_{12,i}, & D_{21,0} + D_{21,\delta i} &= D_{21,i}. \end{aligned}$$

Moreover, by introducing the pseudo input p_f, the pseudo output q_f, and considering $\theta_{f,i} = g_i(z)$ as some measurable parameters ranging within the interval $[0,1]$, the T-S fuzzy plant can be reformulated in a more compacted LFT form as below

$$\begin{bmatrix} \dot{x} \\ q_f \\ e \\ y \end{bmatrix} = \begin{bmatrix} A_0 & E_x & B_{1,0} & B_{2,0} \\ \Delta_x & 0 & \Delta_d & \Delta_u \\ C_{1,0} & E_e & D_{11,0} & D_{12,0} \\ C_{2,0} & E_y & D_{21,0} & 0 \end{bmatrix} \begin{bmatrix} x \\ p_f \\ d \\ \text{sat}(u) \end{bmatrix} \tag{7}$$

$$p_f = \Theta_f q_f \tag{8}$$

where

$$E_x = \begin{bmatrix} I_n & \cdots & I_n & 0_{n \times m n_e} & 0_{n \times m n_y} \end{bmatrix}_{n \times m(n+n_e+n_y)}$$

$$E_e = \begin{bmatrix} 0_{n_e \times mn} & I_{n_e} & \cdots & I_{n_e} & 0_{n_e \times m n_y} \end{bmatrix}_{n_e \times m(n+n_e+n_y)}$$

$$E_y = \begin{bmatrix} 0_{n_y \times mn} & 0_{n_y \times m n_e} & I_{n_y} & \cdots & I_{n_y} \end{bmatrix}_{n_y \times m(n+n_e+n_y)}$$

$$\Delta_x = \begin{bmatrix} A_{\delta 1} \\ \vdots \\ A_{\delta m} \\ C_{1,\delta 1} \\ \vdots \\ C_{1,\delta m} \\ C_{2,\delta 1} \\ \vdots \\ C_{2,\delta m} \end{bmatrix}, \ \Delta_d = \begin{bmatrix} B_{1,\delta 1} \\ \vdots \\ B_{1,\delta m} \\ D_{11,\delta 1} \\ \vdots \\ D_{11,\delta m} \\ D_{21,\delta 1} \\ \vdots \\ D_{21,\delta m} \end{bmatrix}, \ \Delta_u = \begin{bmatrix} B_{2,\delta 1} \\ \vdots \\ B_{2,\delta m} \\ D_{12,\delta 1} \\ \vdots \\ D_{12,\delta m} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Theta_f = \text{diag}\{ & \theta_{f,1} I_n, \cdots, \theta_{f,m} I_n, \\ & \theta_{f,1} I_{n_e}, \cdots, \theta_{f,m} I_{n_e}, \\ & \theta_{f,1} I_{n_y}, \cdots, \theta_{f,m} I_{n_y} \} \end{aligned}$$

and

$$p_f = \begin{bmatrix} p_x \\ p_e \\ p_y \end{bmatrix}, \quad q_f = \begin{bmatrix} q_x \\ q_e \\ q_y \end{bmatrix},$$

$$p_x = \begin{bmatrix} p_{x,1} \\ \vdots \\ p_{x,m} \end{bmatrix}, \quad p_e = \begin{bmatrix} p_{e,1} \\ \vdots \\ p_{e,m} \end{bmatrix}, \quad p_y = \begin{bmatrix} p_{y,1} \\ \vdots \\ p_{y,m} \end{bmatrix},$$
$$q_x = \begin{bmatrix} q_{x,1} \\ \vdots \\ q_{x,m} \end{bmatrix}, \quad q_e = \begin{bmatrix} q_{e,1} \\ \vdots \\ q_{e,m} \end{bmatrix}, \quad q_y = \begin{bmatrix} q_{y,1} \\ \vdots \\ q_{y,m} \end{bmatrix},$$

with

$$\begin{cases} p_{x,i} &= \theta_{f,i} I_n q_{x,i} \\ p_{e,i} &= \theta_{f,i} I_{n_e} q_{e,i} \\ p_{y,i} &= \theta_{f,i} I_{n_y} q_{y,i} \end{cases}$$
$$\begin{cases} q_{x,i} &= A_{\delta i} x + B_{1,\delta i} d + B_{2,\delta i} \mathrm{sat}(u) \\ q_{e,i} &= C_{1,\delta i} x + D_{11,\delta i} d + D_{12,\delta i} \mathrm{sat}(u) \\ q_{y,i} &= C_{2,\delta i} x + D_{21,\delta i} d \end{cases}$$

Using the deadzone nonlinearity, i.e., $\mathrm{dz}(u) = u - \mathrm{sat}(u)$, the state equations of the plant (7)-(8) can be rewritten as

$$\begin{bmatrix} \dot{x} \\ u \\ q_f \\ e \\ y \end{bmatrix} = \begin{bmatrix} A_0 & -B_{2,0} & E_x & B_{1,0} & B_{2,0} \\ 0 & 0 & 0 & 0 & I \\ \Delta_x & -\Delta_u & 0 & \Delta_d & \Delta_u \\ C_{1,0} & -D_{12,0} & E_e & D_{11,0} & D_{12,0} \\ C_{2,0} & 0 & E_y & D_{21,0} & 0 \end{bmatrix} \begin{bmatrix} x \\ p_s \\ p_f \\ d \\ u \end{bmatrix} \quad (9)$$
$$p_s = \mathrm{dz}(u) \quad (10)$$
$$p_f = \Theta_f q_f \quad (11)$$

Now, the property as described in Lemma 1 is to be used to deal with the deadzone nonlinearity.

