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Model Reference Adaptive Iterative Learning Control for Nonlinear Systems Using Observer Design

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Abstract*—In this paper, we propose an observer based model reference adaptive iterative learning control (MRAILC) using model reference adaptive control strategy for more general class of uncertain nonlinear systems with non-canonical form and iteration-varying reference trajectories. Due to the system state vector is assumed to be unmeasurable, a state tracking error observer is applied for state tracking error estimation. Based on the state tracking error observer and a mixed time-domain and s-domain technique, a relative degree one output observation error model whose inputs are some uncertain nonlinearities and filtered signals which is derived to solve the relative degree problem caused by the system states are not measurable. Besides, we also apply some auxiliary signals and an averaging filter to transfer the original output observation error to a new formulation so that we can implement the AILC without using differentiators. The filtered fuzzy neural network (filtered-FNN) using the system state estimation vector as the input vector is applied for approximation of the unknown plant nonlinearities. In order to overcome the lumped uncertainties associated with function approximation error and state estimation error, a normalization signal is applied as a bounding function for designing a robust AILC. The stabilization learning component is used to guarantee the boundedness of internal signals. Based on a Lyapunov like analysis, we show that all the adjustable parameters as well as internal signals remain bounded for all iterations and the norm of output tracking error will asymptotically converge to a tunable residual set.

Keywords*—Adaptive Iterative Learning Control, Observer, Model Reference Adaptive Control, Filtered Fuzzy Neural Network, Nonlinear Systems

I. INTRODUCTION

It is well known that adaptive iterative learning control (AILC) scheme [1], [2] has been widely studied for performing the repeated tracking control of uncertain robotic systems [3], [4], non-Lipschitz nonlinear systems [5], [6] and precision motion systems [7]. In recent years, the fuzzy systems, neural networks or fuzzy neural networks were applied to approximate the plant nonliearties for the design of the state based AILC [8], [9], [10], [11] due to the plant nonlinearties are unknown. It is noted that the most attractive advantages of the AILC schemes in the research field of ILC is that the AILC schemes can be used to deal with three important issues: (1) iteration-varying reference trajectories (2) random large bounded initial resetting error (3) random bounded disturbance. In order to facilitate the design of the state based AILC, the system state vector is necessary assumed to be available for measurement in aforementioned works [3], [4], [7], [8], [9], [10], [11], [12], [13], [14]. But unfortunately, the system state vector of the physical plant is usually unavailable for state measurement. In order to relax the strictest plant assumption in these related AILC works that the system state vector is unavailable for state measurement, the output based AILC schemes [15], [16], [17], [18] for nonlinear systems using only output measurement is still a challenge issue in this research field of ILC. But a comparison with the state based AILC schemes [8], [9], [10], [11], [12], [13], [14] is that most of these related output based AILC schemes [15], [16], [17] are only used to deal with the nonlinear systems satisfying some special structures: (1) canonical form nonlinear systems (2) output-feedback form nonlinear systems.

In this paper, an observer based model reference AILC (MRAILC) using model reference adaptive control strategy is proposed for a more general class of uncertain nonlinear systems with non-canonical form and iteration-varying reference trajectories. In order to deal with the issue that the system state vector is assumed to be unmeasurable, a state tracking error observer is applied for state tracking error estimation. In order to solve the relative degree problem caused by the system state vector is not measurable, we derive a relative degree one output observation error model whose inputs are some uncertain nonlinearities and filtered signals by using the state tracking error observer and a mixed time-domain and s-domain technique. On the other hand, we design some auxiliary signals and an averaging filter to transfer the original output observation error to a new formulation so that the AILC can be implemented without using differentiators. In order to approximate for the unknown plant nonlinearities, the filtered fuzzy neural network (filtered-FNN) using the system state estimation vector as the input vector is applied for approximation of the unknown plant nonlinearities. In addition, a normalization signal is applied as a bounding function for designing a robust learning component in order to overcome the lumped uncertainties are consisted of function approximation error and state estimation error. The stabilization learning component is used to guarantee the boundedness of internal signals. Based on a Lyapunov like analysis, the adaptive laws combining time domain and iteration domain adaptation are determined to ensure the convergence of learning error. Finally, we show that all the adjustable parameters as well as internal signals remain bounded for all iterations and the norm of output tracking error will asymptotically converge to a tunable residual set whose size depends on some design parameters of averaging filter as iteration goes to infinity.

In this paper, $L_{pe}[0,T]$ denotes the set of Lebesgue measurable (or piecewise continuous) real valued (vector) functions with L_{pe} norm [10] and $\|(\cdot)_t\|_\infty = \sup_{\tau \le t}|(\cdot)(\tau)|$ denotes the truncated L_∞ norm of the argument function or vector [19].

II. Problem Formulation and Control Objective

In this paper, we consider a class of nonlinear systems which can perform a given control task repetitively over a finite time interval $[0,T]$ as follows,

$$\begin{aligned} \dot{X}^j(t) &= f(X^j(t)) + b(X^j(t))u^j(t) \\ y^j(t) &= C^\top X^j(t) \end{aligned} \tag{1}$$

where $C^\top = [1 \ \ 0 \ \ \cdots \ \ 0]$. Here $X^j(t) \in \mathcal{R}^{n\times 1}$ is the (transformed) state vector of the system which is not measurable, $u^j(t)$ is the control input, $y^j(t)$ is the system output, $f(X^j(t)) = [f_1(X^j(t)), \cdots, f_n(X^j(t))]^\top \in \mathcal{R}^{n\times 1}$ and $b(X^j(t)) = [b_1(X^j(t)), \cdots, b_n(X^j(t))]^\top \in \mathcal{R}^{n\times 1}$ are unknown real continuous nonlinear function vectors of states, $j \in \mathcal{Z}_+$ denotes the index of iteration. The followings are the most relevant conditions on the nonlinear system throughout this paper.

