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A Novel Relaxed Stabilization Condition for A class of T-S Time-Delay Fuzzy Systems

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***Abstract*—In this paper, the relaxed delay-dependent stabilization problem for a class of Takagi and Sugeno (T-S) fuzzy time-delay systems is explored. By utilizing homogeneous polynomials scheme, Pólya's theorem and some slack matrices, a novel relaxed stabilization condition for a class of T-S fuzzy time-delay systems is proposed in terms of a linear matrix inequalities (LMIs). Finally, an example is given to demonstrate that the proposed stabilization condition can provide a longer allowable delay time than some existing studies.**

I. Introduction

It is known that Takagi and Sugeno (T-S) fuzzy model based control scheme provides a simple and effective design procedure for the stability/stabilization analysis and the controller design of nonlinear systems. T-S fuzzy model consists of a family of local linear-type models connected through nonlinear fuzzy membership functions. For the controller design, the well known parallel distributed compensation (PDC) scheme [1], [2] is adopted to stabilize the overall T-S fuzzy model. By examining the stability and stabilization problems of T-S fuzzy models, the Lyapunov theorem [3], [4] is mainly adopted to yield the stability/stabilization conditions of T-S fuzzy system. In addition, in T-S fuzzy model-based control, many stability/stabilization conditions can be obtained by utilizing linear matrix inequalities (LMIs). Since T-S fuzzy model-based control provides these systematic design procedures, the stability and stabilization problem for T-S fuzzy model has been explored in many studies [5], [6].

In general, systems with time-delay impose difficulties and restrictions on the design of a stabilizing controller. For this reason, the stabilization problems for systems with time-delay become an important topic. Therefore, there are many studies which provides different methods to obtain the stabilization conditions. Generally speaking, the physical systems with time-delay are more complicated than that of systems without time-delays. During the two decades, fuzzy systems with time-delay were investigated in [7]–[12]. In [13] and [14], the Lyapunov-Krasovskii approach and the Lyapunov-Razumikhin functional method are adopted to explore the stability of a class of fuzzy delay systems. [15] proposed a novel result for fuzzy H_∞ design of fuzzy time-delay system. In [16], the authors explored the global exponential fuzzy observer for time-delay fuzzy bilinear systems with disturbances. [17] constructs a novel Lyapunov function to obtain an improved stability criterion. In addition to time-delay stabilization problems of fuzzy systems, the relaxed stabilization conditions for fuzzy time-delay systems are explored in many studies. For example, [18] propounded a partition approach for system with constant delay-time. [19] utilizes the input-output approach to obtain the less conservative stabilization condition. In [20], the Gu discretization technique and strategies to extend the slack matrix variables and a less conservative matrix is obtained.

Following the introduction, the paper is organized as follows. In Section II, some definitions and the stabilization problem for the T-S time-delay fuzzy systems are introduced. By applying the homogeneous polynomials scheme, augmented matrices and Pólya's theorem theorem, the stabilization conditions for the T-S time-delay fuzzy systems can be represented in terms of LMIs form in Section III. In Section IV, a numerical example is provided to demonstrate that the propounded stabilization condition is less conservative than some studies and the proposed control scheme is effective and validity. Finally, in Section V, conclusions are drawn.

The symbol * means transposed elements in LMI that can be obtained by transpose operations which are symbolized by ^{T}. ! denotes factorial for combinatoric expression. Let $K(h)$ be the set of r-tuple defined as below:

$$K(h) = \{\, (\, k_1\, k_2 \cdots\, k_r\,) :\, k_1 + k_2 + \cdots + k_r = h,$$

$$\forall k_i \in I^{+}(\, positive\; integers\,),\, i = 1, 2, \cdots, r\, \},$$

where h is the total number of polynomial degree in μ_i, $i = 1, 2, \cdots, r$. Since the number of rules in fuzzy is r, the number of elements in $K(h)$ is expressed by $J(h) = (r + h - 1)!/\,(h!(r - 1)!)$. For example, $r = 2$

$$\begin{aligned} K(3) &= \{\, (30),\, (21),\, (12),\, (03)\, \} \\ &= \{\, t(1),\, t(2),\, t(3),\, t(4)\, \} \\ &= \{\, (\mu_1^3\mu_2^0),\, (\mu_1^2\mu_2^1),\, (\mu_1^1\mu_2^2),\, (\mu_1^0\mu_2^3)\, \}. \end{aligned}$$

