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Developing T_ω fuzzy DEMATEL method for evaluating green supply chain management practices

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Abstract—**This paper develops a new the weakest t-norm (T_ω) fuzzy Decision Making Trial and Evaluation Laboratory (DEMATEL) to evaluate green supply chain management (GSCM). Traditional DEMATEL is a popular method for making decision. In order to apply a realistic environment the fuzzy DEMATEL had to be developed for analyzing expert judgments with preferences, which are often unclear and ambiguous. The fuzzy DEMATEL usually adopts α-cut arithmetic operations to calculate results. However, general fuzzy DEMATEL model has not really handled fuzzy cause and effect relationships due to the accumulating phenomenon of fuzziness of the α-cut arithmetic. In the numerical example, using novel T_ω fuzzy DEMATEL find out real cause and effect relationships with fuzzy bounds among criteria of GSCM. The T_ω fuzzy DEMATEL can provide more credible information/results and analysis of across quadrants to evaluate cause and effected relationships, because the decision-maker usually wants to accurately grasp uncertain influential factors in the GSCM.**
Keywords—the weakest t-norm; DEMATEL; GSCM

I. INTRODUCTION

Decision Making Trial and Evaluation Laboratory (DEMATEL) ([5], [4]) is a popular decision-making method. The aim of DEMATEL solve complicated world problems regarding issues such as race, hunger, environmental protection, energy, etc [4]. The DEMATEL method is established on graph theory and matrix tools as common analysis method of systematic elements. These elements will be divided into effected group and causal group through analyzing logical relationship and direct impact relations between systematic elements. The obvious advantage of DEMATEL method is the comprehensive inclusion of several experts' viewpoints in a certain industry and analysis of complicated relations between systematic elements applying visualization structure model. DEMATEL technique has been applied in various problem, ranging from manufacturing planning and control to multi criteria decision making and analyzing world challenging such as administration control systems [10], marketing strategy and customer performance ([11], [25]), safety and security measurement [17], success factors of hospital service quality [12], industry material selection process [23], and multiple criteria decision analysis (MCDM) [19].

Nowadays because of the fuzziness of judgment, some researches combine DEMATEL method with fuzzy set theory to solve the complex system problems. Wu and Lee [26] take a high-tech company as an example, combining DEMATEL method with fuzzy logic to segment a set of competencies into the cause group and the effect group. Lin and Wu [16] proposed the fuzzy DEMATEL for group decision-making under fuzzy environment to the R&D project selection of a Taiwanese company. Chang et al. [1] believe that supplier selection has great impact on supply chain management in many corporations, so they analyze the influence factors for supplier selection in the electronic industry using fuzzy DEMATEL method, and draw a conclusion that stable delivery of goods is the key influence factor. Lin [15] applied fuzzy DEMATEL to green supply chain management (GSCM).

In mechanism of fuzzy DEMATEL, fuzzy arithmetic is one main technique for calculating the total relation fuzzy matrix. Past researches usually adopted the α-cut arithmetic. However, theα-cut arithmetic realizes that the fuzziness of the model complicated calculation was fuzzier due to the accumulating phenomenon of fuzziness of the α-cut arithmetic [2]. Furthermore, the fuzzy interval of result could not been investigated or analyzed due to the unjustifiable fuzziness/fuzzy spread of the α-cut arithmetic in fuzzy DEMATEL. In this paper, the novel T_ω fuzzy DEMATEL is developed for improving shortages of fuzzy DEMATEL. The novel T_ω fuzzy DEMATEL will provide more credible information/results and analysis of across quadrants to evaluate cause and effected relationships. Moreover, in order to evidence the performance, this study examined the T_ω fuzzy DEMATEL to GSCM practices, and compare with traditional fuzzy DEMATEL [16]. The GSCM case can refer to [15]. The remainder of this paper is organized as follows. First is the introduction of the DEMATEL and fuzzy DEMATEL. Fuzzy arithmetic operations are given in Section II. Section III introduces constructs of T_ω fuzzy DEMATEL is introduced. In Section IV, the T_ω fuzzy DEMATEL applied to GSCM case. Some concluding remarks are offered in Section V.