- (A1) $b_1(X^j(t)) = \cdots = b_{\rho-1}(X^j(t)) = 0$, $b_\rho(X^j(t)) \neq 0$, $\rho > 1$. The special case of $\rho = 1$ is much simpler and not considered in this paper because of the paper length limitation.
- (A2) The sign of $b_\rho(X^j(t))$ is known. Without loss of generality, we assume $b_\rho(X^j(t)) > 0$.
- (A3) $b(X^j(t))$ is bounded $\forall X^j(t) \in R^{n\times 1}$.
- (A4) $f(X^j(t))$ is bounded if $X^j(t)$ is bounded.

The control objective is to find an iterative learning controller $u^j(t)$ using only output measurement $y^j(t)$ such that $y^j(t)$ can follow an iteration-varying reference output $y^j_m(t)$ as close as possible $\forall t \in [0,T]$ when iteration j approaches infinity. The iteration-varying reference output is generated by the following reference model,

$$\begin{aligned} \dot{X}^j_m(t) &= A_m X^j_m(t) + B_m r^j_m(t) \\ y^j_m(t) &= C^\top X^j_m(t) \end{aligned} \tag{2}$$

where

$$A_m = \begin{bmatrix} -a^m_1 & 1 & 0 & \cdots & 0 \\ -a^m_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -a^m_n & 0 & 0 & \cdots & 0 \end{bmatrix}, \ B_m = \begin{bmatrix} b^m_1 \\ b^m_2 \\ \vdots \\ b^m_n \end{bmatrix}$$

Here $X^j_m(t) \in \mathcal{R}^{n\times 1}$ is the state vector, $r^j_m(t)$ is the reference input, A_m is a Hurwitz matrix. In general, we will choose $b^m_1 = \cdots = b^m_{\rho-1} = 0$ and $b^m_\rho = 1$ according to assumption (A1)and (A2). The reference model $M(s) = C^\top(sI - A_m)^{-1}B_m = \frac{s^{n-\rho} + b^m_{\rho+1}s^{n-\rho-1} + \cdots + b^m_n}{s^n + a^m_1 s^{n-1} + \cdots + a^m_n}$ is a stable and minimum phase system with the required specifications.

III. Observer Design and Observation Error Model

Define the state tracking error vector and output tracking error as $E^j(t) = X^j_m(t) - X^j(t)$ and $e^j(t) = y^j_m(t) - y^j(t)$ respectively, then we can easily derive that

$$\begin{aligned} \dot{E}^j(t) &= A_m E^j(t) + g(X^j(t)) \\ &\quad + h(X^j(t))u^j(t) + B_m u^j(t) \\ e^j(t) &= C^\top E^j(t) \end{aligned} \tag{3}$$

where $g(X^j(t)) \equiv A_m X^j(t) - f(X^j(t)) + B_m r^j_m(t) \in \mathcal{R}^{n\times 1}$ and $h(X^j(t))) \equiv -[b(X^j(t)) + B_m]u^j(t) \in \mathcal{R}^{n\times 1}$ are unknown real continuous bounded nonlinear function vectors of state if $X^j(t)$ is bounded. An observer is designed for the state error estimation vector $\hat{E}^j(t) = X^j_m(t) - \hat{X}^j(t)$ as follows

$$\begin{aligned} \dot{\hat{E}}^j(t) &= A_m \hat{E}^j(t) + K_o(e^j(t) - \hat{e}^j(t)) \\ \widehat{e}^j(t) &= C^\top \widehat{E}^j(t) \end{aligned} \tag{4}$$

where $K_o = [k^o_1, \cdots, k^o_n]^\top \in \mathcal{R}^n$ is the observer gain vector designed such that $A_o = A_m - K_o C^\top$ is Hurwitz. Let $\widetilde{E}^j(t) = E^j(t) - \widehat{E}^j(t)$, the observation error dynamics can be derived as

$$\begin{aligned} \dot{\widetilde{E}}^j(t) &= A_o \widetilde{E}^j(t) + g(X^j(t)) + h(X^j(t))u^j(t) \\ &\quad + B_m u^j(t) \\ \widetilde{e}^j(t) &= C^\top \widetilde{E}^j(t) \end{aligned} \tag{5}$$

Note that $|\widetilde{e}^j(0)| = |e^j(0) - \widehat{e}^j(0)| = |e^j(0)| = \varepsilon^j$. Based on the universal approximation theorem, $g(X^j(t))$ can be approximated by a traditional FNN $W^j(t)^\top O^{(3)}(X^j(t))$. Here $O^{(3)}(X^j(t)) \in \mathcal{R}^{M\times 1}$ is the basis function vector with M being the number of rule nodes and $W^j(t) = [W^j_1(t), \cdots, W^j_n(t)] \in \mathcal{R}^{M\times n}$ is the weight matrix of the output layer. According to the universal approximation theorem, there will exist an optimal weight matrix $W^* = [W^*_1, \cdots, W^*_n] \in \mathcal{R}^{M\times n}$ such that $g(X^j(t)) = W^{*\top}O^{(3)}(X^j(t)) + \epsilon^j(X^j(t))$, where $\epsilon(X^j(t)) \in \mathcal{R}^{n\times 1}$ is the approximation error vector satisfying $\|\epsilon^j(X^j(t))\| \le \epsilon^*$ in a certain compact set. This implies that (5) can be rewritten as

$$\begin{aligned} \dot{\widetilde{E}}^j(t) &= A_o \widetilde{E}^j(t) + W^{*\top}O^{(3)}(\widehat{X}^j(t)) + \delta^j(t) \\ &\quad + h(X^j(t))u^j(t) + B_m u^j(t) \\ \widetilde{e}^j(t) &= C^\top \widetilde{E}^j(t) \end{aligned} \tag{6}$$

where $\delta^j(t) = [\delta^j_1(t), \cdots, \delta^j_n(t)]^\top = W^{*\top}(O^{(3)}(X^j(t)) - O^{(3)}(\widehat{X}^j(t)) + \epsilon(X^j(t)) \in \mathcal{R}^{n\times 1}$ is bounded. The control objective is now transformed into a problem of forcing the output observation error $\widetilde{e}^j(t)$ to converge to zero. We now adopt the mixed use of a time signal and a Laplace transfer function to obtain the explicit expression of $\widetilde{e}^j(t)$ in (6) in time domain with a filtered version as