Lemma 1 ([22]): Let $t(x) = Tx$ be a linear map and suppose $T_i x \in [-\bar{u}_i, \bar{u}_i]$, where T_i denotes ith row of the matrix T. For any u_i, we have $\mathrm{sat}(u_i) \in \mathrm{Co}\{u_i, T_i x\}$ and $\mathrm{dz}(u_i) = \theta_{s,i}(u_i - T_i x)$ for some $\theta_{s,i} \in [0, 1]$.

Therefore, from Lemma 1 the nonlinear equations (10)-(11) can be replaced by

$$\begin{bmatrix} p_s \\ p_f \end{bmatrix} = \begin{bmatrix} \Theta_s & 0 \\ 0 & \Theta_f \end{bmatrix} \left(\begin{bmatrix} u \\ q_f \end{bmatrix} - \begin{bmatrix} H_1 & H_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_k \end{bmatrix} \right) \quad (12)$$

where

$$\Theta_s = \mathrm{diag}\{\theta_{s,1}, \cdots, \theta_{s,n_u}\}$$

under the regional constraint

$$\left| \begin{bmatrix} H_1 & H_2 \end{bmatrix} \begin{bmatrix} x \\ x_k \end{bmatrix} \right|_\ell \le \bar{u}_\ell, \quad (13)$$

where $\ell \in \mathbf{I}[1, n_u]$, and the subscript ℓ specifies the ℓth row of the corresponding matrix, x_k is the state of the controller to be designed. In order to render the fuzzy gain scheduling controller implementable, only the controller state is used to specify the regional constraints for actuator saturation. As a result, we assume $H_1 = 0$ in equations (12) and (13).

In this research, a T-S fuzzy dynamic output feedback controller will be employed to deal with the above T-S fuzzy plant with input saturation. The ith rule of the T-S controller can be described by the following linguistic rule.

$$\begin{aligned} &\text{Rule}\, i: \\ &\text{IF } z_1 \text{ is } M_i^1, \text{ and}, \cdots, z_r \text{ is } M_i^r, \\ &\text{THEN} \quad \begin{cases} \dot{x}_k &= A_{c,i} x_k + B_{c,i} y \\ u &= C_{c,i} x_k + D_{c,i} y \end{cases}, i \in \mathbf{I}[1, m]. \end{aligned} \quad (14)$$

For simplicity, assume $A_{c,i} = A_k, B_{c,i} = B_k, i \in \mathbf{I}[1, m]$ in this paper. By using the same defuzzification method to the plant, the above fuzzy controller can be formulated as below

$$\begin{cases} \dot{x}_k &= A_k x_k + B_k y \\ u &= \sum_{i=1}^m g_i(z)(C_{c,i} x_k + D_{c,i} y) \end{cases}$$

Note that although the above controller is a simplified T-S fuzzy controller, it comprises several commonly used controllers as special cases. For example, if $C_{c,i}$ and $D_{c,i}$ are all constant matrices, it degenerates into a linear dynamical controller. If $C_{c,i} = 0$, it is a static output controller with a scheduling gain, i.e., gain-scheduling static output controller. For the general single-input-single-output case, it can be regarded as a softly switching controller between several linear dynamical controllers with the same poles and different zeros and gains.

By introducing $C_{k,0}, D_{k,0}$ and the pseudo input p_k and pseudo output u_k, the fuzzy controller can be recast in a more compacted LFT formulation as we have done for the T-S fuzzy plant.

$$\begin{bmatrix} \dot{x}_k \\ u_k \\ u \end{bmatrix} = \begin{bmatrix} A_k & 0 & B_k \\ C_k & 0 & D_k \\ C_{k,0} & E & D_{k,0} \end{bmatrix} \begin{bmatrix} x_k \\ p_k \\ y \end{bmatrix} \quad (15)$$
$$p_k = \Theta_k u_k \quad (16)$$

where

$$C_k = \begin{bmatrix} C_{k,1} \\ \vdots \\ C_{k,m} \end{bmatrix}, \qquad D_k = \begin{bmatrix} D_{k,1} \\ \vdots \\ D_{k,m} \end{bmatrix},$$
$$C_{k,i} = C_{c,i} - C_{k,0}, \quad D_{k,i} = D_{c,i} - D_{k,0},$$
$$E = \begin{bmatrix} I_{n_u} & \cdots & I_{n_u} \end{bmatrix}_{n_u \times (m n_u)},$$
$$\Theta_k = \mathrm{diag}\{\theta_{k_1} I_{n_u}, \cdots, \theta_{k_m} I_{n_u}\},$$

and

$$p_k = \begin{bmatrix} p_{k,1} \\ \vdots \\ p_{k,m} \end{bmatrix}, \qquad u_k = \begin{bmatrix} u_{k,1} \\ \vdots \\ u_{k,m} \end{bmatrix},$$
$$p_{k,i} = \theta_{f,i} I_{n_u} u_{k,i}, \quad u_{k,i} = C_{k,i} x_k + D_{k,i} y.$$

