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Towards A Hybrid Approach of Primitive Cognitive Network Process and Fuzzy Cognitive Map for Box Office Analysis

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***Abstract*—Box office analysis is critical to make profitable movies. Various factors have different influences on box office sales. This paper combines Primitive Cognitive Network Process (PCNP) and Fuzzy Cognitive Map (FCM) to measure and analyze the factors of box office. PCNP is a revised approach of Analytic Hierarchy Process (AHP) to quantify the weights of factors to construct a concept in FCM. FCM is used to simulate the influences of the concepts in the network. The proposed hybrid approach can enhance the evaluation and. To show the applicability of PCNP-FCM, an example of box office analysis is illustrated.**

***Keywords*—pairwise comparisons; fuzzy cognitive map; model analysis; box office analysis;**

I. INTRODUCTION

Fuzzy Cognitive Map (FCM) is an extension of Cognitive Map (CM) with fuzzy theory introduced by Kosko [1]. It is a convenient and powerful tool to simulate and analyze models with the causal relationships among distinct concepts. Many applications of FCM are presented in widespread domains including political and social sciences [2], medicine [3], engineering [4], business [5], production systems [6], environment and agriculture [7], and information technology [8]. In parallel with a wide range of applications, the significant achievements of the last decade research are observed into solving the shortcomings and improving the performance of FCM [9].

Generally, two main directions, experts-based method and computational method, are discussed for developing FCM. The experts-based method usually has three steps [10] considered:

--Identification of concepts;

--Identification of causal relationships among these concepts;

--Estimation of strength of the causal relationships.

Identification of the concept values should be a significant step at the beginning of applications of experts-based FCM.

To introduce a new alternative of experts-based FCM, this paper proposes the novel hybrid approach, PCNP-FCM, combining Primitive Cognitive Network Process [11] [12] [13] [14] and Fuzzy Cognitive Map [1] [15]. Primitive Cognitive Network Process (PCNP) is used in measuring the importance of each factor for concept and determining the initial value of concepts of FCM. PCNP addresses how to estimate the concept values in FCM.

PCNP is the basic type of Cognitive Network Process (CNP), which is an approach to rectifying the mathematical representation problem of the perception of the difference of pairwise comparisons in Analytic Hierarchy Process (AHP), which is a popular method for quantifying the subjective judgments to solve experts-based problems and has been proposed by Saaty in 1980 [16]. [17] [18] [19] demonstrated the hybrid use of AHP and FCM. Since PCNP should be an improvement of AHP, the combination of PCNP and FCM is feasible and produces more accurate results. This research proposes the PCNP to measure the initial value of experts-based FCM.

The rest of this paper is organized as follows. Section II presents the proposed details of PCNP-FCM; Section III shows a box office analysis simulation of this proposed hybrid approach; Section IV is conclusion and further study.

II. PCNP-FCM

Primitive Cognitive Network Process is used to quantify the initial concept values which serve as the input of FCM. The input data are an initial concept vector that is a set of the causal concept elements to initialize FCM. The FCM can be used to present fuzzy degrees of causality between causal concepts for analyzing the model. The proposed hybrid approach has three steps: 1) weights judgment by PCNP, 2) adjacency matrix determination in FCM, 3) equilibrium reach from iterative operations.

A. Step 1: Weights judgment by PCNP

In FCM, there are various concepts and each concept may consist of various factors. C_1, C_2, …, C_n indicate the concepts and x_1, x_2, …, x_m indicate the factors of concepts. Primitive Cognitive Network Process [11] [12] [13] [14] determinates the weights of factors which is contributed to calculate the input concepts in PCNP-FCM. The details of PCNP are as below.

To represent the paired comparison between two factors, a measurement scale schema $(\aleph, \bar{X})$ is used. $\aleph$ is a set of linguistic labels describing the paired comparison such as {Equally, Slightly, …, Extremely}. $\bar{X}$ is the numerical representation of as below.

$$\bar{X} = \left\{ \alpha_i = {}^{i\kappa}\!/_{\tau} \mid \forall i \in \{-\tau, \dots, -1, 0, 1, \dots, \tau\}, \kappa > 0 \right\} \quad (1)$$

κ (read kappa) is the normal utility, which is the mean of the individual utility values of the comparison objects, and $\kappa > 0$; by default setting, $Max(\bar{X}) = \kappa$.