$$\begin{aligned} \widetilde{e}^j(t) &= H(s)\Big[-u^j(t)\Big] + \sum_{i=1}^{n} H_i(s)\Big[W^{*\top}_i O^{(3)}(\widehat{X}^j(t)) \\ &\quad + \delta^j_i(t)\Big] + \sum_{i=\rho}^{n} H_i(s)\Big[h_i(X^j(t))u^j(t)\Big] \end{aligned} \tag{7}$$

where $H(s) = C^\top (sI - A_o)^{-1} B_m = \frac{s^{n-\rho}+\cdots+b_n^m}{det(sI-A_o)}$, $H_i(s) = \frac{s^{n-i}}{det(sI-A_o)}$ and $H_i(s) = H(s)\frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}$. The observer gain vector K_o will be chosen such that $det(sI - A_o)$ being any Hurwitz polynomial. The above equation can be further rewritten as

$$\begin{aligned}
&\widetilde{e}^j(t) \\
&= H(s)\Bigg[u^j(t) + \sum_{i=1}^{n} \frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}\Big[W_i^{*\top} O^{(3)}(\widehat{X}^j(t)) \\
&\quad + \delta_i^j(t)\Big] + \sum_{i=\rho}^{n} \frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}\Big[h_i(X^j(t))u^j(t)\Big]\Bigg] \quad (8)
\end{aligned}$$

It should be noted that $\frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}$, $i = 1, \cdots, n$ are stable due to assumption (A1) but may be proper or nonproper. Rewrite (8) as $\widetilde{e}^j(t) = H(s)[u^j(t) + \overline{u}^j(t)]$ where

$$\begin{aligned}
\overline{u}^j(t) &= \sum_{i=1}^{n} \frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}\Big[W_i^{*\top} O^{(3)}(\widehat{X}^j(t)) + \delta_i^j(t)\Big] \\
&\quad + \sum_{i=\rho}^{n} \frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}\Big[h_i(X^j(t))u^j(t)\Big] \quad (9)
\end{aligned}$$

According to the approach of traditional model reference adaptive control [19], it is known that if $\overline{u}^j(t) = 0$ so that $\widetilde{e}^j(t) = H(s)[u^j(t)]$, there exists a constant parameter vector $\Theta = [\theta_1, \cdots, \theta_{2n}]^\top \in \mathcal{R}^{2n\times 1}$ such that the following Laplace algebraic equation will be satisfied:

$$1 - \phi_a(s) - \phi_b(s)H(s) = \theta_{2n} M^{-1}(s)H(s) \quad (10)$$

where $\phi_a(s) = [\theta_1, \cdots, \theta_{n-1}]\frac{a(s)}{\lambda(s)}$, $\phi_b(s) = [\theta_n, \cdots, \theta_{2n-2}]\frac{a(s)}{\lambda(s)} + \theta_{2n-1}$, $a(s) = [s^{n-2}, \cdots, s, 1]^\top$ and $\lambda(s) = s^{n-1} + \lambda_1 s^{n-2} + \cdots + \lambda_{n-1}$ is a monic Hurwitz polynomial will be designed. Since $H(s)$ and $M(s)$ are known transfer functions, $\lambda(s)$ is a known monic Hurwitz polynomial, the constant parameter vector Θ will be a known constant parameter vector by solving the Laplace algebraic equation (10) and we can easily derive that $\theta_{2n} = 1$. Operating both sides of (10) on $u^j(t) + \overline{u}^j(t)$ implies that

$$\begin{aligned}
&\widetilde{e}^j(t) \\
&= M(s)\Big[u^j(t) + \overline{u}^j(t) - \phi_a(s)\Big[u^j(t) + \overline{u}^j(t)\Big] \\
&\quad -\phi_b(s)\Big[\widetilde{e}^j(t)\Big]\Big] \\
&= M(s)\Big[u^j(t) - \Theta^\top w^j(t) + (1 - \phi_a(s))\Big[\overline{u}^j(t)\Big]\Big] \\
&= M(s)\Big[u^j(t) - \Theta^\top w^j(t) \\
&\quad + \sum_{i=1}^{n} P_i(s)\Big[W_i^{*\top} O^{(3)}(\widehat{X}^j(t)) + \delta_i^j(t)\Big] \\
&\quad + \sum_{i=\rho}^{n} P_i(s)\Big[h_i(X^j(t))u^j(t)\Big] \quad (11)
\end{aligned}$$

where $w^j(t) = \Big[\frac{a(s)}{\lambda(s)}[u^j(t)],\ \frac{a(s)}{\lambda(s)}[\widetilde{e}^j(t)],\ \widetilde{e}^j(t)\Big]^\top$, $P_i(s) \equiv \Big(1 - \frac{\theta_1 s^{n-2}+\cdots+\theta_{n-1}}{\lambda(s)}\Big)\frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}$. Since the reference model $M(s)$ is relative degree ρ, we can choose $L(s) = (s+\lambda_2^m)\cdots(s+\lambda_\rho^m)$ as a Hurwitz polynominal with degree $\rho - 1$ such that

$$M(s) = \frac{s^{n-\rho}+\cdots+b_n^m}{det(sI - A_m)} = \frac{1}{(s+\lambda_1^m)(s+\lambda_2^m)\cdots(s+\lambda_\rho^m)}$$

