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Observer-based indirect adaptive supervisory control for unknown time delay system

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Abstract--This paper proposes an indirect adaptive fuzzy neural network controller with state observer and supervisory controller for a class of uncertain nonlinear dynamic time-delay systems. The approximate function of unknown time delay system is inferred by the adaptive time delay fuzzy logic system. The supervisory controller, which can be combined with fuzzy neural network controller, will work when error dynamics is great than a constant which is determined by designer. Therefore, if the system is unstable, the supervisory controller will force the state to be stable. The free parameters of the indirect adaptive fuzzy controller can be tuned on-line by observer based output feedback control law and adaptive laws by means of Lyapunov stability criterion. The resulting of simulation example shows that the performance of nonlinear time-delay chaotic system is fully tracking the reference trajectory. Meanwhile simulation results show that the adaptive control effort of the proposed control scheme is much less due to the assist of the supervisory controller.

Keywords: Adaptive control, fuzzy neural networks (FNN), nonlinear time delay systems, observer and supervisory control.

I. Introduction

Many control systems have the problem of time delay which are infinite dimensional in nature. No matter the presence of pure time delay is in a control or/and state, it always lead to the poor performance and instability. To solve the great challenge of its stability, time delays were pay full attention in academia [1-4]. In order to handle the nonlinear time-delay systems, Lyapunov theory of stability and indirect adaptive fuzzy control have been used.

For the past few years, adaptive control for feedback linearization nonlinear systems has been discussed [5-6]. There are two kind of adaptive control techniques, including direct adaptive control (DAC) and indirect adaptive control (IAC) [7-9]. On the other hand, the fuzzy-neural control which were proven can precise model any nonlinear system due to the ubiquitous approximation theorem. For example, [10-11] shows that the fuzzy-neural network can model unknown functions in dynamic systems effectively. However the nonlinear system and an approximate fuzzy-neural network exists an error which will degenerate the stability and control performance. Besides, the state which cannot be measured is another problem for nonlinear system. Therefore, an adaptive fuzzy-neural control has been proposed to combine with the expert information systematically and the stability which is guaranteed by theoretical analysis [12-18]. Also the concept of the state observer and constructor, which was proposed by Lven Berg, Bath and Bertrand, can solve the problem of the state which cannot be measured. In [12], the unknown nonlinear dynamical systems was successfully control by indirect adaptive control-based on fuzzy-neural network with observer. Also in [13] has been proposed the concept of supervisory control which can be connected with fuzzy-neural network controller. Nevertheless the indirect adaptive control-based on fuzzy-neural network with observer and combine with supervisory control to unknown nonlinear dynamical time delay system has never been shown in any publication.

In this paper, we propose an indirect adaptive control based on fuzzy neural network controller with state observer and supervisory controller for a class of uncertain nonlinear dynamic time-delay systems. The free parameters of the adaptive fuzzy-neural controller can be tuned on-line by an observer- based output feedback control law and adaptive law. Also a supervisory is observed from time to time by a human who, when deeming it necessary, intervenes to modify the control algorithm in some way. Therefore, if the system is unstable, the supervisory controller will force the state to be stability. It is economical design methodology in respect of control efforts.

The rest of paper is organized as follows. The problem formulation is first made in section II. The description of adaptive time delay fuzzy neural networks system is in section III. Indirect adaptive control law design based on fuzzy neural network controller with observer and supervisory controller shows in section IV. Section V shows the simulation examples and the performance of results. Conclusions are given in the last section, section VI.

II. PROBLEM FORMULATION

Consider the nth-order nonlinear dynamical system with time delay of the form

$$\begin{aligned}
&\dot{x}_1 = x_2\\
&\quad\vdots\\
&\dot{x}_{n-1} = x_n\\
&\dot{x}_n = f[x, x(t-\tau_1)\cdots x(t-\tau_r)] + g[x, x(t-\tau_1)\cdots x(t-\tau_r)]u + d\\
&y = x_1\\
&x_i(t) = \Xi(t), t\in[-\varsigma, 0]
\end{aligned} \tag{1}$$

where f and g are unknown but bounded functions and $u\in R$, $y\in R$ are the control input and output of system, respectively. d is the external bounded disturbance and τ_r denotes the time delays. $\Xi(t)$ is the initial state of the system, $\varsigma = \max\{\tau | 1 \le r\}$.

Let

$$f(x,\tau) = f[x, x(t-\tau_1)\cdots x(t-\tau_r)]$$

$$g(x,\tau) = g[x, x(t-\tau_1)\cdots x(t-\tau_r)]$$

We can rewrite (1) in state-space representation

$$\dot{\underline{x}} = A\underline{x} + B[f(x,\tau) + g(x,\tau)u + d]$$

$$y = C^T \underline{x} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

and $\underline{x} = [x_1, x_2, \ldots x_n]^T = [x, \dot{x}, \ldots x^{(n-1)}]^T \in R^n$ is a state vector, but not all x are supposed to be available for measurement. Except for the system output y is assumed to be measurable. Besides, it is required that $g(x,\tau) \ne 0$ for $\underline{x}$ in a certain controllability region to make (2) be controllable.

First of all, the reference signal vector $\underline{y}_r$, the tracking error vector $\underline{e}$ and estimation error vector $\hat{\underline{e}}$ are defined as

$$\underline{y}_r = \left[y_r, \dot{y}_r, \cdots, y_r^{(n-1)}\right]^T \in R^n$$

$$\underline{e} = \underline{y}_r - \underline{x} = \left[e, \dot{e}, \cdots, e^{(n-1)}\right]^T \in R^n$$

$$\hat{\underline{e}} = \underline{y}_r - \hat{\underline{x}} = \left[\hat{e}, \dot{\hat{e}}, \cdots, \hat{e}^{(n-1)}\right]^T \in R^n$$

where $\hat{\underline{x}}$ and $\hat{\underline{e}}$ devote the estimates of $\underline{x}$ and $\underline{e}$, respectively.

If the function $f(x,\tau)$ and $g(x,\tau)$ are known and the system is without the external disturbance *d*, then control law of the certainty equivalent controller is obtained as

$$u^* = \frac{1}{g(x,\tau)}\left[-f(x,\tau) + y_r^{(n)} + \underline{k}^T \underline{e}\right] \tag{3}$$

where $\underline{k} = [k_1, k_2, \cdots, k_n]^T \in R^n$ are chosen such that all roots of the polynomial are in the open left-half plane. However $f(x,\tau)$ and $g(x,\tau)$ are unknown and not all system states $\underline{x}$ can be measured, hence we use fuzzy logic system to model original system function and design an observer to estimate the state vector in the following context.