By combining the plant and the controller together, we obtain the closed-loop system as below

$$\begin{bmatrix} \dot{x}_{cl} \\ q_{cl} \\ e \end{bmatrix} = \begin{bmatrix} A_{cl} & B_{0,cl} & B_{1,cl} \\ C_{0,cl} & D_{00,cl} & D_{01,cl} \\ C_{1,cl} & D_{10,cl} & D_{11,cl} \end{bmatrix} \begin{bmatrix} x_{cl} \\ p_{cl} \\ d \end{bmatrix}, \quad (17)$$
$$p_{cl} = \Theta \left(q_{cl} - H x_c l \right), \quad (18)$$

$$
\begin{bmatrix} A_{cl} & B_{0,cl} & B_{1,cl} \\ C_{0,cl} & D_{00,cl} & D_{01,cl} \\ C_{1,cl} & D_{10,cl} & D_{11,cl} \end{bmatrix}
$$

$$
= \left[\begin{array}{cc|cccc|c}
A_0 + B_{2,0}D_{k,0}C_{2,0} & B_{2,0}C_{k,0} & -B_{2,0} & E_x + B_{2,0}D_{k,0}E_y & B_{2,0}E & & B_{1,0} + B_{2,0}D_{k,0}D_{21,0} \\
B_kC_{2,0} & A_k & 0 & B_kE_y & 0 & & B_kD_{21,0} \\ \hline
D_{k,0}C_{2,0} & C_{k,0} & 0 & D_{k,0}E_y & E & & D_{k,0}D_{21,0} \\
\Delta_x + \Delta_u D_{k,0}C_{2,0} & \Delta_u C_{k,0} & -\Delta_u & \Delta_u D_{k,0}E_y & \Delta_u E & & \Delta_d + \Delta_u D_{k,0}D_{21,0} \\
D_kC_{2,0} & C_k & 0 & D_kE_y & 0 & & D_kD_{21,0} \\ \hline
C_{1,0} + D_{12,0}D_{k,0}C_{2,0} & D_{12,0}C_{k,0} & -D_{12,0} & E_e + D_{12,0}D_{k,0}E_y & D_{12,0}E & & D_{11,0} + D_{12,0}D_{k,0}D_{21,0}
\end{array}\right]
$$

$$
= \left[\begin{array}{cc|cccc|c}
A_0 & 0 & -B_{2,0} & E_x & B_{2,0}E & & B_{1,0} \\
0 & 0 & 0 & 0 & 0 & & 0 \\ \hline
0 & 0 & 0 & 0 & E & & 0 \\
\Delta_x & 0 & -\Delta_u & 0 & \Delta_u E & & \Delta_d \\
0 & 0 & 0 & 0 & 0 & & 0 \\ \hline
C_{1,0} & 0 & -D_{12,0} & E_e & D_{12,0}E & & D_{11,0}
\end{array}\right]
+ \left[\begin{array}{ccc}
0 & 0 & B_{2,0} \\
I & 0 & 0 \\ \hline
0 & 0 & I \\
0 & 0 & \Delta_u \\
0 & I & 0 \\ \hline
0 & 0 & D_{12,0}
\end{array}\right]
\begin{bmatrix} A_k & B_k \\ C_k & D_k \\ C_{k,0} & D_{k,0} \end{bmatrix}
\left[\begin{array}{cc|ccc|c}
0 & I & 0 & 0 & 0 & 0 \\
C_{2,0} & 0 & 0 & E_y & 0 & D_{21,0}
\end{array}\right]
$$

and

$$
x_{cl} = \begin{bmatrix} x \\ x_k \end{bmatrix}, \quad q_{cl} = \begin{bmatrix} u \\ q_f \\ u_k \end{bmatrix}, \quad p_{cl} = \begin{bmatrix} p_s \\ p_f \\ p_k \end{bmatrix}
$$

$$
\Theta = \begin{bmatrix} \Theta_s & 0 & 0 \\ 0 & \Theta_f & 0 \\ 0 & 0 & \Theta_k \end{bmatrix}, \quad H = \begin{bmatrix} 0 & H_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},
$$

III. Main Result

Based on the closed-loop system (17)-(18), we will present the main result of this research in the following theorem.

Theorem 1: Given scalars $\gamma, s > 0$, if there exist positive definite matrices $R, S \in \mathbf{S}_+^n$ and $K \in \mathbf{S}_+^{mn_u}$, $\bar{H} \in \mathbf{R}^{n_u \times n}$ such that

$$
\begin{bmatrix} \mathcal{H}_{\tilde{\Phi}}^{\mathrm{T}} & 0 \\ 0 & I \end{bmatrix} \times
\left[\begin{array}{cccc|c}
RA_0^{\mathrm{T}} + A_0R & \star & \star & \star & \star \\
-B_{2,0}^{\mathrm{T}} - \bar{H} & -2I & \star & \star & \star \\
E_x^{\mathrm{T}} + \Delta_x R & -\Delta_u & -2I & \star & \star \\
C_{1,0}R & -D_{12,0} & E_e & -\gamma^2 I & \star \\ \hline
B_{1,0}^{\mathrm{T}} & 0 & \Delta_d^{\mathrm{T}} & D_{11,0}^{\mathrm{T}} & -I
\end{array}\right]
\times \begin{bmatrix} \mathcal{H}_{\tilde{\Phi}} & 0 \\ 0 & I \end{bmatrix} < 0 \tag{19}
$$

$$
\begin{bmatrix} \mathcal{H}_{\Gamma}^{\mathrm{T}} & 0 \\ 0 & I \end{bmatrix} \times
\left[\begin{array}{ccc|ccc}
A_0^{\mathrm{T}}S + SA_0 & \star & \star & \star & \star & \star \\
E_x^{\mathrm{T}}S + \Delta_x & -2I & \star & \star & \star & \star \\
B_{1,0}^{\mathrm{T}}S & \Delta_d^{\mathrm{T}} & -I & \star & \star & \star \\ \hline
-B_{2,0}^{\mathrm{T}}S & -\Delta_u^{\mathrm{T}} & 0 & -2I & \star & \star \\
E^{\mathrm{T}}B_{2,0}^{\mathrm{T}}S & E^{\mathrm{T}}\Delta_u^{\mathrm{T}} & 0 & E^{\mathrm{T}} & -2K & \star \\
C_{1,0} & E_e & D_{11,0} & -D_{12,0} & D_{12,0}E & -\gamma^2 I
\end{array}\right]
\times \begin{bmatrix} \mathcal{H}_{\Gamma} & 0 \\ 0 & I \end{bmatrix} < 0 \tag{20}
$$