, then we have $M(s)L(s) = \frac{1}{\ell(s)} = \frac{1}{s+\lambda_1^m}$ is a stable relative degree one transfer function. Now, if we let $G_i(s) \equiv \frac{1}{L(s)}P_i(s) = \frac{1}{L(s)}\Big(1 - \frac{\theta_1 s^{n-2}+\cdots+\theta_{n-1}}{\lambda(s)}\Big)\frac{s^{n-i}}{s^{n-\rho}+\cdots+b_n^m}$ being stable due to $L(s)$, $\lambda(s)$ and $s^{n-\rho} + \cdots + b_n^m$ are Hurwitz polynomials but may be proper or nonproper. Then according to (9) and (11), the output observation error model (11) can then be written as

$$\begin{aligned}
\widetilde{e}^j(t) &= \frac{1}{\ell(s)}\Bigg[\frac{1}{L(s)}[u^j(t)] - \Theta^\top \xi^j(t) \\
&\quad + \sum_{i=1}^{n} W_i^{*\top} O_i^{(4)}(\widehat{X}^j(t)) + \delta_L^j(t)\Bigg] \quad (12)
\end{aligned}$$

where $\xi^j(t) = \frac{1}{L(s)}\Big[w^j(t)\Big]$, $O_i^{(4)}(\widehat{X}^j(t)) = G_i(s)\Big[O^{(3)}(\widehat{X}^j(t))\Big]$ and the lumped uncertainties is $\delta_L^j(t) = \sum_{i=\rho}^{n} G_i(s)\Big[h_i(X^j(t)))u^j(t)\Big] + \sum_{i=1}^{n} G_i(s)\Big[\delta_i^j(t)\Big]$, respectively.

IV. Design of Observer-based MRAILC

In next, we define an augmented signal with filtered version as

$$y_a^j(t) = \frac{1}{\ell(s)}\left[v^j(t) - \frac{1}{L(s)}\Big[u^j(t)\Big]\right],\ y_a^j(0) = 0 \quad (13)$$

where $v^j(t)$ is an auxiliary input to be designed later. Then, design an auxiliary error signal as

$$e_a^j(t) = \widetilde{e}^j(t) + y_a^j(t),\ e_a^j(0) = \widetilde{e}^j(0) \quad (14)$$

Substituting (11) and (13) into (14), we can find that

$$\begin{aligned}
e_a^j(t) &= \frac{1}{\ell(s)}\Bigg[v^j(t) - \Theta^\top \xi^j(t) \\
&\quad + \sum_{i=1}^{n} W_i^{*\top} O_i^{(4)}(\widehat{X}^j(t)) + \delta_L^j(t)\Bigg] \quad (15)
\end{aligned}$$

Then the time-domain state space representation of (15) can be derived as

$$\begin{aligned}
\dot{e}_a^j(t) &= -\lambda_1^m e_a^j(t) + v^j(t) - \Theta^\top \xi^j(t) \\
&\quad + \sum_{i=1}^{n} W_i^{*\top} O_i^{(4)}(\widehat{X}^j(t)) + \delta_L^j(t) \quad (16)
\end{aligned}$$

To overcome the uncertainties from initial output tracking error, a new signal $e_\phi^j(t)$ is introduced as follows,

$$e_\phi^j(t) = e_a^j(t) - \phi^j(t)\mathbf{sat}\left(\frac{e_a^j(t)}{\phi^j(t)}\right),\ \phi^j(t) = \varepsilon^j e^{-\lambda_1^m t} \quad (17)$$

where **sat** is a saturation function defined as

$$\mathbf{sat}\left(\frac{e_a^j(t)}{\phi^j(t)}\right)=\begin{cases}1 & \text{if } e_a^j(t)>\phi^j(t)\\ \frac{e_a^j(t)}{\phi^j(t)} & \text{if } |e_a^j(t)|\le\phi^j(t)\\ -1 & \text{if } e_a^j(t)<-\phi^j(t)\end{cases}$$

Note that $0<\varepsilon^j e^{-\beta T}\le\phi^j(t)\le\varepsilon^j$ and $e_\phi^j(0)=0,\forall j\ge 1$. Now differentiate $\frac{1}{2}(e_\phi^j(t))^2$ as follows,

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}(e_\phi^j(t))^2 &= e_\phi^j(t)\Big(\dot{e}_a^j(t)-\mathbf{sgn}\left(e_\phi^j(t)\right)\dot{\phi}^j(t)\Big)\\ &= -\lambda_1^m(e_\phi^j(t))^2+e_\phi^j(t)\Big[v^j(t)-\Theta^\top\xi^j(t)\\ &\quad+\sum_{i=1}^n W_i^{*\top}O_i^{(4)}(\widehat{X}^j(t))+\delta_L^j(t)\Big]\end{aligned}\tag{18}$$

where $\mathbf{sgn}(e_\phi^j(t))$ is the typical signum function. Next, we introduce the normalization signal $m^j(t)$ [20] as follows,

$$m^j(t)=\frac{\delta_2}{s+\delta_1}\Big[1+\big|u^j(t)\big|\Big]\tag{19}$$

where $m^j(0)>\frac{\delta_2}{\delta_1}$, $\delta_1,\delta_2>0$ and $\delta_1<\delta^*$. Here δ^* is the least positive constant such that $G_i(s-\delta^*)$ is stable systems. According to the definition of $\delta_L^j(t)$ defined in (9), we can find that $\sum_{i=1}^n G_i(s)\big[\delta_i^j(t)\big]$ is bounded since $\delta_i^j(t)$ are bounded and $G_i(s)$ is a stable proper or strictly proper transfer function for $i=1,2,\cdots,n$. Furthermore $\sum_{i=\rho}^n G_i(s)\big[h_i(X^j(t))u^j(t)\big]$ will be bounded by $u^j(t)$ since $h_i(X^j(t))$ is bounded and $G_i(s)$ is a strictly proper transfer function for $i=\rho,\cdots,n$. Hence by using Lemma 3.1 in [20], we can prove that $\left|\delta_L^j(t)\right|\le\psi^*(m^j(t)+1)$ for some unknown positive constant ψ^*. Based on the derived error model and the useful signals, we design $u^j(t)$ and $v^j(t)$ as follows