Pairwise Opposite Matrix B is formed from subjective judgments of experts by paired interval scales using $(\aleph, \bar{X})$. $\tilde{B}$ is determined by B as below.

$$\begin{aligned} \tilde{B} = \left[\tilde{b}_{ij}\right] &= \begin{bmatrix} v_1 - v_1 & v_1 - v_2 & \cdots & v_1 - v_m \\ v_2 - v_1 & v_2 - v_2 & \cdots & v_2 - v_m \\ \vdots & \vdots & \ddots & \vdots \\ v_m - v_1 & v_m - v_2 & \cdots & v_m - v_m \end{bmatrix} \\ &= \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mm} \end{bmatrix} = \left[b_{ij}\right] = B \end{aligned} \quad (2)$$

v_i means the importance value of factor x_i, and $b_{ij} \cong v_i - v_j$ is the comparison value between factor x_i and x_j. If $i = j$, then $b_{ij} \cong v_i - v_j = 0$. $\left[b_{ij}\right]$ is given by an expert, and $b_{ij} \in \bar{X}$.

B is validated by the Accordance Index (AI) as below.

$$AI = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} \text{ , where}$$

$$d_{ij} = \sqrt{Mean\left(\left(\frac{1}{\kappa}\left(B_i + B_j^T - b_{ij}\right)\right)^2\right)}, \quad i, j \in (1, \dots, m) \quad (3)$$

$AI \geq 0$ and $m\kappa$ is the population utility. If $AI = 0$, then B is perfectly accordant; if $0 < AI \leq 0.1$, then B is satisfactory; if $AI > 0.1$, then B is unsatisfactory.

When B is valid, the weights of factors can be determined by the Row Average plus the normal Utility (RAU) as below.

$$v_i = \left(\frac{1}{m} \sum_{j=1}^{n} b_{ij}\right) + \kappa \quad (4)$$

In the above equation, the average of each row in B is calculated, and then the κ value is added to each average value. Finally, the individual factor weight is given.

The weights can be normalized by below function.

$$W = \left\{ w_i : w_i = \frac{v_i}{m\kappa}, \forall i \in \{1, \dots, n\} \right\}, \text{ where } \sum_{i=1}^{n} v_i = m\kappa \quad (5)$$

Apart from the weights, the expert also provides the factor scores. Each concept value is multiplication of the score and weight of each factor. The concept value will be used in step 3.

B. *Step 2: Adjacency matrix determination in FCM*

To get the description of weights in a FCM, the linguistic labels in Table I are used to describe the causalities among concepts. The linguistic labels of weights are mapped into crisp numbers in the interval [-1, 1]. Each weight is an element of an adjacency matrix E in the FCM. For the FCM of n concepts, E is an $n \times n$ matrix where the element value e_{ij} is the degree of causality from C_i to C_j presented as below.

$$E = \left(e_{ij}\right)_{1 \leq i \leq n, 1 \leq j \leq n} \quad (6)$$

TABLE I.
LINGUISTIC LABELS

Negatively Very Strong
Negatively Strong
Negatively Medium
Negatively Weak
Zero
Positively Weak
Positively Medium
Positively Strong
Positively Very Strong

To improve reliability of adjacency matrix, the judgment aggregation from a group of experts can be used in FCM. Each expert judges the relationship weights among causal concepts, and presents the results as (6). Each expert can have different weight, w_k, for his/her judgment result, E_k. To get the overall weight of each relationship in FCM, an average of judgment results from L experts is calculated as below.

$$\bar{E} = \frac{\sum_{k=1}^{L} w_k \times E_k}{\sum_{k=1}^{L} w_k} \quad (7)$$

w_k is a scalar value. The overall adjacency matrix $\bar{E}$ will be used in step 3.

C. *Step 3: Equilibrium reach from iterative operations*

The initial concept vector $C^{(0)} = \left[c_1^{(0)}, c_2^{(0)}, \dots, c_i^{(0)}, \dots, c_n^{(0)}\right]$ and the overall adjacency matrix $\bar{E}$ are used as the inputs of FCM to update the concept vector. The new concept vector $C^{(r+1)}$, in the below form [21], is updated by the sum of the previous concept vector $C^{(r)}$ and the vector-matrix dot product operation of the previous concept vector $C^{(r)}$ and the $\bar{E}$.

$$C^{(r+1)} = \left[c_1^{(r+1)} \quad c_2^{(r+1)} \quad \dots \quad c_n^{(r+1)} \right] = C^{(r)} + C^{(r)} \times \bar{E}$$
$$= \left[c_1^{(r)} \quad c_2^{(r)} \quad \dots \quad c_n^{(r)} \right] + \tag{8}$$
$$\left[c_1^{(r)} \quad c_2^{(r)} \quad \dots \quad c_n^{(r)} \right] \times \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & \cdots & e_{nn} \end{bmatrix}$$

The index r is number of iterations.