To ease the presentation, we adopt the notations displayed below:

$$k = k_1 k_2 \cdots k_r$$
$$\mu^k = \mu_1^{k_1}\mu_2^{k_2}\cdots\mu_r^{k_r}$$

$$e_i = 0 \cdots \underbrace{1}_{i^{th}} \cdots 0$$
$$k - e_i = k_1 k_2 \cdots (k_i - 1) \cdots k_r$$
$$\pi(k) = (k_1!)(k_2!) \cdots (k_r!)$$
$$C_{ii}^k(h) = \frac{(h!) k_i (k_i - 1)}{\pi(k)}, C_{ij}^k(h) = \frac{(h!) k_i k_j}{\pi(k)}.$$

II. PRELIMINARIES

The fuzzy model proposed by Takagi-Sugeno is display by fuzzy IF-THEN rules. To begin, consider the following ith rule of T-S fuzzy time-delay system.

Rule i :
IF $\xi_1(t)$ is $M_1(t)$ and $\cdots$ and $\xi_p(t)$ is $M_{ip}(t)$
THEN

$$\dot{x}(t) = A_i x(t) + A_{di} x(t - h(t)) + B_i u(t) \tag{1}$$
$$x(t) = \phi(t), t \in [-max\{h_M\}, 0]$$

where $\xi_1(t)$, $\xi_2(t)$, $\cdots \xi_p(t)$ are premise variables, M_{ij}, $i = 1, 2, \cdots, r$; $j = 1, 2, \cdots,$ r are fuzzy sets, r is fuzzy rules number. $x(t) \in R^n$ is state, $u(t) \in R^n$ is input, and the delay time satisfies $0 \leq h(t) \leq h_M$, $0 \leq \dot{h}(t) \leq h_D$. The matrices A_i, $A_{di} \in R^{n\times n}$ and $B_i \in R^{n\times s}$ and initial vector $\phi(t)$ belongs to the set of continuous functions.

The overall T-S fuzzy time-delay system can be expressed as (2),

$$\begin{aligned}\dot{x}(t) &= \frac{\sum_{i=1}^{r} \beta_i(\xi(t))[A_i x(t) + A_{di} x(t - h(t)) + B_i u(t)]}{\sum_{i=1}^{r} \beta_i(\xi(t))} \\ &= \sum_{i=1}^{r} \mu_i(\xi(t))[A_i x(t) + A_{di} x(t - h(t)) + B_i u(t)] \\ &= A(\mu)x(t) + A_d(\mu)x(t - (h(t)) + B(\mu)u(t)) \end{aligned} \tag{2}$$

where $\xi(t) = [\xi_1(t), \cdots, \xi_n(t)]$, and $M_{ij}(\xi(t))$ is the membership degree of $\xi(t)$, $\beta_i(\xi(t)) = \prod_{i=1}^{p} M_{ij}(\xi(t))$, $\mu_i = \beta_i(\xi(t))/\sum_{i=1} \beta_i(\xi(t))$, $\sum_{i=1}^{r} \mu_i(\xi(t))A_i = A_(\mu)$, $\sum_{i=1}^{r} \mu_i(\xi(t))A_{di} = A_d(\mu)$, and $\sum_{i=1}^{r} \mu_i(\xi(t))B_i = B(\mu)$. Basic properties of $\beta_i(\xi(t))$ are $\beta_i(\xi(t)) \geq 0$ and $\sum_{i=1} \beta_i(\xi(t)) > 0$. It is clear that $\sum_{i=1}^{r} \mu_i(\xi(t)) \geq 0$, and $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$.

