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Fuzzy group decision-making based on variable weighted averaging operators

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***Abstract*—By merging the linguistic quantifier and Borda function, we propose a kind of new method to construct the state variable weight vectors with reward and the variable weight state vector, establish the variable weight synthesis decision making model which grasp the meaning of the linguistic value, and investigate several group decision making models derived by some typical linguistic quantifiers and the OWA operators. Finally, we use one numerical example to illustrate our model reasonable, make the comparison between OWA operator and variable weighted averaging operator, and point out that the quantified guided OWA operator is a pessimistic intendancy decision model, and quantifier guided VWA operator is a optimistic intendancy decision model.**

I. Introduction

Group decision making(GDM) is an important topic in decision making field$[1, 2, 3, 4, 5, 9, 18]$. In group decision making, the key issue is how to aggregate the individual opinions. Based on the complexity of the real world and the difference of the expert's experience, knowledge and information resource, each expert would have different understanding for the decision making problem. Hence, these evaluations proposed by different experts may be different or even have a large difference. Obviously, it is difficult to give a complete consistence for every expert. The more utilizable strategy is to consider "most experts' opinions" or "more than 80% of experts' opinions". Thus, the linguistic words "most" and "more than 80%" transit some meaningful fuzzy information. How to reflect the semantics of the linguistic quantifiers is a crucial issue while we establish the mathematical model of group decision making. Aimed at that all experts have the same level, Kacprzyk$[10, 11]$ proposed a method of synthesizing the experts' evaluations by viewing linguistic quantifier as a fuzzy set. This method has been applied many times by assuming that all experts present their evaluations by utilizing fuzzy preference relations. Herrera et al.$[6, 7, 8]$ aggregated the experts' evaluations with linguistic quantifier guided OWA operator and the semantics of the linguistic value is reflected by the weights of the OWA operator obtained from the linguistic quantifier.

Considering that different experts have different evaluations, we usually allocate the weight to every expert and obtained a weighted average expression. It is called as the traditional weighted average operator or weighted average synthesis. Here, the expert's weight is constant in the decision process, we call it as the constant weight and the weighted average expression as the constant weighted average synthesis. Wang$[17]$ pointed out that the constant weighted decision making model presented any limitations and used an example to illustrate that the constant weighted average synthesis is not appropriate.

Example 1: Suppose the decision to approve an engineering project is dependent on two factors: f_1="feasibility" and f_2="necessity". If the two factors are equality important, then we should assign equal weights to each factor, i.e. $W = (w_1, w_2) = (0.5, 0.5)$. Hence, the weighted average synthesis expression is $M(x_1, x_2) = 0.5x_1 + 0.5x_2$.

Consider two cases: (1) the project is entirely feasible, but its necessity is quite low; and (2) the project may be highly necessity, but it is not feasible. Normally we do not approve the project in either case because the "merit" of the combined effect is quite low. Numerically, let $X = (0.1, 0.9)$ and $X' = (0.5, 0.5)$, then we would expect $M(X) << M(X')$ in more cases. However, using the constant weighted synthesis expression, we have $M(X) = M(X') = 0.5$, this result contradicts common expectations.

Namely, these weights should be adjusted automatically according to the varying of the experts' evaluations. For example, in Olympic games of gymnastics and diving, the athlete's final score is the summing of the remaining referees' marks after canceling the two highest scores and the two lowest scores. It is the same as varying the weights of the referees who give the two highest or two lowest scores to zeros, and only synthesizing the scores of the most referees. Just like linguistic quantifier guided OWA operator, we can view the linguistic quantifier as a fuzzy set and utilize the membership function of the linguistic quantifier to adjust the experts' weight. The adjusted expert weights can reflect the semantics of the linguistic quantifier. The above method is a good idea for establishing fuzzy group decision model. Obviously, the key step of this method is how to finish the weight adjustment. Li et al.$[13]$ proposed a method for adjusting weights utilizing analytic method. Li and Yen$[12]$ viewed decision making as a synthesis process under the frame of factor space, and regard every attribute value as a state of the factor space. All attribute values form a vector which is called a state configuration level of the factor space. Then, under the frame of factor space, each attribute weight