$$
\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0 \tag{21}
$$

$$
\begin{bmatrix} \frac{\bar{u}_\ell^2}{s^2} & (\bar{H})_\ell & 0 \\ \star & R & I \\ \star & \star & S \end{bmatrix} \geq 0, \quad \forall \ell \in \mathbf{I}[1, n_u] \tag{22}
$$

where $\mathcal{H}_{\tilde{\Phi}}$ and $\mathcal{H}_{\Gamma}$ denote matrices whose columns are bases of $\operatorname{Ker}\left[B_{2,0}^{\mathrm{T}}\ I\ \Delta_u^{\mathrm{T}}\ D_{12,0}^{\mathrm{T}}\right]$ and $\operatorname{Ker}\left[C_{2,0}\ E_y\ D_{21,0}\right]$ respectively, then a fuzzy dynamic output feedback controller in the form of (14) of the same order as the plant, will asymptotically stabilize the T-S plant with its ith rule in the form of (1) and render the $\mathcal{L}_2$ gain of the loop system less than γ for any bounded disturbance $d \in \mathcal{W}_s$.

Proof: Using a quadratic Lyapunov function $V(x_{cl}) = x_{cl}^{\mathrm{T}} P x_{cl}$ and a matrix $\Lambda > 0$ commutable with Θ, the Lyapunov condition can be extended as

$$
\dot{V} + \frac{1}{\gamma^2} e^{\mathrm{T}} e - d^{\mathrm{T}} d + p_{cl}^{\mathrm{T}} \Lambda (q_{cl} - p_{cl}) + (q_{cl} - p_{cl})^{\mathrm{T}} \Lambda p_{cl} < 0,
$$

and the set inclusion condition

$$
\begin{aligned}
&\left\{ x_{cl} : \ x_{cl}^{\mathrm{T}} P x_{cl} \leq s^2 \right\} \\
&\subset \left\{ x_{cl} : \ \left| \begin{bmatrix} 0 & H_2 \end{bmatrix} x_{cl} \right|_\ell \leq \bar{u}_\ell, \ell \in \mathbf{I}[1, n_u] \right\}
\end{aligned} \tag{23}
$$

guarantee the stability and $\mathcal{L}_2$ gain performance in the existence of actuator saturation. Therefore, we have

$$
\begin{bmatrix}
A_{cl}^{\mathrm{T}} P + PA_{cl} & \star & \star & \star \\
B_{0,cl}^{\mathrm{T}} P + \Lambda (C_{0,cl} - H) & \Lambda (D_{00,cl} - I) + (D_{00,cl} - I)^{\mathrm{T}} \Lambda & \star & \star \\
B_{1,cl}^{\mathrm{T}} P & D_{01,cl}^{\mathrm{T}} \Lambda & -I & \star \\
C_{1,cl} & D_{10,cl} & D_{11,cl} & -\gamma^2 I
\end{bmatrix} < 0 \tag{24}
$$

Taking equation (17) into consideration, the inequality (24) can be rewritten as

$$\Psi + \Gamma^{\mathrm{T}}\Pi^{\mathrm{T}}\Phi + \Phi^{\mathrm{T}}\Pi\Gamma < 0, \tag{25}$$

where

$$\Psi = \begin{bmatrix} \mathcal{A}^{\mathrm{T}}P + P\mathcal{A} & \star & \star & \star \\ \mathcal{B}_0^{\mathrm{T}}P + \Lambda(\mathcal{C}_0 - H) & \Lambda(\mathcal{D}_{00} - I) + (\mathcal{D}_{00} - I)^{\mathrm{T}}\Lambda & \star & \star \\ \mathcal{B}_1^{\mathrm{T}}P & \mathcal{D}_{01}^{\mathrm{T}}\Lambda & -I & \star \\ \mathcal{C}_1 & D_{10} & \mathcal{D}_{11} & -\gamma^2 I \end{bmatrix},$$

$$\Pi = \begin{bmatrix} A_k & B_k \\ C_k & D_k \\ C_{k_0} & D_{k_0} \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} \mathcal{F} & \mathcal{G}_1 & \mathcal{G}_2 & 0 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \mathcal{W}^{\mathrm{T}}P & \mathcal{E}_1^{\mathrm{T}}\Lambda & 0 & \mathcal{E}_2^{\mathrm{T}} \end{bmatrix},$$

with

$$\mathcal{A} = \begin{bmatrix} A_0 & 0 \\ 0 & 0 \end{bmatrix}, \ \mathcal{C} = \begin{bmatrix} \mathcal{C}_0 \\ \mathcal{C}_1 \end{bmatrix} = \left[\begin{array}{cc} 0 & 0 \\ \Delta_x & 0 \\ 0 & 0 \\ \hline C_{1,0} & 0 \end{array}\right],$$