$$u^j(t)=\frac{L(s)}{F^\rho(\tau s)}[v^j(t)]\tag{20}$$

$$\begin{aligned}v^j(t) &= \Theta^\top\xi^j(t)-\sum_{i=1}^n W_i^j(t)^\top O_i^{(4)}(\widehat{X}^j(t))\\ &\quad-\mathbf{sat}\left(\frac{e_a^j(t)}{\phi^j(t)}\right)\psi^j(t)(m^j(t)+1)\\ &\quad-e_\phi^j(t)\xi^j(t)^\top\xi^j(t)-e_\phi^j(t)(m^j(t)+1)^2\end{aligned}\tag{21}$$

where $W_i^j(t)^\top O^{(4)}(\widehat{X}^j(t))$ is the ith output of a filtered-FNN, $W_i^j(t)$, $i=1,\cdots,n$ is the weight matrix of the network and $\psi^j(t)$ is control parameter. $W_i^j(t)$ and $\psi^j(t)$ are used to compensate for the unknown W_i^*, $i=1,\cdots,n$ and ψ^* respectively. Furthermore, we define $F(\tau s)=(\tau s+1)$ with τ being a small positive constant. In the literature, $\frac{1}{F(\tau s)}$ is referred to as an *averaging filter*, which is obviously a low-pass filter whose bandwidth can be arbitrarily enlarged as τ approaches 0. A set of stable adaptive laws is designed to tune all the control parameters as follows,

$$\begin{aligned}(1-\gamma_1)\dot{W}_i^j(t) &= -\gamma_1 W_i^j(t)+\gamma_1 W_i^{j-1}(t)\\ &\quad+\beta_1 e_\phi^j(t)O_i^{(4)}(\widehat{X}^j(t))\end{aligned}\tag{22}$$

$$\begin{aligned}(1-\gamma_2)\dot{\psi}^j(t) &= -\gamma_2\psi^j(t)+\gamma_2\psi^{j-1}(t)\\ &\quad+\beta_2|e_\phi^j(t)|(m^j(t)+1)\end{aligned}\tag{23}$$

with $W_i^j(0)=W_i^{j-1}(T)$, $i=1,\cdots,n$, $\psi^j(0)=\psi^{j-1}(T)$ for $j\ge 1$, and $0<\gamma_1,\gamma_2<1$, $\beta_1,\beta_2>0$.

V. Analysis of Stability and Convergence

If we define the parameter error as $\widetilde{W}_i(t)=W_i^j(t)-W_i^*$, $i=1,\cdots,n$ and $\widetilde{\psi}^j(t)=\psi^j(t)-\psi^*$ and substitute (21) into (18), we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}e_\phi^j(t)^2\\ &\le -\lambda_1 e_\phi^j(t)^2-e_\phi^j(t)\sum_{i=1}^n\widetilde{W}_i^j(t)^\top O_i^{(4)}(\widehat{X}^j(t))\\ &\quad-|e_\phi^j(t)|\widetilde{\psi}^j(t)(m^j(t)+1)-e_\phi^j(t)^2\xi^j(t)^\top\xi^j(t)\\ &\quad-e_\phi^j(t)^2(m^j(t)+1)^2\end{aligned}\tag{24}$$

Lemma 1 : Consider the nonlinear system (1) performing a repetitive control task. If we apply the observer-based MRAILC (13), (14), (17), (19), (20), and (21) with adaptation laws (22) and (23), then we guarantee that $e_\phi^1(t)$, $e_a^1(t)$, $\widetilde{W}_i^1(t)$, $\widetilde{\psi}^1(t)$ are bounded.

Proof: Choose a Lyapunov-like positive function as

$$\begin{aligned}V_a^j(t) &= \frac{1}{2}e_\phi^j(t)^2+\frac{(1-\gamma_1)}{2\beta_1}\sum_{i=1}^n\widetilde{W}_i^j(t)^\top\widetilde{W}_i^j(t)\\ &\quad+\frac{(1-\gamma_2)}{2\beta_2}\widetilde{\psi}^j(t)^2\end{aligned}$$

then we have

$$\begin{aligned}\dot{V}_a^j(t) &\le \frac{\gamma_1}{2\beta_1}\sum_{i=1}^n\widetilde{W}_i^{j-1}(t)^\top\widetilde{W}_i^{j-1}(t)+\frac{\gamma_2}{2\beta_2}\widetilde{\psi}^{j-1}(t)^2\\ &\equiv V_b^{j-1}(t)\end{aligned}\tag{25}$$

Since $\widetilde{W}_i^0(t)=-W_i^*, i=1,\cdots,n$ and $\widetilde{\psi}^0(t)=-\psi^*$ are bounded for all $t\in[0,T]$ so that if $j=1$, (25) is rewritten as

$$\dot{V}_a^1(t)\le\frac{\gamma_1}{2\beta_1}\sum_{i=1}^n W_i^{*\top}W_i^*+\frac{\gamma_2}{2\beta_2}(\psi^*)^2\tag{26}$$

It readily implies $V_a^1(t), e_\phi^1(t), \widetilde{W}_i^1(t), \widetilde{\psi}^1(t)\in L_{\infty e}[0,T]$, $i=1,\cdots,n$ since $V_a^1(0)$ is bounded.

Lemma 2 : Consider the problem set-up in Lemma 1. The proposed observer-based MRAILC guarantees that $e_\phi^j(T),\widetilde{W}^j(T)$ and $\widetilde{\psi}^j(T)$ are bounded for all $j\ge 1$ as well as $\lim_{j\to\infty}\int_0^T e_\phi^j(t)^2dt=0$ and $\lim_{j\to\infty}e_\phi^j(T)^2=0$.