Let the control u is the sum of $u_I(\hat{\underline{x}} | \underline{\theta})$ and $u_s(\hat{\underline{x}})$ which are used to force the state to be within the constraint

$$u = u_I(\hat{\underline{x}} | \underline{\theta}) + u_s(\hat{\underline{x}}) \tag{4}$$

where $u_I(\hat{\underline{x}} | \underline{\theta})$ is the indirect adaptive control fuzzy neural network controller with observer, and $u_s(\hat{\underline{x}})$ is the output of supervisory controller (described in section IV). By using the certainty equivalent controller (3) can be rewritten as

$$u_I(\hat{\underline{x}}) = \frac{1}{g(\hat{\underline{x}}, \tau | \underline{\theta}_g, \underline{m}_g, \underline{\sigma}_g)}\left[-f(\hat{\underline{x}}, \tau | \underline{\theta}_f, \underline{m}_f, \underline{\sigma}_f) + y_r^{(n)} + \underline{k}^T \hat{\underline{e}}\right] \tag{5}$$

By using (4) and (5) into (2) we can obtain the error dynamic equation

$$\dot{\underline{e}} = A\underline{e} - B\underline{k}^T \hat{\underline{e}} + B\left\{f(\hat{\underline{x}}, \tau | \underline{\theta}_f, \underline{m}_f, \underline{\sigma}_f) - f(\underline{x}, \tau) + \left[g(\hat{\underline{x}}, \tau | \underline{\theta}_g, \underline{m}_g, \underline{\sigma}_g) - g(\underline{x})\right]u_I(\hat{\underline{x}})\right\} - Bd - Bg(\underline{x}, \tau)u_s(\hat{\underline{x}})$$

$$e_1 = C^T \underline{e} \tag{6}$$

From (6), the following observer that estimates the state error vector $\underline{e}$ in (6) is

$$\dot{\hat{\underline{e}}} = A\hat{\underline{e}} - B\underline{k}^T \hat{\underline{e}} + L(e_1 - \hat{e}_1)$$

$$\hat{e}_1 = C^T \hat{\underline{e}} \tag{7}$$

where $L = [l_1, l_2, \cdots, l_n]^T$ is the gain vector of observer.

The observation errors are defined as

$$\tilde{\underline{e}} = \underline{e} - \hat{\underline{e}} = \hat{\underline{x}} - \underline{x}$$

$$\tilde{e}_1 = e_1 - \hat{e}_1 = \hat{x}_1 - x_1$$

Subtracting (7) form (6), we can obtain the error dynamics

$$\dot{\tilde{\underline{e}}} = \left(A - LC^T\right)\tilde{\underline{e}} + B\left\{f(\hat{\underline{x}}, \tau | \underline{\theta}_f, \underline{m}_f, \underline{\sigma}_f) - f(\underline{x}, \tau) + \left[g(\hat{\underline{x}}, \tau | \underline{\theta}_g, \underline{m}_g, \underline{\sigma}_g) - g(\underline{x}, \tau)\right]u_I(\hat{\underline{x}})\right\} - Bd - Bg(\underline{x}, \tau)u_s(\hat{\underline{x}})$$

$$\tilde{e}_1 = C^T \tilde{\underline{e}} \tag{8}$$

where

$$A - LC^T = \begin{bmatrix} -l_1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ -l_2 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -l_{n-1} & 0 & 0 & 0 & \cdots & 0 & 1 \\ -l_n & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

L is the observer gain vector which can be chosen such that the characteristic polynomial of is strictly Hurwitz (i.e., the roots of the closed-loop system are in the open left half-plane) and there exists a positive definite symmetric $n \times n$ matrix *P* which satisfies the Lyapunov equation.

$$\left(A - LC^T\right)^T P + P\left(A - LC^T\right) = -Q \tag{9}$$

where *Q* is an arbitrary positive–definite matrix.

To guarantee our state observer is working. Let us rewrite (7) as

$$\dot{\hat{\underline{e}}} = \hat{A}\hat{\underline{e}} + LC^T \tilde{\underline{e}} \tag{10}$$

where

$$\hat{A} = A - B\underline{k}^T = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ -k_1^c & -k_2^c & -k_3^c & -k_4^c & \cdots & -k_{n-1}^c & -k_n^c \end{bmatrix}$$

is a strictly Hurwitz matrix. Therefore, there exists a positive definite symmetric $n \times n$ matrix $\hat{P}$ which satisfies the Lyapunov equation.

$$\hat{A}^T \hat{P} + \hat{P}\hat{A} = -\hat{Q} \tag{11}$$

where $\hat{Q}$ is an arbitrary $n \times n$ positive–definite matrix

Let $V_{\underline{\hat{e}}} = (1/2)\underline{\hat{e}}^T \hat{P} \underline{\hat{e}}$ then by using (10) and (11) we get

$$\dot{V}_{\underline{\hat{e}}} = \frac{1}{2}\dot{\underline{\hat{e}}}^T \hat{P}\underline{\hat{e}} + \frac{1}{2}\underline{\hat{e}}^T \hat{P}\dot{\underline{\hat{e}}}$$

$$= \frac{1}{2}\left(\hat{A}\underline{\hat{e}} + LC^T\underline{\tilde{e}}\right)^T \hat{P}\underline{\hat{e}} + \frac{1}{2}\underline{\hat{e}}^T \hat{P}\left(\hat{A}\underline{\hat{e}} + LC^T\underline{\tilde{e}}\right)$$

$$= -\frac{1}{2}\underline{\hat{e}}^T \hat{Q}\underline{\hat{e}} + \underline{\hat{e}}^T \hat{P}LC^T\underline{\tilde{e}}$$

We can choose $\hat{Q}$ and L, which are determined by the designer, such that $\dot{V}_{\underline{\hat{e}}} \leq 0$