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_0 & \mathcal{B}_1 \end{bmatrix} = \left[\begin{array}{ccc|c} -B_{2,0} & E_x & B_{2,0}E & B_{1,0} \\ 0 & 0 & 0 & 0 \end{array}\right],$$

$$\mathcal{D} = \begin{bmatrix} \mathcal{D}_{00} & \mathcal{D}_{01} \\ \mathcal{D}_{10} & \mathcal{D}_{11} \end{bmatrix} = \left[\begin{array}{ccc|c} 0 & 0 & E & 0 \\ -\Delta_u & 0 & \Delta_u E & \Delta_d \\ 0 & 0 & 0 & 0 \\ \hline -D_{12,0} & E_e & D_{12,0}E & D_{11,0} \end{array}\right],$$

$$\mathcal{F} = \begin{bmatrix} 0 & I \\ C_{2,0} & 0 \end{bmatrix}, \ \mathcal{G} = \begin{bmatrix} \mathcal{G}_1 & \mathcal{G}_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & E_y & 0 & D_{21,0} \end{array}\right],$$

$$\mathcal{W} = \begin{bmatrix} 0 & 0 & B_{2,0} \\ I & 0 & 0 \end{bmatrix}, \ \mathcal{E} = \begin{bmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{bmatrix} = \left[\begin{array}{ccc} 0 & 0 & I \\ 0 & 0 & \Delta_u \\ 0 & I & 0 \\ \hline 0 & 0 & D_{12,0} \end{array}\right].$$

Applying Elimination Lemma, inequality (25) is equivalent to

$$\begin{cases} \mathcal{N}_\Phi^{\mathrm{T}}\Psi\mathcal{N}_\Phi < 0 \\ \mathcal{N}_\Gamma^{\mathrm{T}}\Psi\mathcal{N}_\Gamma < 0 \end{cases} \tag{26}$$

where $\mathcal{N}_\Gamma$ and $\mathcal{N}_\Phi$ are the null matrices of Γ and Φ, respectively. Note that Φ satisfies with

$$\Phi = \tilde{\Phi} \begin{bmatrix} P & 0 & 0 & 0 \\ 0 & \Lambda & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

with $\tilde{\Phi} = \begin{bmatrix} \mathcal{W}^{\mathrm{T}} & \mathcal{E}_1^{\mathrm{T}} & 0 & \mathcal{E}_2^{\mathrm{T}} \end{bmatrix}$. Therefore, inequality (26) is also equivalent to

$$\begin{cases} \mathcal{N}_{\tilde{\Phi}}^{\mathrm{T}}\tilde{\Psi}\mathcal{N}_{\tilde{\Phi}} < 0 \\ \mathcal{N}_\Gamma^{\mathrm{T}}\Psi\mathcal{N}_\Gamma < 0 \end{cases} \tag{27}$$

where $\mathcal{N}_{\tilde{\Phi}}$ is the null matrix of $\tilde{\Phi}$ and

$$\tilde{\Psi} = \begin{bmatrix} P^{-1}\mathcal{A}^{\mathrm{T}} + \mathcal{A}P^{-1} & \star & \star & \star \\ \Lambda^{-1}\mathcal{B}_0^{\mathrm{T}} + (\mathcal{C}_0 - H)P^{-1} & (\mathcal{D}_{00} - I)\Lambda^{-1} + \Lambda^{-1}(\mathcal{D}_{00} - I)^{\mathrm{T}} & \star & \star \\ \mathcal{B}_1^{\mathrm{T}} & \mathcal{D}_{01}^{\mathrm{T}} & -I & \star \\ \mathcal{C}_1 P^{-1} & \mathcal{D}_{10}\Lambda^{-1} & \mathcal{D}_{11} & -\gamma^2 I \end{bmatrix}.$$

Furthermore, we have

$$\tilde{\Phi} = \begin{bmatrix} \mathcal{W}^{\mathrm{T}} & \mathcal{E}_1^{\mathrm{T}} & 0 & \mathcal{E}_2^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 \\ B_{2,0}^{\mathrm{T}} & 0 & I & -\Delta_u^{\mathrm{T}} & 0 & 0 & D_{12,0}^{\mathrm{T}} \end{bmatrix},$$

and then can derive one of the bases of the kernel space of $\tilde{\Phi}$ as follows

$$\mathcal{N}_{\tilde{\Phi}}^{\mathrm{T}} = \begin{bmatrix} H_{\tilde{\Phi}_1}^{\mathrm{T}} & 0 & H_{\tilde{\Phi}_2}^{\mathrm{T}} & H_{\tilde{\Phi}_3}^{\mathrm{T}} & 0 & 0 & H_{\tilde{\Phi}_4}^{\mathrm{T}} \\ 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}.$$

By assuming

$$P = \begin{bmatrix} S & N \\ N^{\mathrm{T}} & \star_1 \end{bmatrix}, \qquad P^{-1} = \begin{bmatrix} R & M \\ M^{\mathrm{T}} & \star_2 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} L & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & K \end{bmatrix}, \quad \Lambda^{-1} = \begin{bmatrix} U & 0 & 0 \\ 0 & V & 0 \\ 0 & 0 & W \end{bmatrix}.$$

then we can arrive at

$$\tilde{\Psi} = \left[\begin{array}{cc|ccc|c|c} RA_0^{\mathrm{T}} + A_0R & \star & \star & \star & \star & \star & \star \\ M^{\mathrm{T}}A_0^{\mathrm{T}} & 0 & \star & \star & \star & \star & \star \\ \hline -UB_{2,0}^{\mathrm{T}} - H_2M^{\mathrm{T}} & -H_2\star_2 & -2U & \star & \star & \star & \star \\ VE_x^{\mathrm{T}} + \Delta_xR & \Delta_xM & -\Delta_uU & -2V & \star & \star & \star \\ WE^{\mathrm{T}}B_{2,0}^{\mathrm{T}} & 0 & WE^{\mathrm{T}} & WE^{\mathrm{T}}\Delta_u^{\mathrm{T}} & -2W & \star & \star \\ \hline B_{1,0}^{\mathrm{T}} & 0 & 0 & \Delta_d^{\mathrm{T}} & 0 & -I & \star \\ \hline C_{1,0}R & C_{1,0}M & -D_{12,0}U & E_eV & D_{12,0}EW & D_{11,0} & -\gamma^2 I \end{array}\right]$$