Proof: Define a positive function $V^j(T)$ as

$$\begin{aligned}&V^j(T)\\ &=\int_0^T\left(\frac{\gamma_1}{2\beta_1}\sum_{i=1}^n\widetilde{W}_i^j(t)^\top\widetilde{W}_i^j(t)+\frac{\gamma_2}{2\beta_2}\widetilde{\psi}^j(t)^2\right)dt\\ &\quad+\frac{1-\gamma_1}{2\beta_1}\sum_{i=1}^n\widetilde{W}_i^j(T)^\top\widetilde{W}_i^j(T)+\frac{1-\gamma_2}{2\beta_2}\widetilde{\psi}^j(T)^2\end{aligned}\tag{27}$$

Using the technique of integration by parts, we can prove that

$$\begin{aligned} & V^j(T) - V^{j-1}(T) \\ \leq\ & \int_0^T \Bigg(e_\phi^j(t) \sum_{i=1}^n \widetilde{W}_i^j(t)^\top O_i^{(4)}(\widehat{X}^j(t)) \\ & + |e_\phi^j(t)| \widetilde{\psi}^j(t)(m^j(t)+1) \Bigg) dt \\ \leq\ & -\int_0^T \Big(\lambda_1^m e_\phi^j(t)^2 + e_\phi^j(t)^2 \xi^j(t)^\top \xi^j(t) \\ & + e_\phi^j(t)^2 (m^j(t)+1)^2 \Big) dt - \frac{1}{2} e_\phi^j(T)^2 \end{aligned} \tag{28}$$

where we use the integration of (24) from 0 to T and the property of $\frac{1}{2}(e_\phi^j(0))^2 = 0$.

Since $V^1(T)$ is bounded by Lemma 1 and $V^j(T)$ is positive and monotonically decreasing, we conclude by the result of (28) that $V^j(T)$ is bounded for all $j \geq 1$ and will converge as j approaches infinity to some limit value $V(T)$ which is independent of j. Since $V^{j-1}(T) - V^j(T) \leq V^1(T)$, (28) also implies that $\int_0^T e_\phi^j(t)^2 dt$, $\int_0^T e_\phi^j(t)^2 \xi^j(t)^\top \xi^j(t) dt$, $\int_0^T e_\phi^j(t)^2 (m^j(t)+1)^2 dt$ and $e_\phi^j(T)^2$ are bounded for all $j \geq 1$. Furthermore, $\lim_{j\to\infty} \int_0^T e_\phi^j(t)^2 dt = 0$ and $\lim_{j\to\infty} e_\phi^j(T)^2 = 0$.

Lemma 3 : Consider the problem set-up in Lemma 1. The proposed observer-based MRAILC ensures that all the internal signals are bounded.

Proof: Integrating (25) from 0 to t, we have

$$V_a^j(t) \ \leq \ V_a^j(0) + \int_0^T V_b^{j-1}(t) dt \tag{29}$$

Since $V^j(T)$ defined in (27) is bounded and $V_a^j(0)$ is bounded by using Lemma 2, we can conclude from (29) that $\int_0^T V_b^{j-1}(t)dt$ is bounded and hence, $V_a^j(t)$, $e_\phi^j(t)$, $\widetilde{W}_i^j(t)$, $\widetilde{\psi}^j(t)$, $e_a^j(t)$, $\in L_{\infty e}[0,T]$.

However, the boundedness of $V_a^j(t)$, $e_\phi^j(t)$, $\widetilde{W}_i^j(t)$, $\widetilde{\psi}^j(t)$, $e_a^j(t)$, can not guarantee the boundedness of $m^j(t)$ and input $u^j(t)$. In order to show the boundedness of $m^j(t)$ and $u^j(t)$ for all $t \in [0,T]$, we first note that $\int_0^t e_\phi^j(t')^2 \xi^j(t')^\top \xi^j(t') dt'$, $\int_0^t e_\phi^j(t')^2 (m^j(t')+1)^2 dt' \in L_{\infty e}[0,T]$. Now we adopt some techniques given in chapter 2 of [19]. Consider $u^j(t)$ in (20) as follows,

$$\begin{aligned} u^j(t) &= \frac{L(s)}{F^\rho(\tau s)}[v^j(t)] \\ &= \frac{L(s)}{F^\rho(\tau s)}\Bigg[\sum_{i=1}^n W_i^j(t)^\top O_i^{(4)}(\widehat{X}^j(t)) \\ & \quad + \textbf{sat}\left(\frac{e_a^j(t)}{\phi^j(t)}\right) \psi^j(t)(m^j(t)+1) \Bigg] \\ & \quad + \frac{sL(s)}{F^\rho(\tau s)}\Bigg[\int_0^t e_\phi^j(t')\xi^j(t')^\top \xi^j(t') dt' \\ & \quad + e_\phi^j(t')(m^j(t')+1)^2 dt' \Bigg] \end{aligned} \tag{30}$$

Since $W_i^j(t)$, $O_i^{(4)}(\widehat{X}^j(t))$, $\psi^j(t)$, $\int_0^t e_\phi^j(t')\xi^j(t')^\top \xi^j(t') dt'$ and $\int_0^t e_\phi^j(t')(m^j(t')+1)^2 dt'$ are bounded for $t \in [0,T]$, and $\frac{L(s)}{F^\rho(\tau s)}$ is strictly proper stable transfer function, $\frac{sL(s)}{F^\rho(\tau s)}$ is proper stable transfer function, (30) implies that $u^j(t)$ will satisfy

$$|u^j(t)| \leq k_1 \left(\|(\xi^j)_t\|_\infty + \|(m^j)_t\|_\infty \right) + k_1 \tag{31}$$

for some $k_1 > 0$ by lemma 2.6 in [19]. Now, we investigate the filtered signal $\xi^j(t)$. By definition,

$$\begin{aligned} & \xi^j(t) \\ =\ & \frac{1}{L(s)}[u^j(t)] \\ \equiv\ & \left[\frac{a(s)}{L(s)\lambda(s)}[u^j(t)], \frac{a(s)}{L(s)\lambda(s)}[\widetilde{e}^j(t)], \frac{1}{L(s)}[\widetilde{e}^j(t)] \right]^\top \\ =\ & \left[\xi_1^j(t), \xi_2^j(t), \xi_3^j(t) \right]^\top \end{aligned} \tag{32}$$