III. The Description of Adaptive Time delay FNN System

Fuzzy logic system, which was proven that has the characteristic of approaching a definition nonlinear function, was widely utilized in control theme. In this paper, we construct the adaptive time delay fuzzy logic system to approach two time delay system $f(x,\tau)$ and $g(x,\tau)$. By adjusting the parameters of weights $\underline{\theta}_f$, $\underline{\theta}_g$, center $\underline{m}_f$, $\underline{m}_g$ and the width $\underline{\sigma}_f$, $\underline{\sigma}_g$, we can obtain

$$\hat{f}(\underline{\hat{x}},\tau \mid \underline{\theta}_f, \underline{m}_f, \underline{\sigma}_f) = \underline{\theta}_f^T \underline{\xi}_f(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$$

$$\hat{g}(\underline{\hat{x}},\tau \mid \underline{\theta}_g, \underline{m}_g, \underline{\sigma}_g) = \underline{\theta}_g^T \underline{\xi}_g(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$$

where $\underline{\xi}_f(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$ and $\underline{\xi}_g(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$ are the fuzzy function $\underline{\xi}_f(\hat{x},\hat{x}(t-\tau_1)\cdots\hat{x}(t-\tau_r),\underline{m}_f,\underline{\sigma}_f)$ and $\underline{\xi}_g(\hat{x},\hat{x}(t-\tau_1)\cdots\hat{x}(t-\tau_r),\underline{m}_g,\underline{\sigma}_g)$,respectively.

then the output of the fuzzy logic systems with central average defuzzifier can be express as:

$$\sum_{j=1}^{p}\theta_j\left[\prod\nolimits_{i=1}^{n}\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,\underline{m}_j,\underline{\sigma}_j)\right] \Big/ \sum_{j=1}^{p}\left[\prod\nolimits_{i=1}^{n}\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,\underline{m}_j,\underline{\sigma}_j)\right]$$

where

$$\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,\underline{m}_j,\underline{\sigma}_j)$$
$$= \mu_{F_i^j}(\underline{\hat{x}}_i,\underline{m}_j,\underline{\sigma}_j)\mu_{F_i^j}(\underline{\hat{x}}_i(t-\tau),\underline{m}_j,\underline{\sigma}_j)...\mu_{F_i^j}(\underline{\hat{x}}_i(t-\tau_r),\underline{m}_j,\underline{\sigma}_j)$$

and $\mu_{F_i^j}(\underline{\hat{x}}_i,\underline{m}_j,\underline{\sigma}_j)$ is the membership function

The overall configuration of adaptive time delay FNN is shown in figure 1.Based on the Lyapunov approach, the adaptive laws can be developed to adjust the parameters of adaptive time delay FNN to attenuate the tracking error and external disturbance.

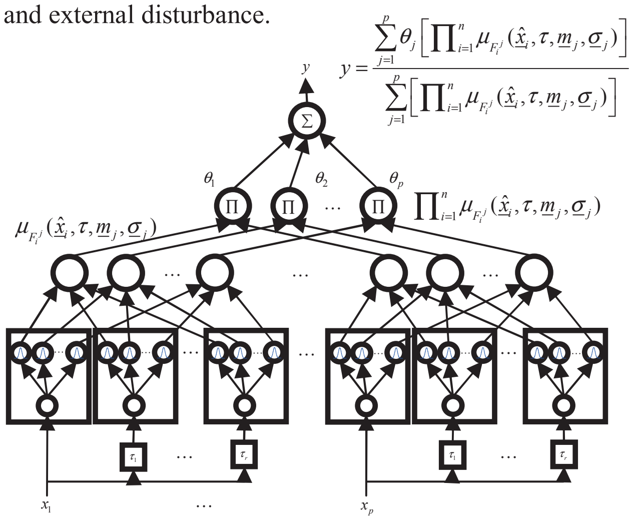

Fig. 1 The configuration of adaptive time delay FNN

IV. Indirect Adaptive Control-Based Fuzzy Neural Network Controller with Observer and Supervisory Controller

In order to design the supervisory controller, first we define $V_{\underline{\tilde{e}}} = (1/2)\underline{\tilde{e}}^T P \underline{\tilde{e}}$ and by using (9), we have $\dot{V}_{\underline{\tilde{e}}} = (1/2)\dot{\underline{\tilde{e}}}^T P \underline{\tilde{e}} + (1/2)\underline{\tilde{e}}^T P \dot{\underline{\tilde{e}}}$ then considering the error dynamics (8) we obtain.

$$\dot{V}_{\underline{\tilde{e}}} = \frac{1}{2}\Big\{\left(A - LC^T\right)\underline{\tilde{e}} + B\big\{f(\underline{\hat{x}},\tau \mid \underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f) - f(\underline{x},\tau)$$
$$+\left[g(\underline{\hat{x}},\tau \mid \underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g) - g(\underline{x},\tau)\right]u_I(\underline{\hat{x}})\big\} - Bd$$
$$-Bg(\underline{x},\tau)u_s(\underline{\hat{x}})\Big\} P\underline{\tilde{e}} + \frac{1}{2}\underline{\tilde{e}}^T P\Big\{\left(A - LC^T\right)\underline{\tilde{e}}$$
$$+B\big\{f(\underline{\hat{x}},\tau \mid \underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f) - f(\underline{x},\tau)$$
$$+\left[g(\underline{\hat{x}},\tau \mid \underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g) - g(\underline{x},\tau)\right]u_I(\underline{\hat{x}})\big\}$$
$$-Bd - Bg(\underline{x},\tau)u_s(\underline{\hat{x}})\Big\}$$
$$= -\frac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} + \underline{\tilde{e}}^T PB\big\{f(\underline{\hat{x}},\tau \mid \underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f) - f(\underline{x},\tau)$$
$$+\left[g(\underline{\hat{x}},\tau \mid \underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g) - g(\underline{x},\tau)\right]u_I(\underline{\hat{x}})\big\} - \underline{\tilde{e}}^T PBd$$
$$-\underline{\tilde{e}}^T PBg(\underline{x},\tau)u_s(\underline{\hat{x}})$$
$$\leq -\frac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} + \left|\underline{\tilde{e}}^T PB\right|\Big\{\left|f(\underline{\hat{x}},\tau \mid \underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f)\right| + \left|f(\underline{x},\tau)\right|$$
$$+\left|g(\underline{\hat{x}},\tau \mid \underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g)u_I(\underline{\hat{x}})\right| + \left|g(\underline{x},\tau)u_I(\underline{\hat{x}})\right| + |d|\Big\}$$
$$-\underline{\tilde{e}}^T PBg(\underline{x},\tau)u_s(\underline{\hat{x}}) \tag{12}$$

We need the following hypothesis, in order to design u_s such that $\dot{V}_{\underline{\tilde{e}}} \leq 0$.

Hypothesis: we can determine functions $f^U(x,\tau), g^U(x,\tau)$ and $g_L(x,\tau)$ such that

$$f^U(x,\tau) \approx f^U(\hat{x},\tau) < \infty$$
$$|f(x,\tau)| \le f^U(x,\tau) \approx f^U(\hat{x},\tau)$$
$$g_L(\hat{x},\tau) \approx g_L(x,\tau) \le g(x,\tau) \le g^U(x,\tau) \approx g^U(\hat{x},\tau)$$
$$g^U(x,\tau) \approx g^U(\hat{x},\tau) < \infty$$
$$g_L(\hat{x},\tau) \approx g_L(x,\tau) > 0$$

This is because that we can choose L in (7) to let $\underline{x} \approx \underline{\hat{x}}$. Also external disturbance is bounded, express as $|d| \le d_m$, where d_m is the upper bound of noise.