Assuming $\bar{H} = H_2 M^T$, then we derive from $\mathcal{N}_{\tilde{\Phi}}^{\mathrm{T}} \tilde{\Psi} \mathcal{N}_{\tilde{\Phi}} < 0$ that

$$\begin{bmatrix} H_{\tilde{\Phi}_1}^{\mathrm{T}} & H_{\tilde{\Phi}_2}^{\mathrm{T}} & H_{\tilde{\Phi}_3}^{\mathrm{T}} & H_{\tilde{\Phi}_4}^{\mathrm{T}} & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \times \begin{bmatrix} RA_0^{\mathrm{T}} + A_0 R & \star & \star & \star & \star \\ -UB_{2,0}^{\mathrm{T}} - \bar{H} & -2U & \star & \star & \star \\ VE_x^{\mathrm{T}} + \Delta_x R & -\Delta_u U & -2V & \star & \star \\ C_{1,0} R & -D_{12,0} U & E_e V & -\gamma^2 I & \star \\ B_{1,0}^{\mathrm{T}} & 0 & \Delta_d^{\mathrm{T}} & D_{11,0}^{\mathrm{T}} & -I \end{bmatrix} \times \begin{bmatrix} H_{\tilde{\Phi}_1} & 0 \\ H_{\tilde{\Phi}_2} & 0 \\ H_{\tilde{\Phi}_3} & 0 \\ H_{\tilde{\Phi}_4} & 0 \\ 0 & I \end{bmatrix} < 0 \tag{28}$$

Since

$$\Gamma = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 \\ C_{2,0} & 0 & 0 & E_y & 0 & D_{21,0} & 0 \end{bmatrix}$$

$\mathcal{N}_{\Gamma}^{\mathrm{T}}$, one of the bases of the kernel space of Γ, can be described as follows

$$N_{\Gamma} = \begin{bmatrix} H_{\Gamma_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ H_{\Gamma_2} & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ H_{\Gamma_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

Recall that

$$\Psi = \left[\begin{array}{cc|c|cc|c|c} A_0^{\mathrm{T}} S + S A_0 & \star & \star & \star & \star & \star & \star \\ N^{\mathrm{T}} A_0 & 0 & \star & \star & \star & \star & \star \\ \hline -B_{2,0}^{\mathrm{T}} S & -B_{2,0}^{\mathrm{T}} N - L H_2 & -2L & \star & \star & \star & \star \\ E_x^{\mathrm{T}} S + J\Delta_x & E_x^{\mathrm{T}} N & -J\Delta_u & -2J & \star & \star & \star \\ E^{\mathrm{T}} B_{2,0}^{\mathrm{T}} S & E^{\mathrm{T}} B_{2,0}^{\mathrm{T}} N & E^{\mathrm{T}} L & E^{\mathrm{T}} \Delta_u^{\mathrm{T}} J & -2K & \star & \star \\ \hline B_{1,0}^{\mathrm{T}} S & B_{1,0}^{\mathrm{T}} N & 0 & \Delta_d^{\mathrm{T}} J & 0 & -I & \star \\ \hline C_{1,0} & 0 & -D_{12,0} & E_e & D_{12,0} E & D_{11,0} & -\gamma^2 I \end{array}\right],$$

we obtain from $\mathcal{N}_{\Gamma}^{T} \Psi \mathcal{N}_{\Gamma} < 0$ that

$$\begin{bmatrix} H_{\Gamma_1}^{\mathrm{T}} & H_{\Gamma_2}^{\mathrm{T}} & H_{\Gamma_3}^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \times \begin{bmatrix} A_0^{\mathrm{T}} S + S A_0 & \star & \star & \star & \star & \star \\ E_x^{\mathrm{T}} S + J\Delta_x & -2J & \star & \star & \star & \star \\ B_{1,0}^{\mathrm{T}} S & \Delta_d^{\mathrm{T}} J & -I & \star & \star & \star \\ -B_{2,0}^{\mathrm{T}} S & -\Delta_u^{\mathrm{T}} J & 0 & -2L & \star & \star \\ E^{\mathrm{T}} B_{2,0}^{\mathrm{T}} S & E^{\mathrm{T}} \Delta_u^{\mathrm{T}} J & 0 & E^{\mathrm{T}} L & -2K & \star \\ C_{1,0} & E_e & D_{11,0} & -D_{12,0} & D_{12,0} E & -\gamma^2 I \end{bmatrix} \times \begin{bmatrix} H_{\Gamma_1} & 0 & 0 & 0 \\ H_{\Gamma_2} & 0 & 0 & 0 \\ H_{\Gamma_3} & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} < 0. \tag{29}$$

By choosing L and J to be I, and consequently $U = I$ and $V = I$, the inequality (28) confirms inequality (19), and the inequality (29) confirms inequality (20).

The inequality (21) is a equivalent condition of $P > 0$ and $P^{-1} > 0$. The proof of inequality (22) is similar to that in [23] to insure the set inclusion condition (23). ■

The feasibility problem (19)-(22) can be solved as the following optimization problem

$$\begin{array}{rl} \min\limits_{R,S,J,\bar{H}} & \gamma^2 \\ \text{s. t.} & (19) - (22) \end{array} \tag{30}$$

Moreover, given any feasible solution to the above LMI constraints, the state-space gains of a corresponding controller can be determined via a constructive procedure as follows:

Step 0 Obtain R, S, K, and $\bar{H}$ by solving the optimization problem (30).

Step 1 Select M, N matrices such that $MN^{\mathrm{T}} = I - RS$.

Step 2 Compute H_2 by $H_2 = \bar{H} M^{-\mathrm{T}}$.