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should be affected by the state configuration level and varied with the changing of the state configuration level. The law of the weights varying is characterized by variable weight state vector. Therefore, we can finish the weights transferring via constructing appropriate variable weight state vector. Variable weight synthesis approach is widely used in the fields of multiple attribute decision making, fuzzy inference, and so on. For example, Zhang and Li[25] applied the variable weighted synthesis in fuzzy inference and constructed some implication operators. In this paper, we use Borda function and linguistic quantifier, propose a kind of new method to construct the state variable weight vectors with reward and the variable weight state vector, establish the variable weight synthesis decision making model and investigate several group decision making models derived by some typical linguistic quantifiers and the OWA operators. Finally, we use one numerical example to illustrate our model reasonable, make the comparison between OWA operator and variable weighted averaging operator.

The organization of our work is as follows. In section 2, some basic notions are reviewed. In section 3, we constructed a kind of new variable weight state vector with reward and established a variable weight group decision making model guided by linguistic quantifier. In section 4, we used a numerical example to illustrate our proposed method reasonable. The conclusion is given in the last section.

II. Preliminaries

Throughout this paper, we use $X = (x_1, x_2, \cdots, x_n)$ to denote the state value vector and satisfy $x_1 \geq x_2 \geq \cdots \geq x_n$, $W = (w_1, w_2, \cdots, w_n)$ stands for the constant weight vector. $\mathbf{R}$ and $\mathbf{R}^n$ are the set of all real numbers and $n-$dimension real numbers, respectively, $B(X) = B(x_1, x_2, \cdots, x_n)$ expresses balance function, and $W^B(X) = (w_1(X), w_2(X), \cdots, w_n(X)) = (w_1(x_1, x_2, \cdots, x_n), w_2(x_1, x_2, \cdots, x_n), \cdots, w_n(x_1, x_2, \cdots, x_n))$ stands for the variable weight vector generated by the balance function $B(X)$ and the constant weight vector W.

Definition 1[12]*:* A mapping $W = (w_1, w_2, \cdots, w_n)$ from $[0,1]^n$ to $[0,1]^n$,

$$\begin{gathered} w_j : [0,1]^n \to [0,1] \\ (x_1, x_2, \cdots, x_n) \mapsto w_j(x_1, x_2, \cdots, x_n) \\ j = 1, 2, \cdots, n \end{gathered}$$

is a variable weight vector with reward, if W satisfies the following properties:

(w.1) $\sum_{j=1}^{n} w_j(x_1, x_2, \cdots, x_n) = 1$;

(w.2) The function $w_j(x_1, x_2, \cdots, x_n)$ is continuous with respect to every variable $x_i, i, j = 1, 2, \cdots, n$;

(w.3) The function $w_j(x_1, x_2, \cdots, x_n)$ is monotonically increasing with respect to the variable $x_j, j = 1, 2, \cdots, n$.

Thus, $W(X) = (w_1(X), w_2(X), \cdots, w_n(X)) = (w_1(x_1, x_2, \cdots, x_n), w_2(x_1, x_2, \cdots, x_n), \cdots, w_n(x_1, x_2, \cdots, x_n))$ is called the variable weight vector with reward.

Definition 2: A mapping $M_n:$ from $[0,1]^n$ to $[0,1]$,

$$\begin{aligned} (x_1, x_2, \cdots, x_n) &\mapsto M_n(x_1, x_2, \cdots, x_n) \\ &= \sum_{i=1}^{n} w_i(x_1, x_2, \cdots, x_n) x_i \end{aligned}$$

is called the variable weight average. For simplicity, we denote it as VWA.