By (12) we can define

$$\underline{\tilde{e}}^T PBg(\underline{x},\tau)u_s(\underline{\hat{x}}) = |\underline{\tilde{e}}^T PB| \{ |f(\underline{\hat{x}},\tau|\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f)| + |f(\underline{x},\tau)| + |g(\underline{\hat{x}},\tau|\underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g)u_I(\underline{\hat{x}})| + |g(\underline{x},\tau)u_I(\underline{\hat{x}})| + |d| \}$$

Based on the above hypothesis the supervisory controller is chosen as

$$u_s(\underline{\hat{x}}) = \frac{|\underline{\tilde{e}}^T PB|}{\underline{\tilde{e}}^T PB} \frac{1}{g(\underline{x},\tau)} \{ |f(\underline{\hat{x}},\tau|\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f)| + |f(\underline{x},\tau)| + |g(\underline{\hat{x}},\tau|\underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g)u_I(\underline{\hat{x}})| + |g(\underline{x},\tau)u_I(\underline{\hat{x}})| + |d| \}$$

$$= \operatorname{sgn}(\underline{\tilde{e}}^T PB) \frac{1}{g_L(\underline{\hat{x}},\tau)} \{ |f(\underline{\hat{x}},\tau|\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f)| + f^U(\hat{x},\tau) + |g(\underline{\hat{x}},\tau|\underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g)u_I(\underline{\hat{x}})| + g^U(\underline{x},\tau)|u_I(\underline{\hat{x}})| + d_m \} \quad (13)$$

Then inserting (13) into (12) we have

$$\begin{aligned}\dot{V}_{\underline{\tilde{e}}} &\le -\frac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} + |\underline{\tilde{e}}^T PB| \{ |f(\underline{\hat{x}},\tau|\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f)| + |f(\underline{x},\tau)| \\ &\quad + |g(\underline{\hat{x}},\tau|\underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g)u_I(\underline{\hat{x}})| + |g(\underline{x},\tau)u_I(\underline{\hat{x}})| + |d| \} \\ &\quad - |\underline{\tilde{e}}^T PB| \frac{g(\underline{x},\tau)}{g_L(\underline{\hat{x}},\tau)} \{ |f(\underline{\hat{x}},\tau|\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f)| + f^U(\underline{x},\tau) \\ &\quad + |g(\underline{\hat{x}},\tau|\underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g)u_I(\underline{\hat{x}})| + g^U(\underline{x},\tau)|u_I(\underline{\hat{x}})| + d_m \} \\ &\le -\frac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} + |\underline{\tilde{e}}^T PB| \{ [|f(\underline{x},\tau)| + |g(\underline{x},\tau)u_I(\underline{\hat{x}})|] \\ &\quad - \frac{g(\underline{x},\tau)}{g_L(\underline{\hat{x}},\tau)} [f^U(\underline{\hat{x}},\tau) + g^U(\underline{\hat{x}},\tau)|u_I(\underline{\hat{x}})|] \} \\ &\le -\frac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} \le 0\end{aligned}$$

let

$$\begin{aligned}&\hat{f}(\underline{\hat{x}},\tau|\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f) - f(\underline{\hat{x}},\tau|\underline{\theta}_f^*,\underline{m}_f^*,\underline{\sigma}_f^*) = \\ &\underline{\tilde{\theta}}_f^T [\underline{\xi}_f(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f) - \underline{\xi}_{m_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{m}_f \\ &- \underline{\xi}_{\sigma_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\sigma}_f] \\ &+ \underline{\theta}_f^T [\underline{\xi}_{m_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\tilde{m}}_f + \underline{\xi}_{\sigma_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\tilde{\sigma}}_f] \\ &+ \underline{\tilde{\theta}}_f^T [\underline{\xi}_{m_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{m}_f^* + \underline{\xi}_{\sigma_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\sigma}_f^*]\end{aligned} \quad (14)$$

$$\begin{aligned}&\hat{g}(\underline{\hat{x}},\tau|\underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g) - g(\underline{\hat{x}},\tau|\underline{\theta}_g^*,\underline{m}_g^*,\underline{\sigma}_g^*) = \\ &\underline{\tilde{\theta}}_g^T [\underline{\xi}_g(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g) - \underline{\xi}_{m_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{m}_g \\ &- \underline{\xi}_{\sigma_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\sigma}_g] \\ &+ \underline{\theta}_g^T [\underline{\xi}_{m_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\tilde{m}}_g + \underline{\xi}_{\sigma_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\tilde{\sigma}}_g] \\ &+ \underline{\tilde{\theta}}_g^T [\underline{\xi}_{m_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{m}_g^* + \underline{\xi}_{\sigma_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\sigma}_g^*]\end{aligned} \quad (15)$$

where $\underline{\tilde{\theta}}_f = \underline{\theta}_f - \underline{\theta}_f^*$, $\underline{\tilde{m}}_f = \underline{m}_f - \underline{m}_f^*$, $\underline{\tilde{\sigma}}_f = \underline{\sigma}_f - \underline{\sigma}_f^*$

$\underline{\tilde{\theta}}_g = \underline{\theta}_g - \underline{\theta}_g^*$, $\underline{\tilde{m}}_g = \underline{m}_g - \underline{m}_g^*$, $\underline{\tilde{\sigma}}_g = \underline{\sigma}_g - \underline{\sigma}_g^*$

In addition $\underline{\xi}_{m_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$ and $\underline{\xi}_{\sigma_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$ are partial derivatives of $\underline{\xi}_f(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$ with respect to $\underline{m}_f$ and $\underline{\sigma}_f$ respectively; and so as $\underline{\xi}_{m_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$ and $\underline{\xi}_{\sigma_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$ are partial derivatives of $\underline{\xi}_g(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$ with respect to $\underline{m}_g$ and $\underline{\sigma}_g$ respectively.