Step 3 Calculate Π from (31) as an LMI feasibility problem to obtain the controller gains.

$$\Psi + \Gamma^{\mathrm{T}} \Pi^{\mathrm{T}} \Phi + \Phi^{\mathrm{T}} \Pi \Gamma < 0 \tag{31}$$

IV. Examples

To illustrate the proposed approach, consider the problem of balancing an inverted pendulum on a cart. Recall the equations of motion for the pendulum

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g \sin(x_1) - amlx_2^2 \sin(2x_1)/2 - a\cos(x_1) u}{4l/3 - aml\cos^2(x_1)}$$

where x_1 denotes the angle of the pendulum from the vertical and x_2 is the angular velocity; $g = 9.8m/s^2$ is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart; $a = 1/(m+M)$. We choose $m = 2.0$kg, $M = 8.0$kg, $2l = 1.0$m and the $\bar{u} = 50$N in the simulation.

In order to approximate the above system by a T-S fuzzy model in the form of (1), we choose x_1 as the premise variable, d as the disturbance force on the cart, e and y both as the angle of the pendulum from the vertical. Then we have

$$
\begin{aligned}
&\text{Rule 1}: \\
&\text{IF } x_1 \text{ is about } 0, \\
&\text{THEN} \quad \begin{cases} \dot{x} &= A_1 x + B_{1,1} d + B_{2,1}\mathrm{sat}(u) \\ e &= C_1 x \\ y &= C_2 x \end{cases} ; \\
&\text{Rule 2}: \\
&\text{IF } x_1 \text{ is about } \pm\pi/8, \\
&\text{THEN} \quad \begin{cases} \dot{x} &= A_2 x + B_{1,2} d + B_{2,2}\mathrm{sat}(u) \\ e &= C_1 x \\ y &= C_2 x \end{cases} .
\end{aligned}
$$

where

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3-aml} & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ \frac{8g/\pi\sin(\pi/8)}{4l/3-aml\cos^2(pi/8)} & 0 \end{bmatrix}, \\
B_{1,1} &= \begin{bmatrix} 0 \\ \frac{-a}{4l/3-aml} \end{bmatrix}, \quad B_{1,2} = \begin{bmatrix} 0 \\ \frac{-a\cos(\pi/8)}{4l/3-aml\cos^2(pi/8)} \end{bmatrix}, \\
B_{2,1} &= \begin{bmatrix} 0 \\ \frac{-a}{4l/3-aml} \end{bmatrix}, \quad B_{2,2} = \begin{bmatrix} 0 \\ \frac{-a\cos(\pi/8)}{4l/3-aml\cos^2(pi/8)} \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}.
\end{aligned}
$$

Membership functions for Rules 1 and 2 are chosen to be triangular membership functions shown as Fig. 1 in this example. By solving the synthesis condition, we determine

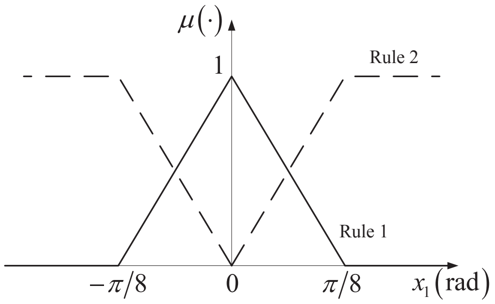

Fig. 1. Membership Functions

an optimal value $\gamma = 0.1173$ for $s = 0.1$. By applying some constraints on the LMI variables, we can obtain a relaxed performance level of $\gamma = 0.1306$. The corresponding controller matrices are also obtained using the controller construction algorithm.

$$
\Pi = \left[\begin{array}{rr|r} -51.8066 & -56.5815 & 368.7278 \\ 46.2804 & -28.9313 & 580.3190 \\ \hline 0.0001 & 0.0002 & -0.0017 \\ 0.0001 & 0.0002 & -0.0017 \\ \hline 345.0838 & 175.3478 & 1.0371 \end{array}\right]
$$

In our simulation, the initial angle is set on $\pi/8$, and the disturbance d is chosen to be a pulse force starting at $10sec$ and ending at $11sec$ with the disturbance magnitude of $25\,\mathrm{N}$. The response of the closed-loop system for $s = 0.1$ is shown in Fig. 2, while the control input is presented in Fig. 3.

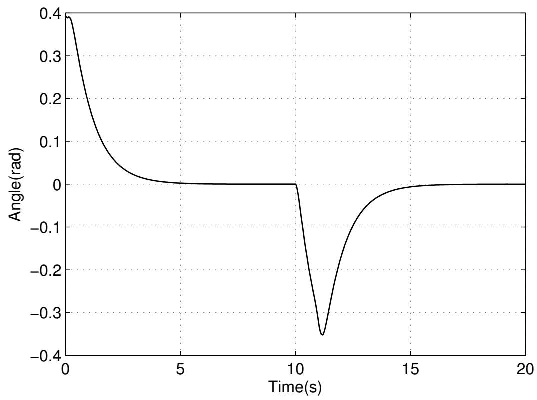

Fig. 2. Angle of the pendulum

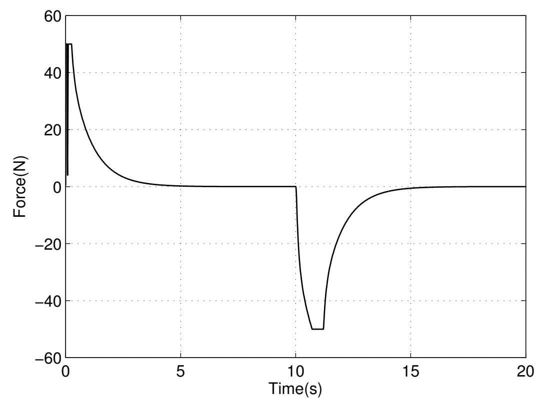

Fig. 3. Control input

It is observed from the above figures that the fuzzy dynamic output feedback controller succeeds in rejecting the disturbance as well as keeping the closed-loop system stable while the actuator has limited amplitude.