The value of the new concept vector is kept in defined boundaries by the threshold function. The FCM can use various threshold functions, such as bivalent threshold function, trivalent threshold function, sigmoid threshold function, and hyperbolic tangent threshold function. Both sigmoid function and hyperbolic tangent function can rescale the concept values in the interval [0, 1]. For a simple illustration, this paper chooses the hyperbolic tangent function as threshold function as below.

$$f(x) = \begin{cases} 0, & x \le 0; \\ \tanh\left(\dfrac{x}{2}\right), & x > 0. \end{cases} \tag{9}$$

By combining the (8) and (9), the new concept vector C at time $r+1$ is calculated by the below equation.

$$C^{(r+1)} = f\left(C^{(r)} + W^T \times C^{(r)}\right) \tag{10}$$

Explicitly,

$$c_i^{(r+1)} = f\left(c_j^{(r)} + \sum_{\substack{j=1 \\ j \ne i}}^{n} c_j^{(r)} \times e_{ji} \right) \tag{11}$$

After N iteration, FCM will reach equilibrium such that the FCM is cyclic [22], when the new concept vector is equal to the previous concept vector, i.e. $C^{(r)} = C^{(r-1)}$ such that $r=N$.

III. Box Office Analysis

This section demonstrates how the proposed PCNP-FCM approach can be applied in box office analysis. Six concepts and their relations are illustrated in the FCM, shown in Fig.1. The description is presented as below.

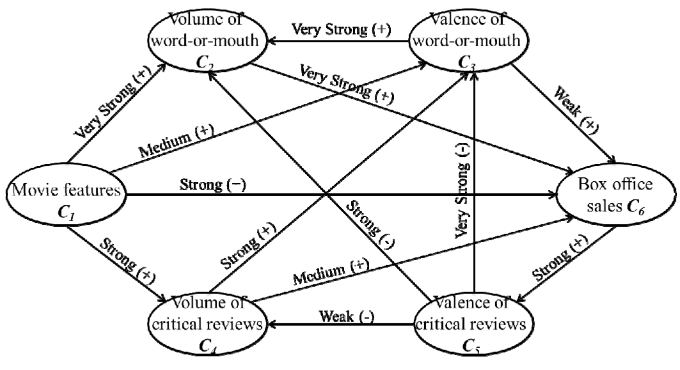

Fig. 1. The FCM model of box office sales analysis

- C_1 *Movie features*: The quality of a movie which is confirmed once the movie is made.
- C_2 *Volume of word-or-mouth*: The total amount of word-or-mouth interactions which is updated iteratively before equilibrium.
- C_3 *Valence of word-or-mouth*: The nature of word-or-mouth messages which is updated iteratively before equilibrium.
- C_4 *Volume of critical reviews*: The total amount of critical reviews which is updated iteratively before equilibrium.
- C_5 *Valence of critical reviews*: The nature of critical reviews which is updated iteratively before equilibrium.
- C_6 *Box office sales*: The total production of movie ticket is updated iteratively before equilibrium.

The concept of movie features C_1 is chosen to examine its impact to other concepts, and is determined by PCNP. Five factors of C_1 in FCM are presented in Fig. 2. Influential director x_1 or strong star power x_2 can make the high quality movie and increase the value of C_1. Motion Picture Association of America (MPAA) rating x_3 classifies movies according to the suitability of movie to different audience segments. High suitability can increase the value of C_1. Large production budget x_4 can increase the value of C_1. Fascinating storyline x_5 can increase the value of C_1.