In order to simplify formula ,we define the following variables

$$T_{f1}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f) = [\underline{\xi}_f(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f) - \underline{\xi}_{m_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{m}_f - \underline{\xi}_{\sigma_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\sigma}_f] \quad (16)$$

$$T_{f2}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f) = [\underline{\xi}_{m_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\tilde{m}}_f + \underline{\xi}_{\sigma_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\tilde{\sigma}}_f] \quad (17)$$

$$T_{g1}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g) = [\underline{\xi}_g(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g) - \underline{\xi}_{m_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{m}_g - \underline{\xi}_{\sigma_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\sigma}_g] \quad (18)$$

$$T_{g2}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g) = [\underline{\xi}_{m_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\tilde{m}}_g + \underline{\xi}_{\sigma_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\tilde{\sigma}}_g] \quad (19)$$

Equation (8) can be rewritten as

$$\begin{aligned}\dot{\underline{\tilde{e}}} &= (A - LC^T)\underline{\tilde{e}} + B\{ f(\underline{\hat{x}},\tau|\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f) - f(\underline{\hat{x}},\tau|\underline{\theta}_f^*,\underline{m}_f^*,\underline{\sigma}_f^*) \\ &+ f(\underline{\hat{x}},\tau|\underline{\theta}_f^*,\underline{m}_f^*,\underline{\sigma}_f^*) - f(\underline{x},\tau) + [g(\underline{\hat{x}},\tau|\underline{\theta}_g^*,\underline{m}_g^*,\underline{\sigma}_g^*) \\ &- g(\underline{\hat{x}},\tau|\underline{\theta}_g^*,\underline{m}_g^*,\underline{\sigma}_g^*) + g(\underline{\hat{x}},\tau|\underline{\theta}_g^*,\underline{m}_g^*,\underline{\sigma}_g^*) - g(\underline{x},\tau)] u_I(\underline{\hat{x}}) \} \\ &- Bd - Bg(\underline{x},\tau)u_s(\underline{\hat{x}})\end{aligned} \quad (20)$$

The minimum approximation error is defined as

$$\begin{aligned}\omega &= \underline{\tilde{\theta}}_f^T [\underline{\xi}_{m_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{m}_f^* + \underline{\xi}_{\sigma_f}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\sigma}_f^*] \\ &+ [f(\underline{\hat{x}},\tau|\underline{\theta}_f^*,\underline{m}_f^*,\underline{\sigma}_f^*) - f(\underline{x},\tau)] + \{ \underline{\tilde{\theta}}_g^T [\underline{\xi}_{m_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{m}_g^* \\ &+ \underline{\xi}_{\sigma_g}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\sigma}_g^*] + [g(\underline{\hat{x}},\tau|\underline{\theta}_g^*,\underline{m}_g^*,\underline{\sigma}_g^*) - g(\underline{x},\tau)] u_I(\underline{\hat{x}}) - d\end{aligned} \quad (21)$$

By using (14)-(19) and (21) into (20) the error dynamics (20) can be expressed as

$$\begin{aligned}\dot{\underline{\tilde{e}}} &= (A - LC^T)\underline{\tilde{e}} + B\{ \underline{\tilde{\theta}}_f^T T_{f1}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f) \\ &+ \underline{\theta}_f^T T_{f2}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f) + [\underline{\tilde{\theta}}_g^T T_{g1}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g) \} \\ &+ \underline{\theta}_g^T T_{g2}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)] u_I(\underline{\hat{x}}) \} + B\omega - Bg(\underline{x},\tau)u_s(\underline{\hat{x}})\end{aligned} \quad (22)$$

The Lyapunov function candidate is defined as

$$V=\frac{1}{2}\underline{\tilde{e}}^T P\underline{\tilde{e}}+\frac{1}{2r_{f1}}\underline{\tilde{\theta}}_f^T\underline{\tilde{\theta}}_f+\frac{1}{2r_{g1}}\underline{\tilde{\theta}}_g^T\underline{\tilde{\theta}}_g+\frac{1}{2r_{f2}}\underline{\tilde{m}}_f^T\underline{\tilde{m}}_f$$
$$+\frac{1}{2r_{g2}}\underline{\tilde{m}}_g^T\underline{\tilde{m}}_g+\frac{1}{2r_{f3}}\underline{\tilde{\sigma}}_f^T\underline{\tilde{\sigma}}_f+\frac{1}{2r_{g3}}\underline{\tilde{\sigma}}_g^T\underline{\tilde{\sigma}}_g+\frac{1}{2}\sum_{i=1}^{r}\int_{t-\tau_i}^{t}e^T(v)e(v)dv \quad (23)$$

Differentiating (23) with respect to time along the trajectory (22) we obtain

$$\dot{V}=\frac{1}{2}\dot{\underline{\tilde{e}}}^T P\underline{\tilde{e}}+\frac{1}{2}\underline{\tilde{e}}^T P\dot{\underline{\tilde{e}}}+\frac{1}{r_{f1}}\underline{\tilde{\theta}}_f^T\dot{\underline{\tilde{\theta}}}_f+\frac{1}{r_{g1}}\underline{\tilde{\theta}}_g^T\dot{\underline{\tilde{\theta}}}_g+\frac{1}{r_{f2}}\underline{\tilde{m}}_f^T\dot{\underline{\tilde{m}}}_f$$
$$+\frac{1}{r_{g2}}\underline{\tilde{m}}_g^T\dot{\underline{\tilde{m}}}_g+\frac{1}{r_{f3}}\underline{\tilde{\sigma}}_f^T\dot{\underline{\tilde{\sigma}}}_f+\frac{1}{r_{g3}}\underline{\tilde{\sigma}}_g^T\dot{\underline{\tilde{\sigma}}}_g+\frac{1}{2}\sum_{i=1}^{r}e^T(t)e(t)$$
$$-\frac{1}{2}\sum_{i=1}^{r}e^T(t-\tau_i)e(t-\tau_i) \quad (24)$$