Definition 3[12]*:* A mapping $S = (S_1, S_2, \cdots, S_n)$ from $[0,1]^n$ to $[0,1]^n$,

$$\begin{gathered} S_j : [0,1]^n \to [0,1] \\ (x_1, x_2, \cdots, x_n) \mapsto S_j(x_1, x_2, \cdots, x_n) \\ j = 1, 2, \cdots, n \end{gathered}$$

is a state variable weight vector with penalty, if S satisfies the following properties:

(s.1) Every $S_j(x_1, x_2, \cdots, x_n)$ is continuous with respect to every variable $x_i, i, j = 1, 2, \cdots, n$;

(s.2) $x_i \geq x_j \Rightarrow S_i(x_1, x_2, \cdots, x_n) \leq S_j(x_1, x_2, \cdots, x_n)$;

(s.3) The mapping $W : [0,1]^n \to [0,1]^n$ given by

$$W(x_1, x_2, \cdots, x_n) = \frac{W \cdot S(x_1, x_2, \cdots, x_n)}{\sum_{j=1}^{n} w_j S_j(x_1, x_2, \cdots, x_n)} \tag{1}$$

is a variable weight vector with penalty, where $W = (w_1, w_2, \cdots, w_n)$ is a constant weight vector, and $W \cdot S(x_1, x_2, \cdots, x_n) = (w_1 S_1(x_1, x_2, \cdots, x_n), w_2 S_2(x_1, x_2, \cdots, x_n), \cdots, w_n S_n(x_1, x_2, \cdots, x_n))$.

The mapping S is a state variable weight vector with reward if it satisfies (s.1) and the following two properties:

(s.2') $x_i \geq x_j \Rightarrow S_i(x_1, x_2, \cdots, x_n) \geq S_j(x_1, x_2, \cdots, x_n)$;

(s.3') The mapping $W : [0,1]^n \to [0,1]^n$ given by

$$W(x_1, x_2, \cdots, x_n) = \frac{W \cdot S(x_1, x_2, \cdots, x_n)}{\sum_{j=1}^{n} w_j S_j(x_1, x_2, \cdots, x_n)} \tag{2}$$

is a variable weight vector with reward, where $W = (w_1, w_2, \cdots, w_n)$ is a constant weight vector, and $W \cdot S(x_1, x_2, \cdots, x_n) = (w_1 S_1(x_1, x_2, \cdots, x_n), w_2 S_2(x_1, x_2, \cdots, x_n), \cdots, w_n S_n(x_1, x_2, \cdots, x_n))$.

Theorem 1: Let $S(X) = (S_1(X), S_2(X), \cdots, S_n(X))$ be the variable weight state vector with reward and $W = (w_1, w_2, \cdots, w_n)$ be the constant weight vector. If $W(X) = (w_1(X), w_2(X), \cdots, w_n(X))$ is the variable weight vector obtained from Eq. (1), and $x_i > x_j$, then $w_i(X)/w_i \geq w_j(X)/w_j$.

Theorem 2: Assume that $X = (x_1, x_2, \cdots, x_n)$ is a state vector and $W = (w_1, w_2, \cdots, w_n)$ is a constant weight vector. Let $S(X) = (S_1(X), S_2(X), \cdots, S_n(X))$ be the variable weight state vector with reward and $W(X) = (w_1(X), w_2(X), \cdots, w_n(X))$ be a variable weight vector obtained from Eq. (1). If $x_1 \geq x_2 \geq \cdots \geq x_n$, then

$w_i(X)/w_i \geq 1$ if and only if $\sum_{j=1}^{n} w_j(S_i(X) - S_j(X)) \leq \sum_{j=1}^{i-1} w_j(S_j(X) - S_i(X))$.

Proof: Known by the definition of the variable weight state vector with reward, we have,

$$\begin{aligned}\frac{w_i(X)}{w_i} \geq 1 \quad & \Leftrightarrow S_i(X) \geq \sum_{j=1}^{n} w_j S_j(X) \\ & \Leftrightarrow \sum_{j=1}^{n} w_j S_i(X) \geq \sum_{j=1}^{n} w_j S_j(X) \\ & \Leftrightarrow \sum_{j=1}^{n} w_j (S_i(X) - S_j(X)) \\ & \leq \sum_{j=1}^{n} w_j (S_j(X) - S_i(X))\end{aligned}$$

Since Yager[20] introduced OWA operator, some researchers have successfully applied it in decision making and support systems. For example, Yager[22] investigated the families of OWA operators, Xu[19] investigated some methods for determining OWA weights.