The fuzzy controller for the T-S fuzzy time-delay systems (1) can be formulated as follows:

$$\begin{aligned} u(t) &= \sum_{k\in K(1)} \mu^k F_k x(t) \\ &= F(\mu)x(t) \end{aligned} \tag{3}$$

By substituting the (3) into (2), the overall closed-loop fuzzy system can be obtained as

$$\dot{x}(t) = (A(\mu) + B(\mu)F(\mu))x(t) + A_d(\mu)x(t - h(t)) \tag{4}$$

In next section, the relaxed stabilization of (4) will be discussed.

III. MAIN RESULTS

Before proceeding with the following theorem, we firstly give the following results, which will be used in the proof of the theorem.

Lemma 1: [11] For any constant matrices Q_{11}, Q_{22}, $Q_{12} \in R$, $Q_{11} \geq 0$, $Q_{22} \geq 0$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0$, scalar $\tau(t) \leq \bar{\tau}$, and vector function $\dot{x} : [-\bar{\tau}, 0] \to R^n$ such that the following integration is well defined, then

$$-\tau \int_{t-\bar{\tau}}^{t} \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds$$
$$\leq \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ \int_{t-\bar{\tau}}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ Q_{22} & -Q_{22} & Q_{12}^T \\ -Q_{12} & Q_{12} & -Q_{11} \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ \int_{t-\bar{\tau}}^{t} x(s)ds \end{bmatrix}. \tag{5}$$

Lemma 2: [21] A system is quadratically stabilizable if and only if there exist a symmetric positive definite matrix Q, Z_k, $k \in K(1)$ and a sufficiently large $d \in N$ such that

$$R^k = \sum_{k' \in K(d)} \sum_{i=1}^{N} \frac{d!}{\pi(k')} \Big(\frac{(k_i - k_i')}{\pi(k_i - k')} (A_i Q + *) + (B_i Z_{k-k'-e_i} + *) \Big) < 0$$

where, $k \in K(d+2)$ and controller gain $F(\hat{\mu})$ = $Z(\hat{\mu})Q^{-1}$, $Z_k \in R^{n\times m}$, $Z(\hat{\mu}) = \sum_{k\in K(1)} Z_k \mu^k$.

Lemma 3: (Pólya's theorem) [22] For a positive integer r, $\triangle_r$: $(\mu_1, \cdots, \mu_r) \mid \mu_i \geq 0$, $\sum_{i=1}^{r} \mu_i = 1$. If a real homogeneous polynominal $F(\mu_1, \cdots, \mu_r)$ is positive definite, then for a sufficiently large d all the coefficients of

$$(\mu_1 + \cdots + \mu_r)^d F(\mu_1, \cdots, \mu_r)$$

are positive.

Theorem 1: If there exist positive constant d_2, d_3, and positive definite symmetric matrix $\overline{R}$, positive definite matrices $\overline{P}_{11}$, $\overline{P}_{22}$, $\overline{Q}_{11}$, $\overline{Q}_{22}$, real matrices X, $\overline{P}_{12}$, $\overline{Q}_{12}$, and $\begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ * & \overline{P}_{22} \end{bmatrix} \geq 0$, $\begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ * & \overline{Q}_{22} \end{bmatrix} \geq 0$ with $0 \leq h(t) \leq h_M$, $0 \leq \dot{h}(t) \leq h_D$ and a sufficiently large d such that the following LMIs conditions hold, then the T-S fuzzy time-delay system is quadratically stablilize via the fuzzy controller $F(\mu) = \bar{F}(\mu)X^{-1}$, where $\bar{F}(\mu) = \sum_{k\in K(1)} \mu^k \bar{F}_k$.