Substituting (22) into (24) $\dot{V}$ can rewritten as

$$\dot{V}=\frac{1}{2}\Big\{\left(A-LC^T\right)\underline{\tilde{e}}+B\Big\{\underline{\tilde{\theta}}_f^T T_{f1}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$$
$$+\underline{\theta}_f^T T_{f2}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)+\Big[\underline{\tilde{\theta}}_g^T T_{g1}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$$
$$+\underline{\theta}_g^T T_{g2}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\Big]u_I(\underline{\hat{x}})\Big\}+B\omega-Bg(\underline{x},\tau)u_s(\underline{\hat{x}})\Big\}^T P\underline{\tilde{e}}$$
$$+\frac{1}{2}\underline{\tilde{e}}^T P\Big\{\left(A-LC^T\right)\underline{\tilde{e}}+B\Big\{\underline{\tilde{\theta}}_f^T T_{f1}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$$
$$+\underline{\theta}_f^T T_{f2}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)+\Big[\underline{\tilde{\theta}}_g^T T_{g1}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$$
$$+\underline{\theta}_g^T T_{g2}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\Big]u_I(\underline{\hat{x}})\Big\}+B\omega-Bg(\underline{x},\tau)u_s(\underline{\hat{x}})\Big\}$$
$$+\frac{1}{r_{f1}}\underline{\tilde{\theta}}_f^T\dot{\underline{\tilde{\theta}}}_f+\frac{1}{r_{g1}}\underline{\tilde{\theta}}_g^T\dot{\underline{\tilde{\theta}}}_g+\frac{1}{r_{f2}}\underline{\tilde{m}}_f^T\dot{\underline{\tilde{m}}}_f+\frac{1}{r_{g2}}\underline{\tilde{m}}_g^T\dot{\underline{\tilde{m}}}_g+\frac{1}{r_{f3}}\underline{\tilde{\sigma}}_f^T\dot{\underline{\tilde{\sigma}}}_f$$
$$+\frac{1}{r_{g3}}\underline{\tilde{\sigma}}_g^T\dot{\underline{\tilde{\sigma}}}_g+\frac{1}{2}\sum_{i=1}^{r}e^T(t)e(t)-\frac{1}{2}\sum_{i=1}^{r}e^T(t-\tau_i)e(t-\tau_i)$$
$$\le\frac{1}{2}\underline{\tilde{e}}^T(Q-rI)\underline{\tilde{e}}$$
$$+\underline{\tilde{\theta}}_f^T\left[T_{f1}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\left(B^TP\underline{\tilde{e}}\right)+\frac{1}{r_{f1}}\dot{\underline{\tilde{\theta}}}_f\right]$$
$$+\underline{\tilde{\theta}}_g^T\left[T_{g1}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\left(B^TP\underline{\tilde{e}}\right)u_I(\underline{\hat{x}})+\frac{1}{r_{g1}}\dot{\underline{\tilde{\theta}}}_g\right]$$
$$+\underline{\tilde{m}}_f^T\left[\underline{\xi}_{m_f}^T(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\theta}_f\left(B^TP\underline{\tilde{e}}\right)+\frac{1}{r_{f2}}\dot{\underline{\tilde{m}}}_f\right]$$
$$+\underline{\tilde{m}}_g^T\left[\underline{\xi}_{m_g}^T(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\theta}_g\left(B^TP\underline{\tilde{e}}\right)u_I(\underline{\hat{x}})+\frac{1}{r_{g2}}\dot{\underline{\tilde{m}}}_g\right]$$
$$+\underline{\tilde{\sigma}}_f^T\left[\underline{\xi}_{\sigma_f}^T(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\theta}_f\left(B^TP\underline{\tilde{e}}\right)+\frac{1}{r_{f3}}\dot{\underline{\tilde{\sigma}}}_f\right]$$
$$+\underline{\tilde{\sigma}}_g^T\left[\underline{\xi}_{\sigma_g}^T(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\theta}_g\left(B^TP\underline{\tilde{e}}\right)u_I(\underline{\hat{x}})+\frac{1}{r_{g3}}\dot{\underline{\tilde{\sigma}}}_g\right]$$
$$+\underline{\tilde{e}}^T PB\omega-\underline{\tilde{e}}^T PBg(\underline{x},\tau)u_s(\underline{\hat{x}}) \quad (25)$$

where $\dot{\underline{\tilde{\theta}}}_f=\dot{\underline{\theta}}_f$, $\dot{\underline{\tilde{\theta}}}_g=\dot{\underline{\theta}}_g$, $\dot{\underline{\tilde{m}}}_f=\dot{\underline{m}}_f$, $\dot{\underline{\tilde{m}}}_g=\dot{\underline{m}}_g$, $\dot{\underline{\tilde{\sigma}}}_f=\dot{\underline{\sigma}}_f$ and $\dot{\underline{\tilde{\sigma}}}_g=\dot{\underline{\sigma}}_g$.

If we choose the adaptive law as

$$\dot{\underline{\theta}}_f=-r_{f1}T_{f1}(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\left(B^TP\underline{\tilde{e}}\right) \quad (26)$$

$$\dot{\underline{\theta}}_g=-r_{g1}T_{g1}(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\left(B^TP\underline{\tilde{e}}\right)u_I(\underline{\hat{x}}) \quad (27)$$

$$\dot{\underline{m}}_f=-r_{f2}\underline{\xi}_{m_f}^T(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\theta}_f\left(B^TP\underline{\tilde{e}}\right) \quad (28)$$

$$\dot{\underline{m}}_g=-r_{g2}\underline{\xi}_{m_g}^T(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\theta}_g\left(B^TP\underline{\tilde{e}}\right)u_I(\underline{\hat{x}}) \quad (29)$$

$$\dot{\underline{\sigma}}_f=-r_{f3}\underline{\xi}_{\sigma_f}^T(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)\underline{\theta}_f\left(B^TP\underline{\tilde{e}}\right) \quad (30)$$

$$\dot{\underline{\sigma}}_g=-r_{g3}\underline{\xi}_{\sigma_g}^T(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)\underline{\theta}_g\left(B^TP\underline{\tilde{e}}\right)u_I(\underline{\hat{x}}) \quad (31)$$

Substituting adaptive laws (26)~(31) into (25) we have

$$\dot{V}=-\frac{1}{2}\underline{\tilde{e}}^T(Q-rI)\underline{\tilde{e}}+\underline{\tilde{e}}^T PB\omega-\underline{\tilde{e}}^T PBg(\underline{x},\tau)u_s(\underline{\hat{x}}) \quad (32)$$

Because $\underline{\tilde{e}}^T PBg(\underline{x},\tau)u_s(\underline{\hat{x}})\ge 0$, and ω is very small, we can choose r such that $\dot{V}\le 0$ is satisfied. Therefore, the tracking performance can be achieved.

Summarizing the above analysis, the procedure of observer-based indirect adaptive fuzzy control with supervisory control can be presented as follow steps.

Step 1) Specify the feedback and observer gain vector K and L, such that the characteristic matrices $A-LC^T$ and $A-B\underline{k}^T$ are strictly Hurwitz matrices, respectively.

Step 2) Specify a positive definite $n\times n$ matrix Q and solve the Lyapunov equation (9) to obtain a positive definite $n\times n$ symmetric matrix P.

Step 3) Solve the state error (7) to obtain estimate state vector $\underline{\hat{x}}$.