Definition 4[20]*:* An ordered weighted averaging(OWA) operator of $n-$dimension is a mapping $f:\ [0,1]^n \rightarrow [0,1]$ that has an associated $n-$vector $W = (w_1, w_2, \cdots, w_n)$ such that $w_i \in [0,\ 1]$ and $\sum_{i=1}^{n} w_i = 1$, where $f(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} w_i y_i$, and y_i is the ith largest of the $x_j, j = 1, 2, \cdots, n$.

Remark: Just like in OWA operator, the weight associated to each attribute value is changed discontinuously, thus we can omit the condition of w.2) and only reserve the conditions of w.1) and w.3) in definition 1 in real decision making.

In order to extend the two valued logic to linguistic fuzzy logic, Zadeh[24] introduced the concept of linguistic quantifier and distinguished them with absolute quantifiers such as "much more than 2", and relative or proportional quantifiers such as "most", "about half". Some researchers[15, 16, 23] introduced the linguistic value and linguistic information in group decision making. Absolute quantifier is a fuzzy set Q of $[0, +\infty)$, for any $r \in [0, \infty)$, the membership degree $Q(r)$ expresses the degree to which r satisfies the concept conveyed by the linguistic quantifier Q. Relative quantifier is a fuzzy set Q of the unit interval $[0,\ 1]$, for any $r \in [0,\ 1]$, the membership degree $Q(r)$ indicates the degree to which r matches the semantic of the linguistic quantifier Q. Relative quantifier Q satisfies the condition such as $Q(0) = 0$ and there exists $r_0 \in [0,\ 1], Q(r_0) = 1$.

Furthermore, in order to obtain the OWA weight vectors, Yager[21] introduced the regular increasing monotone(RIM) quantifier.

Definition 5[21]*:* A fuzzy subset Q of $[0,\ 1]$ is called regular increasing monotone(RIM) quantifier if the fuzzy set Q satisfies $Q(0) = 0, Q(1) = 1$, and $Q(x) \geq Q(y)$ if $x > y$.

Let Q be a RIM quantifier, then the OWA weight vector can be obtained by Rager[18] as follows.

$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n}), \quad i = 1, 2, \cdots, n \tag{3}$$

It is interesting that Liu and Lou[14] investigated the equivalence of some approaches to the OWA operator and RIM quantifier determination.

III. Fuzzy group decision making model based on linguistic quantifier and variable weight

A. QG-VWA group decision making model

For a group decision making problem, let A be an alternative to be evaluated, $e_1, e_2, \cdots, e_n$ be the experts participating in the decision making, $w_1, w_2, \cdots, w_n$ be weights of the experts, and x_i be the evaluation given by experts $e_i, i = 1, 2, \cdots, n$. Decision maker should synthesize the individual judgment value into a group's evaluation by applying an appropriate decision making model. Considering the complication of the real world, decision maker usually gives some feasible conditions to satisfy with the alternative in the synthesizing process such as "most experts", "80% of the expert's opinions", and so on. Apparently, the terms "most" and "about 80%" transfer some vague information. How to grasp the semantics of the terms is a key issue to be solved for this kind of group decision making[1]. In the following, we establish a fuzzy group decision making model based on linguistic quantifier and variable weight theory. Via the weights transferring among the experts, this decision model can grasp the semantics of the linguistic term fully.

At first, we reorder the expert's evaluations $x_1, x_2, \cdots, x_n$ in descending order and get a new vector $Y = (y_1, y_2, \cdots, y_n)$, where y_i is the $i-$th largest element of $x_i, i = 1, 2, \cdots, n$. Later, by applying the Borda function which is usually used in group decision making[19], we define an order value for every element y_i as $b_i = n+1-i, i = 1, 2, \cdots, n$. If experts give the same evaluation, without loss of generality, assume that they are the elements $y_k, y_{k+1}, \cdots, y_{k+1-m}$ of vector Y, then we let $b_i = \frac{1}{m}\sum_{i=k}^{k+m-1}(n-i+1), i = k, k+1, \cdots, k+m-1$. Suppose we wish to consider Q experts' evaluations in the synthesis process, where Q is a RIM linguistic quantifier, and its membership function is $Q(x)$. Let $z_j = 1 - Q(1 - \frac{k_{k_j}}{n})$, where k_j is the index value of x_j after reordering the experts' judgment value vector $X = (x_1, x_2, \cdots, x_n)$ with descendent order, then $z_j = 1-Q(1-\frac{k_{k_j}}{n})$. We call $Z_X^Q = (z_1, z_2, \cdots, z_n)$ quantifier guided order index vector of X. Finally, we construct a vector $s(X) = (s_1(X), s_2(X), \cdots, s_n(X))$, where variable weight state $s_i(X) = z_i^\alpha, \alpha > 0, i = 1, 2, \cdots, n$. Particularly, we order $o^\alpha = 0$. Then, we have the following conclusion.