$$\nabla < 0 \tag{6}$$

where

$$\nabla = diag\left[\nabla^{t(1)}, \nabla^{t(2)}, ..., \nabla^{t(J(d+2))}\right]$$

$$\nabla^k = \begin{bmatrix} \nabla(1,1) & \nabla(1,2) & \nabla(1,3) & \nabla(1,4) \\ * & \nabla(2,2) & \nabla(2,3) & \nabla(2,4) \\ * & * & \nabla(3,3) & \nabla(3,4) \\ * & * & * & \nabla(4,4) \end{bmatrix} < 0$$

$$\begin{aligned} \nabla(1,1) &= \frac{2!}{\pi(k-k!)}\left[\overline{P}_{12} + * + \overline{R} + h_M^2\overline{Q}_{11} - \overline{Q}_{22}\right] \\ &+ \sum_{i=1}^{r} \frac{1!(k_i-k_i')}{\pi(k-k')}\left[A_iX^T + *\right] \\ &+ \sum_{i=1}^{r}\left[B_i\overline{F}_{k-k'-e_i} + *\right] \end{aligned}$$

$$\begin{aligned} \nabla(1,2) &= \frac{2!}{\pi(k-k')}\left[-(1-h_D)\overline{P}_{12} + \overline{Q}_{22}\right] \\ &+ \sum_{i=1}^{r} \frac{1!(k_i-k_i')}{\pi(k-k')}\left[A_{di}X^T + d_2XA_i^T\right] \\ &+ \sum_{i=1}^{r}\left[d_2\overline{F}_{k-k'-e_i}^T B_i^T\right] \end{aligned}$$

$$\nabla(1,3) = \frac{2!}{\pi(k-k')}\left[\overline{P}_{22}^T - \overline{Q}_{12}^T\right]$$

$$\begin{aligned} \nabla(1,4) &= \frac{2!}{\pi(k-k')}\left[\overline{P}_{11} + h_M^2\overline{Q}_{12} - X^T\right] \\ &+ \sum_{i=1}^{r} \frac{1!(k_i-k_i')}{\pi(k-k')}\left[d_3XA_i^T\right] \\ &+ \sum_{i=1}^{r}\left[d_3\overline{F}_{k-k'-e_i}^T B_i^T\right] \end{aligned}$$

$$\begin{aligned} \nabla(2,2) &= \frac{2!}{\pi(k-k')}\left[-(1-h_D)\overline{R} - \overline{Q}_{22}\right] \\ &+ \sum_{i=1}^{r} \frac{1!(k_i-k_i')}{\pi(k-k')}\left[d_2A_{di}X^T + *\right] \end{aligned}$$

$$\nabla(2,3) = \frac{2!}{\pi(k-k')}\left[-(1-h_D)\overline{P}_{22}^T + \overline{Q}_{12}^T\right]$$

$$\begin{aligned} \nabla(2,4) &= \frac{2!}{\pi(k-k')}\left[-d_2X^T\right] \\ &+ \sum_{i=1}^{r} \frac{1!(k_i-k_i')}{\pi(k-k')}\left[d_3XA_{di}^T\right] \end{aligned}$$

$$\nabla(3,3) = \frac{2!}{\pi(k-k')}\left[-\overline{Q}_{11}\right]$$

$$\nabla(3,4) = \frac{2!}{\pi(k-k')}\left[\overline{P}_{12}^T\right]$$

$$\nabla(4,4) = \frac{2!}{\pi(k-k')}\left[h_M^2\overline{Q}_{22} - d_3X - d_3X^T\right].$$

Proof : Let us consider a Lyapunov function

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{7}$$

where

$$V_1(t) = \rho^T(t)P\rho(t) \tag{8}$$

$$V_2(t) = \int_{t-h(t)}^{t} x^T(s)Rx(s)ds \tag{9}$$

$$V_3(t) = h_M\int_{t-h_M}^{t}\left(s-(t-h_M)\right)\varepsilon^T(s)Q\varepsilon(s)ds \tag{10}$$

$\rho(t) = \left[x^T(t)\left(\int_{t-h(t)}^{t} x(s)ds\right)^T\right]^T$, $P = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22}\end{bmatrix}$, $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22}\end{bmatrix}$, $\varepsilon(s) = [\ x^T(s) \quad \dot{x}^T(s)\ ]^T$.