Suppose that the corresponding state-space realization of each element of $\xi^j(t)$ is A_{zi}, B_{zi}, C_{zi} with state variable $z_i^j(t)$, $i = 1,2,3$. Together with the observation error dynamics (6) and normalization signal (19), we construct an extended dynamic equation as follows:

$$\begin{aligned} \begin{bmatrix} \dot{\widetilde{E}}^j(t) \\ \dot{m}^j(t) \\ \dot{z}_1^j(t) \\ \dot{z}_2^j(t) \\ \dot{z}_3^j(t) \end{bmatrix} &= \begin{bmatrix} A_o & 0 & 0 & 0 & 0 \\ 0 & -\delta_1 & 0 & 0 & 0 \\ 0 & 0 & A_{z1} & 0 & 0 \\ 0 & 0 & 0 & A_{z2} & 0 \\ 0 & 0 & 0 & 0 & A_{z3} \end{bmatrix} \begin{bmatrix} \widetilde{E}^j(t) \\ m^j(t) \\ z_1^j(t) \\ z_2^j(t) \\ z_3^j(t) \end{bmatrix} \\ & \quad + \begin{bmatrix} q_1^j(t) \\ q_2^j(t) \\ q_3^j(t) \\ q_4^j(t) \\ q_5^j(t) \end{bmatrix} \end{aligned} \tag{33}$$

where $q_1^j(t) = W^{*\top} O^{(3)}(\widehat{X}^j(t)) + \delta^j(t) + h(X^j(t))u^j(t) + B_m u^j(t)$, $q_2^j(t) = -\delta_2(1+|u^j(t)|)$, $q_3^j(t) = B_{z1}u^j(t)$, $q_4^j(t) = B_{z2}\widetilde{e}^j(t)$, $q_5^j(t) = B_{z3}\widetilde{e}^j(t)$. Let $X_a^j(t)$ be the state vector of the extended dynamic equation (33). Taking norms on (33) will yield

$$\begin{aligned} \|\dot{X}_a^j(t)\| &\leq k_2\|X_a^j(t)\| + k_2|u^j(t)| + k_2 \\ &\leq k_3\|(X_a^j)_t\|_\infty + k_3 \end{aligned} \tag{34}$$

for some $k_2, k_3 > 0$ since $|q_1^j(t)| \leq \gamma_1 \left(\|(\xi^j)_t\|_\infty + \|(m^j)_t\|_\infty \right) + \gamma_1$ and $|\widetilde{e}^j(t)| = |C^\top \widetilde{E}^j(t)| \leq \|X_a^j(t)\|$. This implies that $X_a^j(t)$ is regular [19] and $m^j(t)$ is a smooth signal. Together with the result of $\int_0^t \left(e_\phi^j(t')\xi^j(t')^\top \xi^j(t') + e_\phi^j(t')(m^j(t')+1)^2 \right) dt' \in L_{\infty e}[0,T]$, we can now conclude that $\xi^j(t), m^j(t) \in L_{\infty e}[0,T]$.

Due to $W_i^j(t)$, $O_i^{(4)}(\widehat{X}^j(t))$, $\psi^j(t)$, $\xi^j(t)$, $m^j(t)$, $e_\phi^j(t) \in L_{\infty e}[0,T]$, we have $v^j(t) \in L_{\infty e}[0,T]$ (by (21)). The facts of $v^j(t) \in L_{\infty e}[0,T]$ and $\frac{L(s)}{F^\rho(\tau s)}$ being a strictly proper stable transfer function implies that $u^j(t) \in L_{\infty e}[0,T]$ (by (20)). By the facts of $v^j(t), u^j(t) \in L_{\infty e}[0,T]$, we can easily prove that all the internal signals are bounded.

Theorem 1 : Consider the system set-up in Lemma 1. The proposed observer-based MRAILC guarantees the tracking performance and system stability as follows :

(T1) $\lim_{j\to\infty} e_\phi^j(t)^2 = 0$, for all $t \in [0, T]$.

(T2) $\lim_{j\to\infty} |e_a^j(t)| \le e^{-\lambda_1^m t}\varepsilon^\infty$, for all $t \in [0, T]$.

(T3) $\lim_{j\to\infty} |\widetilde{e}^j(t)| \le e^{-\lambda_1^m t}\varepsilon^\infty + \tau k_4$, for all $t \in [0, T]$ and for some $k_4 > 0$.

(T4) Let δ and k_5 be the positive constants such that the transition matrix $\Phi(t)$ of A_c satisfies $|\Phi(t)| \le k_5 e^{-\delta t}$. Then there exists a positive constant k_6 such that $\lim_{j\to\infty} |\widehat{e}^j(t)| \le k_6 \left(\varepsilon^\infty \frac{e^{-\lambda_1^m t} - e^{-\delta t}}{\delta - \lambda_1} + \tau k_4 \frac{1-e^{-\delta t}}{\delta}\right)$, for all $t \in [0, T]$.

(T5) $\lim_{j\to\infty} |e^j(t)| = |e^\infty(t)| \le e^{-\lambda_1^m t}\varepsilon^\infty + \tau k_4 + k_6 \left(\varepsilon^\infty \frac{e^{-\lambda_1^m t} - e^{-\delta t}}{\delta - \lambda_1^m} + \tau k_4 \frac{1-e^{-\delta t}}{\delta}\right)$, for all $t \in [0, T]$.