Theorem 3: Let $X = (x_1, x_2, \cdots, x_n)$ and Q be a RIM linguistic quantifier, $Z_X = (z_1, z_2, \cdots, z_n)$ is a Q guided order index vector of X, then $s^Q(X) = (z_1^\alpha, z_2^\alpha, \cdots, z_n^\alpha), \alpha > 0$ is a variable weight state vector with reward.

Proof: We need only to prove that if $x_i > x_j$ then $s_i(X) \geq s_j(X)$. And since $s_i(X) = z_i^\alpha, s_j(X) = z_j^\alpha$, thus, we need only to prove $z_i \geq z_j$. Therefore, known by the condition $x_i > x_j$, we have $b_i > b_j$, hence we have $Q(1-\frac{b_i}{n}) \leq Q(1-\frac{b_j}{n})$, so we have $z_i \geq z_j$.

We complete the proof of Theorem 3.

For group decision making, with the expert's evaluation vector X and variable weight state vector $S(X)$, we can obtain variable weights $w_i^Q(X), i = 1, 2, \cdots, n$ corresponding to the experts by Eq.1. Furthermore we can establish the following group decision making model based on variable weight

$$M_n(X) = \sum_{i=1}^{n} w_i^Q(X) x_i \ . \tag{4}$$

We call formula Eq. 2 as Quantifier Guided Variable Weight Average(QG-VWA).

B. Comparison between QG-VWA and QG-OWA

In the following, we analyze the variable weights obtained from the variable weight state vector with reward which are constructed by several typical linguistic quantifiers. Simultaneously, the results of QG-VWA and QG-OWA are compared.

(1) Linguistic quantifiers "all" and "there exists"

For convenience, we denote the quantifiers "all" and "there exists" by Q_* and Q^*, respectively, they are represented by the fuzzy subsets in the following .

$$Q_*(x) = \begin{cases} 1, & x = 1 \\ 0, & x \neq 1 \end{cases}$$

$$Q^*(x) = \begin{cases} 0, & x = 0 \\ 1, & x \neq 0 \end{cases}$$

For Q_*, we have $\frac{b_i}{n} \neq 0$ for any i, then $z_i = 1$. Hence $s_i(X) = 1, i = 1, 2, \cdots, n$. Known by Eq. (1), we have $w_i^{Q_*}(X) = w_i, i = 1, 2, \cdots, n$. It indicates that the QG-VWA in this situation is degenerated to the popular weighted average. For QG-OWA operator, we have $Q_*\text{G-OWA}(X) = \min_i x_i$. The above results show that QG-OWA operator is a more pessimistic decision model, and QG-VWA is a more optimistic decision model.

(2) Linguistic quantifier Q_β"at least $100 \times \beta\%, \beta \in [0,\ 1]$"

The quantifier "at least $100 \times \beta\%$" is represented by the fuzzy subset in the following.