V. Conclusions

In this paper, a T-S fuzzy dynamic output feedback controller with the same antecedents as in the T-S fuzzy plants is proposed to guarantee the stability of the closed-loop T-S fuzzy system as well as providing H_∞ performance. By using the deadzone function, both the T-S fuzzy plant and the T-S fuzzy controller can be transformed into the form of linear fractional transformation. Within the framework of robust control, the problem of stability and performance is cast as a convex optimization problem which can be approached by solving a set of linear matrix inequalities. If the solution to the LMIs is feasible, a dynamic output feedback controller can be systematically constructed which could guarantee the asymptotical stability of the closed-loop system as well as the prescribed disturbance/error attenuation performance. The inverted pendulum system is used to demonstrate the effectiveness of the proposed method.

REFERENCES

[1] Y. Hao, *Fuzzy Control and Modeling: Analytical Foundations and Applications.* Wiley-IEEE Press, New York, USA. 2000.

[2] K. M. Passino, and S. Yurkovich, *Fuzzy Control.* Addison-Wesley, Inc., Menlo Park, CA. 1998.

[3] T. Takagi, and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics*, vol. SMC-15, no. 1, pp. 116-132, Feb. 1985.

[4] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 5, pp. 676-697, Oct. 2006.

[5] H. K. Lam, and F. H. F. Leung, "Stability analysis of fuzzy control systems subject to uncertain grades of membership," *IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics*, vol. 35, no. 6, pp. 1322-1325, Dec. 2005.

[6] S. K. Nguang, and P. Shi, "Robust H_∞ output feedback control design for fuzzy dynamic systems with quadratic D stability constraints: An LMI approach," *Information Sciences*, vol. 176, no. 15, pp. 2161-2191, Aug. 2006.

[7] S. Zhou, G. Feng, J. Lam, and S. Xu, "Robust H_∞ control for discrete-time fuzzy systems via basis-dependent Lyapunov functions," *Information Sciences*, vol. 174, no. 3-4, pp. 197-217, Aug. 2005.

[8] H.K. Lam, "Stability analysis of sampled-data fuzzy controller for nonlinear systems based on switching TC-S fuzzy model," *Nonlinear Analysis: Hybrid Systems*, vol. 3, no. 4, pp. 418-432, Nov. 2009.

[9] H.K. Lam, and L.D. Seneviratne, "Tracking control of sampled-data fuzzy-model-based control systems," *IET Control Theory and Applications*, vol. 3, no. 1, pp. 56-67, Jan. 2009.

[10] J. C. Lo, and Y. C. Wang, "Stabilization of fuzzy retarded systems with input and state delays," *IEEE World Congress on Computational Intelligence,* Barcelona, Spain, 2010: 2920-2925.

[11] H. Han, and T. Koshiro, "Adaptive T-S Fuzzy Controller Design using Fuzzy Approximators," *IEEE World Congress on Computational Intelligence,* Barcelona, Spain2010: 581-587.

[12] K. Tanaka, H.O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach.* John Wiley & Sons, Inc., New York, USA, 2001.

[13] T. Hu and Z. Lin, *Control Systems with Actuator Saturation: Analysis and Design.* Springer, London, UK, 2001.

[14] Z. Lin, *Low Gain Feedback.* Springer, London, UK, 1998.

[15] S. Tarbouriech and G. Garcia (eds), *Control of Uncertain Systems with Bounded Inputs.* Springer, London, UK, 1997.

[16] D.S. Bernstein and A.N. Michel, "A chronological bibliography on saturating actuators," *International Journal of Robust and Nonlinear Control*, vol. 5, no. 5, pp. 375-380, Aug. 1995.

[17] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs," *IEEE Transanctions on Fuzzy Systems,* vol. 6, no. 2, pp. 250-265, May. 1998.

[18] Y. Y. Cao, and Z. L. Lin, "Robust stability analysis and fuzzy-scheduling control for nonlinear systems subject to actuator saturation," *IEEE Transanctions on Fuzzy Systems,* vol. 11, no. 1, pp. 57-67, Feb. 2003.

[19] C. S. Tseng, and B. S. Chen, "H_∞ fuzzy control design for nonlinear systems subject to actuator saturation, " *IEEE International Conference on Fuzzy Systems,* Vancouver, CA, 2006: 783-788.

[20] H. N. Wu, and H. X. Li, "H_∞ fuzzy observer based control for a class of nonlinear distributed parameter systems with control constraints," *IEEE Transanctions on Fuzzy Systems,* vol. 16, no. 2, pp. 502-516, Apr. 2008.

[21] T. Zhang, G. Feng, H. Liu, "Piecewise fuzzy anti-windup dynamic output feedback control of nonlinear processes with amplitude and rate actuator saturations," *IEEE Transactions on Fuzzy Systems,* vol. 17, no. 2, pp. 253-264, Apr. 2009.

[22] T. Hu, A.R. Teel, and L. Zaccarian, "Stability and performance for saturated systems via quadratic and non-quadratic Lyapunov functions," *IEEE Transactions on Automatic Control,* vol. 51, no. 11, pp. 1770-1786, Nov. 2006.

[23] X. Ban and F. Wu, "Gain scheduling output feedback controller design for saturated linear plants," *Chinese Control Conference,* Hefei, China, 2012: 89-94.