The time derivative of Lyapunov function (7) becomes

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \tag{11}$$

where

$$\begin{aligned} \dot{V}_1(t) &= \rho^T(t)P\dot{\rho} \\ &= 2\begin{bmatrix} x(t) \\ \left(\int_{t-h(t)}^{t} x(s)ds\right)\end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22}\end{bmatrix} \\ &\quad\times\begin{bmatrix} \dot{x}(t) \\ x(t)-(1-\dot{h}(t))x(t-h(t))\end{bmatrix} \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{V}_2(t) &= x^T(t)Rx(t) \\ &\quad -(1-\dot{h}(t))x^T(t-h(t))Rx(t-h(t)) \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{V}_3(t) &= h_M^2\begin{bmatrix} x(t) \\ \dot{x}(t)\end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22}\end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(t)\end{bmatrix} \\ &-h_M\int_{t-h_M}^{t}\begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22}\end{bmatrix}\begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix}ds. \end{aligned} \tag{14}$$

According to $0 \le \dot{h}(t) \le h_D$, one has the following results

$$\begin{aligned} \dot{V}_1(t) &\le 2\begin{bmatrix} x(t) \\ \int_{t-h(t)}^{t} x(s)ds\end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22}\end{bmatrix} \\ &\quad\times\begin{bmatrix} \dot{x}(t) \\ x(t)-(1-h_D)x(t-h(t))\end{bmatrix} \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{V}_2(t) &\le x^T(t)Rx(t) \\ &\quad -\left(1-h_D(t)\right)x^T(t-h(t))Rx(t-h(t)). \end{aligned} \tag{16}$$

By Lemma 2, one has the following results from (14).

$$\dot{V}_3(t) \le h_M^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$
$$+ \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \int_{t-h(t)}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ Q_{22} & -Q_{22} & Q_{12}^T \\ -Q_{12} & Q_{12} & -Q_{11} \end{bmatrix}$$
$$\times \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \int_{t-h(t)}^{t} x(s)ds \end{bmatrix}. \quad (17)$$

In addition, the following result is satisfied.

$$\begin{aligned} 0 \quad = \quad & 2\left[x^T(t)T_1 + x^T(t-h(t))T_2 + \dot{x}^T(t)T_3\right] \\ & \times[(A(\mu)+B(\mu)F(\mu))x(t) \\ & +A_d(\mu)x(t-h(t)) - \dot{x}(t)]. \end{aligned} \quad (18)$$

Concluding by (15), (16), (17) and (18), we can obtain

$$\dot{V}(t) \le \Phi^T(t)\Theta\Phi(t) \quad (19)$$

where

$$\begin{aligned} \Phi^T(t) \quad = \quad & \big[\ x^T(t) \quad x^T(t-h(t)) \\ & (\textstyle\int_{t-h(t)}^{t} x(s)ds)^T \quad \dot{x}^T(t) \ \big] \end{aligned} \quad (20)$$

$$\Theta = \begin{bmatrix} \Theta(1,1) & \Theta(1,2) & \Theta(1,3) & \Theta(1,4) \\ * & \Theta(2,2) & \Theta(2,3) & \Theta(2,4) \\ * & * & \Theta(3,3) & \Theta(3,4) \\ * & * & * & \Theta(4,4) \end{bmatrix} \quad (21)$$

$$\Theta(1,1) = P_{12} + * + R + h_M^2 Q_{11} - Q_{22} + T_1 A(\mu) + * \\ + T_1 B(\mu)F(\mu) + *$$

$$\Theta(1,2) = -(1-h_D)P_{12} + Q_{22} + T_1 A_d(\mu) + A^T(\mu)T_2^T \\ + F^T(\mu)B^T(\mu)T_2^T$$

$$\Theta(1,3) = P_{22}^T - Q_{12}^T$$

$$\Theta(1,4) = P_{11} + h_M^2 Q_{12} - T_1 + A^T(\mu)T_3^T \\ + F^T(\mu)B^T(\mu)T_3^T$$