$$Q_\beta(x) = \begin{cases} 1, & x \geq \beta \\ 0, & x < \beta \end{cases}$$

If $1 - \frac{b_i}{n} \geq \beta$, namely, $i \geq 1 + n\beta$, then $z_i = 0$. Otherwise, if $1 - \frac{b_i}{n} < \beta$ or $i < 1 + n\beta$, then $z_i = 1$. It indicates that the weights are distributed again among the experts whose evaluations are the 1st, 2nd, $\cdots$, $[1 + n\beta]$-th. Known by Theorem 1, the larger the expert's evaluation, the greater the varying ratio of the expert's weight. Furthermore, we give some analysis on the influence of β on the variable weights. If β is near with 1 enough, then, for any i, we have $z_i = 1$. Hence, we have that the QG-VWA reduces to the popular weighted average. It's the same with the QG-VWA derived from quantifier Q_*. On the other hand, from the viewpoint of semantics, while β closes with 1 enough, the semantics of Q_β is similar with the meaning of "all", namely, the semantics of Q_*. From the two different viewpoints, we get the same result. Similarly, if β closes with 0 enough, let i_0 be an integer, such that $x_{i_0} = \max_i x_i$. Then $z_{i_0} = 1$ and $z_i = 0$ for any $i \neq i_0$. It implies that the Q_βG-VWA is max operator when $\beta \to 0$. Namely, $\lim_{\beta \to 0} Q_\beta\text{G-VWA}(X) = \max_i x_i$. On the other hand, from the viewpoint of semantics of linguistic, while $\beta \to 0$, the semantic of Q_β is similar with the meaning of "there exists", namely, the semantic of Q^*. The two results are the same as. The above results give some illustrations of the rationality of the decision model (4).

For OWA operator derived from Q_β, if $n\beta \leq j < 1 + n\beta$, then we have $w_j = 1$, else $w_j = 0$. It indicates that $Q_\beta\text{G-OWA}(X) = x_j$, where $n\beta \leq j < 1 + n\beta$. Hence, this operator describes the semantic of "at least $100 \times \beta, \beta \in [0,\ 1]$ percent of" by only one expert's evaluation. From this view of point, we can also see that QG-OWA operator is a more pessimistic decision model, and QG-VWA is a more optimistic decision model.

IV. Numerical Example

Suppose we need to select an alternative from four alternatives A, B, C, D. Five experts, e_1, e_2, e_3, e_4, e_5, take part in the decision making. The experts' judgments are listed in Table 1 as follow.

TABLE I. The five expert's judgements

	e_1	e_2	e_3	e_4	e_5
A	0.85	0.76	0.8	0.85	0.9
B	0.78	0.83	0.82	0.9	0.85
C	0.9	0.85	0.78	0.83	0.74
D	0.85	0.82	0.73	0.88	0.82

Assume that the weights of the five experts are $0.3, 0.15, 0.2, 0.2, 0.15$, respectively, and we aggregate the evaluations of the experts with the strategy of considering "the most of the evaluations". The fuzzy set "most" is defined as[7]:

$$Q(x) = \begin{cases} 0, & 0 \leq x \leq 0.3 \\ 2(x - 0.3), & 0.3 < x \leq 0.8 \\ 1, & 0.8 < x \leq 1 \end{cases}$$

In the following, we introduce the method of obtaining the decision value by the decision model presented in this paper. We only give process of obtaining the evaluation of A, the others are similar.

Firstly, the expert's judgment value vector is $X = (0.85, 0.76, 0.80, 0.85, 0.90)$. With the descending result of X, we have $b_1 = 3.5, b_2 = 1, b_3 = 2, b_4 = 3.5, b_5 = 5$. Hence, we obtain the quantifier guided order index vector of X as $Z_X^Q = (1, 0, 0.4, 1, 1)$. Assume that the parameter $\alpha = 0.5$ in the variable weight state vector, then we construct the following variable weight state vector $s(X) = (z_1^{0.5}, z_2^{0.5}, \cdots, z_5^{0.5})$. By variable weight Eq.(1), we get the variable weight vector $w_A(X) = (0.3864, 0, 0.1629, 0.2576, 0.1932)$.

Similarly, we have

$$w_B(X) = (0, 0.2197, 0.2071, 0.2368, 0.2456),$$

$$w_C(X) = (0.3972, 0.1986, 0.1675, 0.2368, 0),$$

$$w_D(X) = (0.4096, 0.1586, 0, 0.2731, 0.1586).$$

Finally, by decision making model of QG-VWA, we have $\mathrm{VM}_A = 0.8515, \mathrm{VM}_B = 0.7741, \mathrm{VM}_C = 0.8534, \mathrm{VM}_D = 0.8487$. Therefore, the final ranking is $C \succ A \succ D \succ B$, where "$\succ$" denotes "better than or dominate".