Step 4) Specify a positive definite $n\times n$ matrix $\underline{\hat{Q}}$ and solve the Lyapunov equation (11) to obtain a positive definite $n\times n$ symmetric matrix P.

Step 5) Define the membership function $\mu_{F_i^j}(\underline{\hat{x}}_i,\underline{m}_j,\underline{\sigma}_j)$ for $i=1,2,...,p$ and compute the fuzzy basis functions $\underline{\xi}_f(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$ and $\underline{\xi}_g(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$. Then the output of fuzzy logic system are constructed as $\hat{f}(\underline{\hat{x}},\tau\,|\,\underline{\theta}_f,\underline{m}_f,\underline{\sigma}_f)=\underline{\theta}_f^T\underline{\xi}_f(\underline{\hat{x}},\tau,\underline{m}_f,\underline{\sigma}_f)$ and $\hat{g}(\underline{\hat{x}},\tau\,|\,\underline{\theta}_g,\underline{m}_g,\underline{\sigma}_g)=\underline{\theta}_g^T\underline{\xi}_g(\underline{\hat{x}},\tau,\underline{m}_g,\underline{\sigma}_g)$.

Step 6) Obtain the control from equation (5) and apply to plant, then compute the adaptive law (26)~(31) to adjust the parameters of weights $\underline{\theta}_f$, $\underline{\theta}_g$, center $\underline{m}_f$, $\underline{m}_g$ and the width $\underline{\sigma}_f$, $\underline{\sigma}_g$.

V. Simulation Example

In this section, we will apply our observer indirect adaptive fuzzy neural network controller for a nonlinear time delay system.

The dynamic equations of nonlinear time delay system is given by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = 1.8x_1 - 0.1x_2 - x_1^3 + 0.02x_1(t-0.001) + 0.04x_1^2(t-0.001) + 0.06x_2(t-0.001) - 1.1\cos(0.4t) + u(t) + d(t)$$

Also the external disturbance is assumed to be a sine wave with amplitude ± 0.5, period 2π, and step size $h = 0.001$.

According to the design procedure, the design is given as the following steps.

Step 1) The observer and feedback gain vectors are chosen as $L = [89\ 184]$, and $K = [1\ 2]$, respectively.

Step 2) We select Q in (9) as $\begin{bmatrix} 10 & 13 \\ 13 & 28 \end{bmatrix}$, then after solving (9), the positive definite symmetric 2×2 matrix P in (9) is $\begin{bmatrix} 29 & -14 \\ -14 & 7 \end{bmatrix}$.

Step 3) Solve (7) to obtain $\underline{\hat{x}}$.

Step 4) we select $\hat{\underline{Q}}$ in (11) as $\begin{bmatrix} 40 & 25 \\ 25 & 30 \end{bmatrix}$ and $\hat{A} = \begin{bmatrix} 0 & -1 \\ -4 & -4 \end{bmatrix}$ in (11). Therefore the positive definite symmetric 2×2 matrix $\hat{P}$ in (11) is $\begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$.

Step 5) The following membership functions for $\hat{x}_i$ $i = 1,2$ are selected as:

$$\mu F_i^1 = \exp(-(\frac{\hat{x}_i + 1.5}{0.15})^2)$$

$$\mu F_i^2 = \exp(-(\frac{\hat{x}_i + 0.25}{0.15})^2)$$

$$\mu F_i^3 = \exp(-(\frac{\hat{x}_i}{0.15})^2)$$

$$\mu F_i^4 = \exp(-(\frac{\hat{x}_i - 0.25}{0.15})^2)$$

$$\mu F_i^5 = \exp(-(\frac{\hat{x}_i - 1.5}{0.15})^2)$$

To cover whole cases, we apply 25 fuzzy rules, the initial values of $\underline{x}(0)$ and $\underline{\hat{x}}(0)$ are given as $[0.2\ 0.2]^T$ and $[-0.15\ 0]^T$, respectively. Hence u_i is constructed.

Step 6) Compute the adaptive law (26) to (31).

The trajectories of the states x_1 and $\hat{x}_1$ are shown in Fig. 2 and Fig. 3 shows that the estimation state $\hat{x}_1$ takes very short time to catch up the system state x_1.

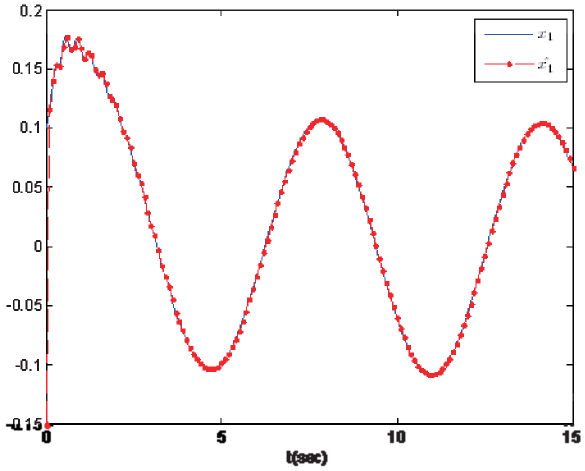

Fig. 2 The trajectories of the states x_1 and $\hat{x}_1$

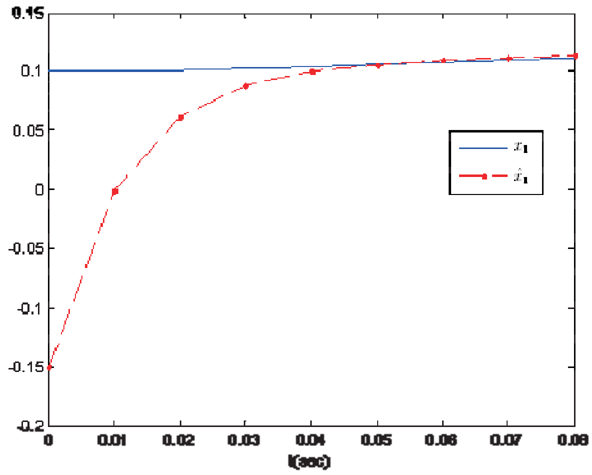

Fig. 3 The trajectories of the states x_1 and $\hat{x}_1$ t=0-0.08s

The tracking performance also is very good as shown in Fig. 4,in which y_r is the reference output and y is the system output trajectories. Fig. 5 shows the reference output $\dot{y}_r$ and the system output trajectories $\dot{y}$.