Proof : Based on Lemma 1, Lemma 2 and Lemma 3, we can conclude the results of (T1) and (T2) by using similar argument for Barbalat's lemma (e.g., Lemma 3.2.6 in [21]). For (T3), substituting (20) into (13), we can find that $e_a^j(t)$ actually satisfies

$$\begin{aligned} e_a^j(t) &= \widetilde{e}^j(t) - y_a^j(t) \\ &= \widetilde{e}^j(t) - \frac{1}{\ell(s)}\left(1 - \frac{1}{F^\rho(\tau s)}\right)\left[v^j(t)\right] \\ &\triangleq \widetilde{e}^j(t) - R^j(t) \end{aligned} \tag{35}$$

Since $v^j(t)$ is bounded and the H_∞ norm of $\|\frac{1}{s}(1 - \frac{1}{F^\rho(\tau s)})\|_\infty = n\tau$ and $\|\frac{s}{\ell(s)}\|_\infty$ is bounded, we can conclude that

$$|R^j| \le \left\|\frac{1}{s}\left(1 - \frac{1}{F^\rho(\tau s)}\right)\right\|_\infty \left\|\frac{s}{\ell(s)}\right\|_\infty \|(v^j)_t\|_\infty \le \tau k_4$$

for some $k_4 > 0$. Taking norms on (35), we find that

$$|\widetilde{e}^j(t)| \le |e_a^j(t)| + |R^j(t)| \le |e_a^j(t)| + \tau k_4$$

As iteration goes to infinity,

$$\lim_{j\to\infty} |\widetilde{e}^j(t)| \le e^{-\lambda_1^m t}\varepsilon^\infty + \tau k_4$$

Finally, the results of (T4) and (T5) can be achieved by the similar technique in [10].

VI. Simulation Example

In this section, we consider a strict-feedback nonlinear system [22] whose dynamic equation is given as follows,

$$\begin{aligned} \dot{x}_1^j(t) &= 0.1(x_1^j(t))^2 + (1 + 0.1\sin(x_1^j(t)))x_2^j(t)) \\ \dot{x}_2^j(t) &= 0.2e^{-x_2^j(t)} + x_1^j(t)\sin(x_2^j(t)) \\ &\quad + (1 + 0.3\cos(x_1^j(t)))u^j(t) \\ y^j(t) &= x_1^j(t) \end{aligned}$$

where $X^j(t) = [x_1^j(t), x_2^j(t)]^\top \in \mathcal{R}^{2\times 1}$ is the state vector of the system, $u^j(t) \in \mathcal{R}$ is the control input, $y^j(t) \in \mathcal{R}$ is the system output. Here the input gain function is chosen as $(1 + 0.3\cos(y^j(t))$ rather than 1 such that it will be more general than the one in [22]. The iteration-varying reference trajectory dynamics is designed as in (2) with $a_1^m = 2, a_2^m = 1, b_1^m = 0, b_2^m = 1, r_m^j(t) = \sin(t) + 0.01\sin(2\pi j/5)$. The control objective is to let system output $y^j(t)$ track $y_m^j(t)$ as close as possible over a finite time interval $[0, 30]$ with only $y^j(t)$ is measurable. Furthermore, we design $det(sI - A_m) = (s+1)^2$, $det(sI - A_o) = (s+2)^2$, $\lambda(s) = (s+1)$, $\ell(s) = s+1$ and $L(s) = (s+1)$, respectively. Since $H(s) = \frac{1}{(s+2)^2}$ and $M(s) = \frac{1}{(s+1)^2}$ are known transfer functions, the constant parameter vector $\Theta = [\theta_1, \theta_2, \theta_3, \theta_4]^\top = [2, -1, -4, 1]^\top$ will be a known constant parameter vector by solving the Laplace algebraic equation (10) so that we can easily design $G_i(s) = \frac{s-1}{(s+1)^2}s^{2-i}$, $i = 1, 2$. In addition, the normalization signal $m^j(t)$ is designed with $\delta_1 = 0.02$, $\delta_2 = 0.04$, $m^j(0) = 5$. To guarantee a satisfied tracking performance, the adaptation algorithms in (22) and (23) are designed with $\gamma_1 = \gamma_2 = 0.5$ and $\beta_1 = \beta_2 = 500$, respectively. Besides, the averaging filter in (20) is given with $\tau = 0.01$. The nice learning performance is shown in Figure 1. It is clear that the effectiveness of the learning controller can be achieved by the proposed AILC.

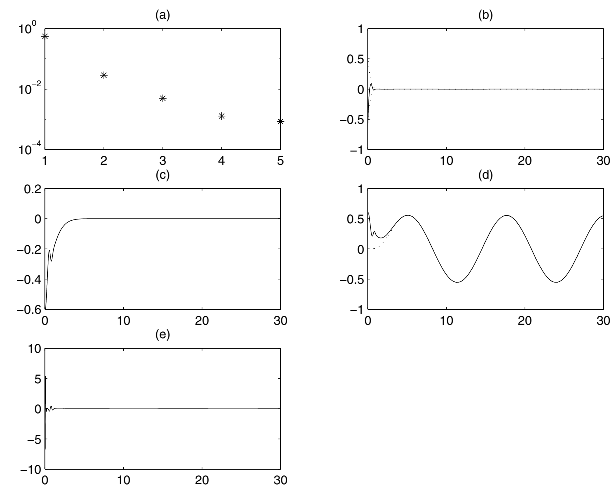

Figure 1 : (a) $\sup_{t\in[0,30]} |e_\phi^j(t)|$ versus j; (b) $e_a^5(t)$ (solid line) and $\pm\phi^5(t)$ (dotted lines) versus t; (c) $y^5(t)$ (solid line) and $y_m^5(t)$ (dotted line) versus t; (d) $e^5(t)$ (solid line) versus t; (e) $u^5(t)$ versus t.

VII. Conclusion

An observer based MRAILC for repeated tracking control is proposed for more general class of uncertain nonlinear systems with non-canonical form and iteration-varying reference trajectories in this paper. Since the system state vector is assumed to be unmeasurable, a state tracking error observer is designed to estimate the unknown system state vector. Besides, a relative degree one output observation error model based on the tracking error observer and a mixed time-domain and s-domain technique is derived for the design of the MRAILC. By using a technique of averaging filter, a filtered fuzzy neural learning component is used to approximate the unknown plant nonlinearities, a robust learning component is designed to compensate for the lumped uncertainties and a stabilization learning component is used to guarantee the boundedness of internal signals, respectively. Finally, we use a Lyapunov like analysis to derive adaptive laws and study stability and learning performance. All adjustable parameters as well as the internal signals will remain bounded. Furthermore, asymptotically convergence of output tracking error to a tunable residual set is shown as iteration goes to infinity.

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