If we adopt the constant weighted averaging operator, then we have $\mathrm{M}_A = 0.8340, \mathrm{M}_B = 0.8300, \mathrm{M}_C = 0.8305, \mathrm{M}_D = 0.8230$. The final ranking of the alternatives is $A \succ C \succ B \succ D$.

If we adopt the QG-OWA, then the corresponding weight vector is $W_{OWA} = (0, 0.2, 0.4, 0.4, 0)$. Furthermore, we get four synthetical evaluations of the four alternatives, $\mathrm{OWAM}_A = 0.83, \mathrm{OWAM}_B = 0.83, \mathrm{OWAM}_C = 0.814, \mathrm{OWAM}_D = 0.826$, and the final ranking of the alternatives is $A \succ B \succ D \succ C$.

Thus, we obtain three different ranking results by applying three decision making models. When the constant weighted averaging operator is adopted, the linguistic quantifier doesn't give any effect in the synthesizing process. It presents the rank of $A \succ C \succ B \succ D$. However, if we consider the effect of the linguistic quantifier and adopt the QG-VWA, since this is a more optimistic decision model, the experts who give larger judgments will make greater influence on the decision, then the ranking is changed as $C \succ A \succ D \succ B$. For example, the ranking result of $A \succ C$ by using constant weighed averaging operator is changed as $C \succ A$. This is because "most" experts' judgment values of C are larger than the corresponding experts' judgment values of A. On the contrary, if we adopt QG-OWA operator, since it is a pessimistic decision model, the maximum element is rejected off the synthesis.This leads to different result with the two previous results. For instance, the result of $C \succ D$ is changed as $D \succ C$. It is because we only consider three judgment values 0.85, 0.78, 0.83, given by e_2, e_3 and e_4 respectively, when we investigate alternative C, and utilize the judgment values 0.85, 0.82, 0.82, given by experts e_1, e_2, e_5 respectively, when we investigate alternative D.

V. Conclusions

Since variable weight synthesis considers both the factor weights and the configuration of factor states, it shows more scientific than the constant weighted synthesis in many real problems. In practice, it makes that the variable weight synthesis model can better meet different decision requirements to adopt different strategies of weight varying according to different real problems. In this paper, we merge linguistic quantifier and Borda function, propose a kind of new method to construct the state variable weight vectors with reward and the variable weight state vector, establish the variable weight synthesis decision making model and investigate several group decision making models derived by some typical linguistic quantifiers and the OWA operators. Finally, we use one numerical example to illustrate our model reasonable, make the comparison between OWA operator and variable weighted averaging operator.

Of course, balance function is an important concept in the theory of variable weight decision making. It can help us to scientifically grasp the action mechanism of the balance function for applying the variable weight synthesis. In this paper, we develop the research from analyzing the mechanism of the state variable weight vector to the mechanism of the balance function. We not only discuss the balance on the configuration of the state values, but also analyze the affection on the decision attitude of the decision maker including the weights transferring and the synthesis decision value. These results can help us to propose some principles for selecting balance function in decision making procedure.

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References

[1] S. Alonso, F.J. Cabrerizo, F. Chiclana, F. Herrera and E. Herrera-Viedma, "Group decision making with incomplete fuzzy linguistic preference relations," *International Journal of Intelligent Systems*, vol. 24, pp. 201-222, 2009.

[2] F. Chiclana, E. Herrera-Viedma, F. Herrera and S. Alonso, "Some induced ordered weighted averaging operators and their use for solving group decision-making problems based on fuzzy preference relations," *European Journal of Operational Research*, vol. 182, pp. 383-399, 2007.

[3] F. Chiclana, J.M. Tapia Garcia, M.J. Del Moral and E. Herrera-Viedma, "A statistical comparative study of different similarity measures of consensus in group decision making," *Information Sciences*, vol. 221, pp. 110-123, 2013.

[4] S.J. Chuu, "Interactive group decision making using a fuzzy linguistic approach for evaluating the flexibility in a supply chain," *European Journal of Operational Research*, vol. 213, pp. 279-289, 2011.