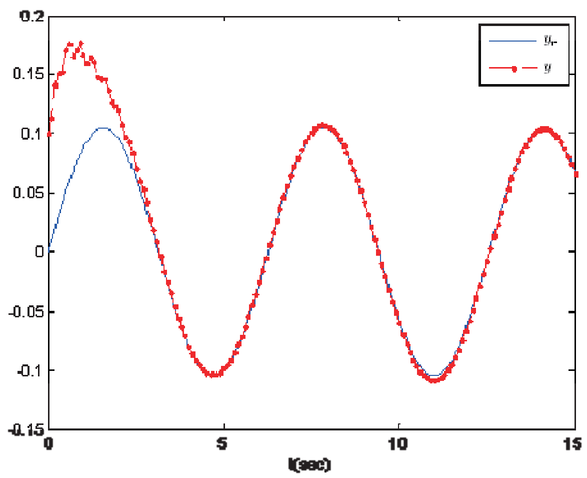

Fig. 4 The reference output y_r and output trajectories y

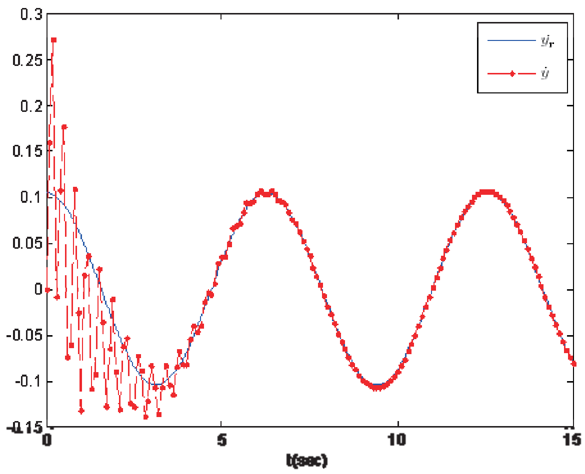

Fig. 5 The reference output $\dot{y}_r$ and output trajectories $\dot{y}$

The all control input is shown in Fig. 6. Fig. 7 shows the control input u_I only. Fig. 8 shows the supervisory control u_s and we can see that the supervisory control only works in the beginning period. Soon after, the FNN controller can stabilize the system and the supervisory control will be deactivated.

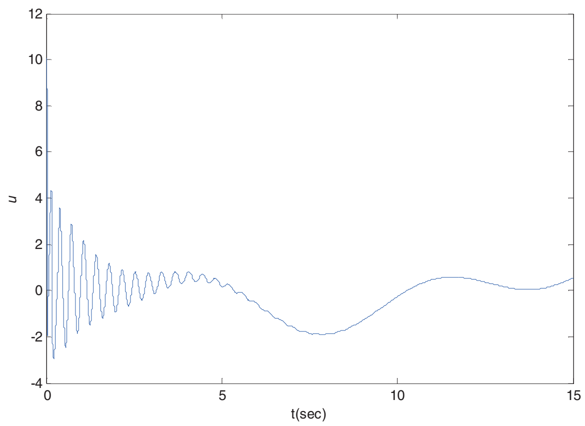

Fig. 6 The trajectories of control input($u_I + u_s$)

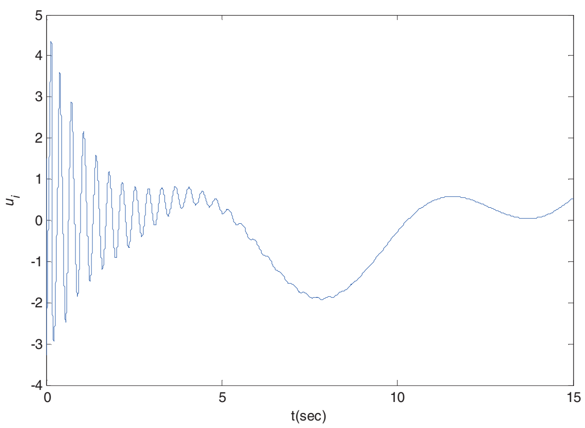

Fig. 7 The trajectories of control input(u_I only)

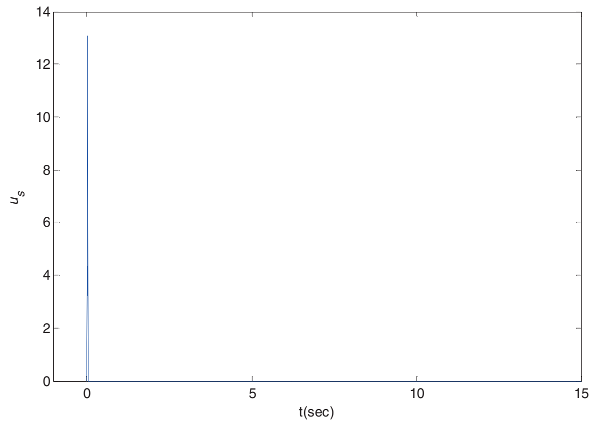

Fig. 8 The trajectories of the supervisory control u_s

Fig. 9 shows the graph of $\dot{V}(t)$. Owing to the maximum value of $\dot{V}(t)$ is -9.9992e-013, i.e. it always be negative value. Consequently the stability is confirmed.

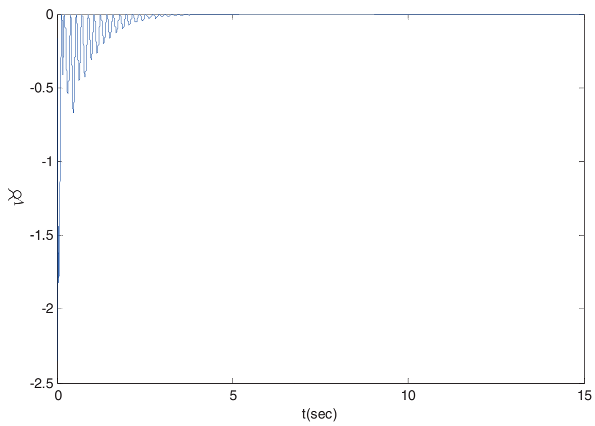

Fig. 9 The graph of $\dot{V}(t)$

VI. Conclusions

Due to the fact that IAC has the advantage of combining with different forms of controllers and parameter observer, we have proposed a method for designing a IAC-based FNN controller with observer and supervisory controller for a certain class of unknown nonlinear time delay systems, in which only the system output can be measured. Simulation results confirm that the supervisory controller will be activated as long as the system tends to be unstable controlled by FNN controller. In other words, if the FNN controller works well, the supervisory controller will be deactivated. Meanwhile, the simulation results not only show the adaptive control effort of the proposed control scheme is much less due to the assist of the supervisory controller, but also show how the supervisory control force the state to be within the constraint set and how the adaptive FNN controller learned to regain control.

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