$$\Theta(2,2) = -(1-h_D)R - Q_{22} + T_2 A_d(\mu) + *$$

$$\Theta(2,3) = -(1-h_D)P_{22}^T + Q_{12}^T$$

$$\Theta(2,4) = -T_2 + A_d^T(\mu)T_3^T, \Theta(3,3) = -Q_{11}$$

$$\Theta(3,4) = P_{12}^T, \Theta(4,4) = h_M^2 Q_{22} - T_3 - T_3^T$$

In order for $\dot{V}(t) < 0$, $\forall \neq 0$, $\Theta < 0$ should be satisfied. By utilizing homogeneous polynomial technique, Lemma 2, and pre- and post-multiplying both side of (21) with $diag\begin{bmatrix} X & X & X & X \end{bmatrix}$, and defining $T_1^{-1} = X$, $T_2 = d_2 T_1$, $T_3 = d_2 T_1$, $\bar{P}_{11} = XP_{11}X$, $\bar{P}_{12} = XP_{12}X$, $\bar{P}_{22} = XP_{22}X$, $\bar{Q}_{11} = XP_{11}X$, $\bar{Q}_{12} = XP_{12}X$, $\bar{Q}_{22} = XP_2X$, $\bar{R} = XRX$, $\bar{F}(\mu) = F(\mu)X$, we have the following results,

$$\sum_{k \in K(2)} \mu^k \Psi^k = \Psi(\mu) < 0 \quad (22)$$

$$\Psi^k = \begin{bmatrix} \Psi(1,1) & \Psi(1,2) & \Psi(1,3) & \Psi(1,4) \\ * & \Psi(2,2) & \Psi(2,3) & \Psi(2,4) \\ * & * & \Psi(3,3) & \Psi(3,4) \\ * & * & * & \Psi(4,4) \end{bmatrix}$$

$$\Psi(1,1) = \frac{2!}{\pi(k)}\left[\overline{P}_{12} + * + \overline{R} + h_M^2\overline{Q}_{11} - \overline{Q}_{22}\right] \\ + \sum_{i=1}^{r} \frac{1!k_i}{\pi(k)}\left[A_i X^T + *\right] + \sum_{i=1}^{r}\left[B_i \overline{F}_{k-e_i} + *\right]$$

$$\Psi(1,2) = \frac{2!}{\pi(k)}\left[-(1-h_D)\overline{P}_{12} + \overline{Q}_{22}\right] \\ + \sum_{i=1}^{r} \frac{1!k_i}{\pi(k)}\left[A_{di}X^T + d_2 X A_i^T\right] + \sum_{i=1}^{r}\left[d_2 \overline{F}_{k-e_i}^T B_i^T\right]$$

$$\Psi(1,3) = \frac{2!}{\pi(k)}\left[\overline{P}_{22}^T - \overline{Q}_{12}^T\right]$$

$$\Psi(1,4) = \frac{2!}{\pi(k)}\left[\overline{P}_{11} + h_M^2\overline{Q}_{12} - X^T\right] \\ + \sum_{i=1}^{r} \frac{1!k_i}{\pi(k)}\left[d_3 X A_i^T\right] + \sum_{i=1}^{r}\left[d_3 \overline{F}_{k-e_i}^T B_i^T\right]$$

$$\Psi(2,2) = \frac{2!}{\pi(k)}\left[-(1-h_D)\overline{R} - \overline{Q}_{22}\right] \\ + \sum_{i=1}^{r} \frac{1!k_i}{\pi(k)}\left[d_2 A_{di} X^T + *\right]$$

$$\Psi(2,3) = \frac{2!}{\pi(k)}\left[-(1-h_D)\overline{P}_{22}^T + \overline{Q}_{12}^T\right]$$

$$\Psi(2,4) = \frac{2!}{\pi(k)}\left[-d_2 X^T\right] + \sum_{i=1}^{r} \frac{1!k_i}{\pi(k)}\left[d_3 X A_{di}^T\right]$$

$$\Psi(3,3) = \frac{2!}{\pi(k)}\left[-\overline{Q}_{11}\right], \Psi(3,4) = \frac{2!}{\pi(k)}\left[\overline{P}_{12}^T\right]$$

$$\Psi(4,4) = \frac{2!}{\pi(k)}\left[h_M^2\overline{Q}_{22} - d_3 X - d_3 X^T\right]$$