[5] C. Fu and S.L. Yang, "An attributive weight based on feedback model for multiple attributive group decision analysis problem with group consensus requirement in evidential reasoning context," *European Journal of Operational Research*, vol. 212, pp. 179-189, 2011.

[6] F. Herrera and E. Herrera-Viedma, "Aggregation operators for linguistic weighted information," *IEEE Transactions on Systems Man and Cybernetics*, vol. 22, pp. 646-656, 1997.

[7] F. Herrera, E. Herrera-Viedma and J.L. Verdegay, "Direct approach processes in group decision making using linguistic OWA operators," *Fuzzy Sets and Systems*, vol. 79, pp. 175-190, 1996.

[8] E. Herrera-Viedma, F. Chiclana, F. Herrera and S. Alonso, "Group decision-making model with incomplete fuzzy preference relations based on additive consistency," *IEEE Transactions on Systems Man and Cybernetics*, vol. 37, pp. 176-189, 2007.

[9] Y.S. Huang, W.C. Chang, W.H. Li and Z.L. Lin, "Aggregation of utility-based indiviual preferences for group decision making," *European Journal of Operational Research*, vol. 229, pp. 462-469, 2013.

[10] J. Kacprzyk, "Group decision making with a fuzzy linguistic majority," *Fuzzy Sets and Systems*, vol. 18, pp. 105-118, 1986.

[11] J. Kacprzyk, M. Fedrizzi and H. Nurmi, "Group decision making and consensus under fuzzy preferences and fuzzy majority," *Fuzzy Sets and Systems*, vol. 49, pp. 21-31, 1992.

[12] H.X. Li and V.C. Yen, *Fuzzy Sets and Fuzzy Decision-Making.* CRC Press, Boca Raton, Florida, 1995.

[13] H.X. Li, L.X. Li, J.Y. Wang, Z.W. Mo and Y.D. Li, "Fuzzy decision making based on variable weights," *Mathematical and Computer Modelling,* vol. 39, pp. 163-179, 2004.

[14] X.W. Liu and H.W. Lou, “On the equivalence of some approaches to the OWA operator and RIM quantifier determination,” *Fuzzy Sets and Systems,* vol. 159, pp. 1673-1688, 2007.

[15] D. Meng and Z. Pei, “On weighted unbalanced linguistic aggregation operators in group decision making,” *Information Sciences*, vol. 223, pp. 31-41, 2013.

[16] I.J. Perez, F.J. Cabrerizo and E. Herrera-Viedma, “Group decision making problem in a linguistic and dynamic contex,” *Expert Systems with Applications,* vol. 38, pp. 1675-1688, 2011.

[17] P.Z. Wang, *Fuzzy Sets and The Falling Shadow of Random Sets.* Beijing Normal University Press, Beijing, 1985. (in Chinese)

[18] D.V. Winterfeldt and W. Edwards, *Decision Analysis and Behavioral Research.* Cambridge University Press, Cambridge, 1986.

[19] Z.S. Xu, “An overview of metnods for determining OWA weights,” *International Journal of Intelligent Systems,* vol. 20, pp. 843-865, 2005.

[20] R.R. Yager, “On ordered weighted averaging aggregation operators in multiciteria decision making,” *IEEE Transactions on Systems Man and Cybernetics,* vol. 18, pp. 183-190, 1988.

[21] R.R. Yager, “Quantifier guided aggregation using OWA operators,” *International Journal of Intelligent Systems,* vol. 11, pp. 49-73, 1996.

[22] R.R. Yager, “Families of OWA operators,” *Fuzzy Sets and Systems,* vol. 59, pp. 125-148, 1993.

[23] R.R. Yager, “Multi-agent negotiation using linguistically expressed mediation rules,” *Group Decision and Negotiation,* vol. 16, pp. 1-23, 2007.

[24] L.A. Zadeh, “A computational approach to fuzzy quantifiers in natural languages,” *Computers and Mathematics with Applications,* vol. 9, pp. 149-184, 1983.

[25] Y.Z. Zhang and H.X. Li, “Variable weighted synthesis inference method for fuzzy reasoning and fuzzy systems,” *Computers and Mathematics with Applications,* vol. 52, pp. 305-322, 2006.