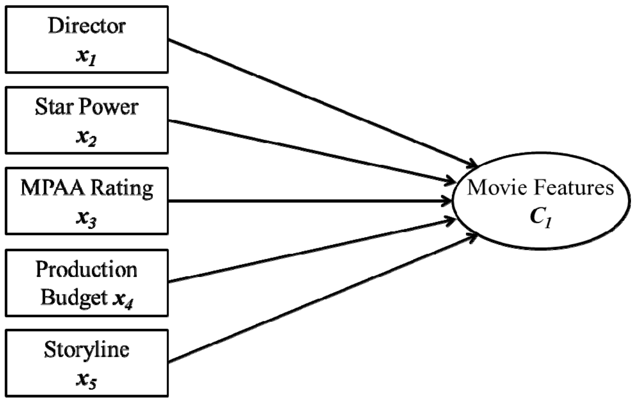

Fig. 2. Five factors of movie features

Demonstration of PCNP-FCM to this problem is shown as follows.

A. Step 1: Weights judgement by PCNP

The weights of these five factors in Fig. 2 are measured by PCNP. κ is set as 8 to determine the numerical representation by (1). The Pairwise Opposite Matrix with numerical representation is calculated by (2) and showed in Table II.

AI is calculated by (3). AI = 0.0346 means the matrix is satisfactory as AI < 0.1. After computed by (4), the weights of factors are normalized by (5) and shown in Table II.

Table III shows the direct rating scores for the five factors of movie A by an expert judgement. The range of the rating scale is [0, 1]. A high score 0.9 of storyline means that movie A has a much fascinating storyline. A low score 0.4 of MPAA rating means the less suitability of movie A to the audience.

TABLE II.
PAIRWISE OPPOSITE MATRIX FOR IMPORTANCE OF FIVE FACTORS AND THEIR WEIGHTS

Factors	Director	Star Power	MPAA Rating	Production Budget	Storyline	**Weight**
Director	0	-1	-3	-5	-7	0.12
Star Power	1	0	-2	-3	-6	0.15
MPAA Rating	3	2	0	-1	-4	0.2
Production Budget	5	3	1	0	-3	0.23
Storyline	7	6	4	3	0	0.3
AI = 0.0346						

TABLE III.
SCORES OF MOVIE A

Director	0.6
Star Power	0.8
MPAA Rating	0.4
Production Budget	0.5
Storyline	0.9

The movie features value is 0.657, which is derived from multiplication between each factor score in Table III and its corresponding weight in Table II. In FCM, it is used as an initial concept value in step 3.

B. Step 2: Adjacency matrix determination in FCM

To simplify the illustration, instead of a group of experts, one expert determines the weights among causal concepts in FCM. The overall adjacency matrix in (6) is presented as below and will be used in step 3.

$$E=\begin{Bmatrix} 0 & 1 & 0.5 & 0.75 & 0 & 0.75 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0.25 \\ 0 & 0 & 0.75 & 0 & 0 & 0.5 \\ 0 & -0.75 & -1 & -0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.75 & 0 \end{Bmatrix} \quad (12)$$

The element value e_{ij} is the degree of causality from C_i to C_j. For example, e_{12}=1 means that C_1 has a very strong positive influence in C_2.

Step 3: Equilibrium reach from iterative operations

Movie features concept C_1 is defined as an invariable before movie released and does not change after movie released. In the FCM, concept C_1 serves as input. The first element value in the initial concept vector is $C_1 = c_1^{(0)} = 0.657$ (i.e. the value of movie features C_1). The rest of the elements are set to 0. The initial concept vector is showed as below.

$$[0.657 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad (13)$$

The rest of the concepts are updated iteratively by (10). When FCM will reach equilibrium, the updated values of the rest of the concepts are the output of FCM. For the simulation for box office analysis, after 3 iterations, the final concept vector is stabilized as below.

$$[0.657 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \quad (14)$$

It can be interpreted as follows. After iteratively updating with the quite large initial value (0.657) of the movie features concept C_1, all concepts interact with each other. The concept of movie features C_1 keeps at the initial value, and the rest of concepts, i.e. volume of word-or-mouth C_2, valence of word-or-mouth C_3, volume of critical reviewers C_4, valence of critical reviewers C_5, and box office sales C_6, finally reach a maximal point (1).

IV. CONCLUSION AND FRUTHER STUDY

This paper proposes a hybrid approach of PCNP-FCM for simulating and analyzing the causal relationships in a network. To present the applicability of this hybrid approach, a box office analysis is demonstrated. The future research will investigate two main directions: exploration and further improvement of FCM and more applications.

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