By utilizing Lemma 3 to (22), we can obtain

$$\begin{aligned}
&\left(\sum_{i=1}^{r}\mu_i\right)^d \Psi(\mu) \\
&= \sum_{k\in(d+2)} \mu^k \sum_{k'\in(d)} \frac{d!}{\pi(k')}\cdot\nabla \\
&= \sum_{k\in(d+2)} \mu^k \cdot \nabla^k < 0. \qquad (23)
\end{aligned}$$

Concluding by above discussion, we can obtain (6). This completes the proof of the theorem.

IV. Simulation

Let us consider the following T-S fuzzy time-delay system given in [7]–[11].

Rule $1:$ **IF** $x_1(t)$ is $M_1(t)$

$$\textbf{THEN } \dot{x}(t) = A_1x(t) + A_{d1}x(t-h(t)) + B_1u(t) \qquad (24)$$

Rule $2:$ **IF** $x_1(t)$ is $M_2(t)$

$$\textbf{THEN } \dot{x}(t) = A_2x(t) + A_{d2}x(t-h(t)) + B_2u(t) \qquad (25)$$

where $A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$,
$A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}$,
$B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

The membership functions of Example 1 are defined as: $\mu_1(t) = (1/(1+\exp(-2x_1+0.5)))$, $\mu_2(t) = 1-\mu_1(t)$. TABLE I shows the comparisons results of the maximal allowable delay time with six different studies. From TABLE I, It is readily seen that the proposed stabilization conditions provide the maximum allowable delay time than the other existing methods.

By utilizing the LMIs toolbox to solve the convex optimization problem in Theorem 1 using with $d_2 = 0.15$ and $d_3 = 1.65$, one can obtain the controller gain as:

$$\begin{aligned}
F_{10} &= [50.2596 \ \ -147.0573]\,, \\
F_{01} &= [53.6308 \ \ -156.0181]\,.
\end{aligned}$$

The state responses by applying the obtained controller with $x(0) = [2 \quad 1]^T$, $d_2 = 0.15$, $d_2 = 1.65$ and $h_M = 1.217$ seconds is shown in the Fig. 1. Simulation results show that the trajectories of the fuzzy time-delay systems converge to the equilibrium state after some transient times.

TABLE I
COMPARISONS AMONG VARIOUS METHODS

Methods	Maximum allowed h_M
Theorem 2 of [7]	0.1524s
Theorem 1 of [8]	0.2302s
Corollary 2 of [9]	0.2574s
Theorem 1 of [9]	0.2664s
Theorem 1 of [10]	0.9s
Corollary 2 of [11]	1.05s
Theorem 2 of this paper	**1.217s**

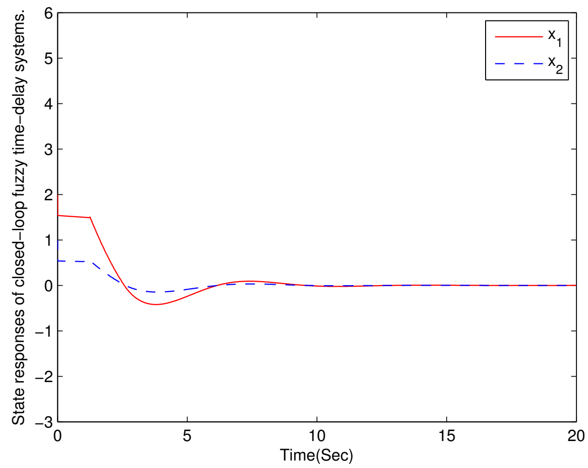

Fig. 1. The state responses for T-S fuzzy time-delay systems.

V. Conclusion

In this paper, the novel relaxed stabilization conditions for the T-S fuzzy time-delay systems are explored. Based on homogenuous polynomial technique, and Pólya's theorem, a delay-dependent relaxed stabilization is formulated in terms of LMIs. Finally, an example is illustrated to demonstrate that the proposed method can provide the maximal allowable delay than some existing methods.

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