II. T_ω Fuzzy operations

The basic concepts and definition of the weakest t-norm arithmetic operations will be introduced in the section. The definition of fuzzy number has been introduced in Appendix A. In Zadeh's extension principle [27], if we generalize by using a general norm T that replaces the original 'min', and the binary T norm on the interval [0, 1], it is said to be a triangular norm (or called t-norm) if it is associative, commutative, and monotonous in [0, 1] and $T(x, 1) = x$ for every $x \in$ [0, 1]. Then Zadeh's sup-min operator can be written as:

$$(\widetilde{A} \circ \widetilde{B})(z) = \sup_{x \circ y = z} T(\widetilde{A}(x), \widetilde{B}(y)), \tag{1}$$

Moreover, each t-norm may be shown to satisfy the following inequalities:

..,

where

$$T_\omega(a_2, b_2) = \begin{cases} a_2, & \text{if } b_2 = 1, \\ b_2, & \text{if } a_2 = 1, \\ 0, & \text{otherwise}, \end{cases}$$

T_ω is the weakest t-norm. The importance of t-norms, e.g. $\min(a_2, b_2)$, $a_2 \cdot b_2$, $\max(0, a_2 + b_2 - 1)$, $T_\omega(a_2, b_2)$, has been shown in ([3], [9]). The addition, subtraction, multiplication, and division of T_ω fuzzy arithmetic can see these researches ([2], [7], [8], [9],[13], [20]).

Let $\widetilde{A}$ and $\widetilde{B}$ be FN on $\Re$ and be written as (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively. The fuzzy operations of T_ω can be shown as follows:

(1) Addition of T_ω:

$$\begin{aligned} \widetilde{A} +_{T_\omega} \widetilde{B} = {} & (a_2 + b_2 - \max(a_2 - a_1,\ b_2 - b_1), \\ & a_2 + b_2,\ a_2 + b_2 + \max(a_3 - a_2,\ b_3 - b_2)). \end{aligned} \tag{2}$$

(2) Subtraction of T_ω:

$$\begin{aligned} \widetilde{A} -_{T_\omega} \widetilde{B} = {} & (a_2 - b_2 - \max(a_2 - a_1,\ b_3 - b_2), \\ & a_2 - b_2,\ a_2 - b_2 + \max(b_2 - b_1,\ a_3 - a_2)). \end{aligned} \tag{3}$$

(3) Multiplication of T_ω:

$$\widetilde{A} \times_{T_\omega} \widetilde{B} = \begin{cases} (a_2 b_2 - \max((a_2 - a_1) b_2,\ (b_2 - b_1) a_2)),\ a_2 b_2, \\ a_2 b_2 + \max((a_3 - a_2) b_2,\ (b_3 - b_2) a_2))), & \text{for } a_2 > 0,\ b_2 > 0, \\ (a_2 b_2 - \max((a_3 - a_2) b_2,\ (b_3 - b_2) a_2),\ a_2 b_2, \\ a_2 b_2 + \max((a_2 - a_1) b_2,\ (b_2 - b_1) a_2))), & \text{for } a_2 < 0,\ b_2 < 0, \\ (-(a_2 - a_1) b_2,\ 0,\ (a_3 - a_2) b_2), & \text{for } a_2 = 0,\ b_2 > 0, \\ ((a_3 - a_2) b_2,\ 0, -(a_2 - a_1) b_2), & \text{for } a_2 = 0,\ b_2 < 0, \\ (0, 0, 0), & \text{for } a_2 = 0,\ b_2 = 0, \\ (a_2 b_2 - \max((a_2 - a_1) b_2, -(b_3 - b_2) a_2),\ a_2 b_2, \\ a_2 b_2 + \max((a_3 - a_2) b_2,\ -(b_2 - b_1) a_2)). & \text{for } a_2 < 0,\ b_2 > 0\ . \end{cases} \tag{4}$$

(4) Division of T_ω:

$$\widetilde{A} /_{T_\omega} \widetilde{B} = \begin{cases} (a_2 / b_2 - \max((a_2 - a_1) / b_2, a_2 (1/b_2 - 1/b_3)), a_2 / b_2, \\ a_2 / b_2 + \max((a_3 - a_2) / b_2, a_2 (1/b_1 - 1/b_2))) & \text{for } a_2 > 0,\ b_2 > 0 \text{ and } b_1 > 0, \\ (a_2 / b_2 - \max((a_2 - a_3) / b_2, a_2 (1/b_2 - 1/b_1)), a_2 / b_2, \\ a_2 / b_2 + \max((a_1 - a_2) / b_2, a_2 (1/b_3 - 1/b_2))), & \text{for } a_2 < 0,\ b_2 < 0 \text{ and } b_3 < 0, \\ (-(a_2 - a_1) / b_2, 0, (a_3 - a_2) / b_2), & \text{for } a_2 = 0,\ b_2 > 0 \text{ and } b_1 > 0, \\ (-(a_2 - a_3) / b_2, 0, (a_1 - a_2) / b_2), & \text{for } a_2 = 0,\ b_2 < 0 \text{ and } b_3 < 0, \\ (a_2 / b_2 - \max((a_2 - a_3) / b_2, a_2 (1/b_2 - 1/b_3)), a_2 / b_2, \\ a_2 / b_2 + \max((a_1 - a_2) / b_2, a_2 (1/b_1 - 1/b_2))), & \text{for } a_2 > 0,\ b_2 < 0 \text{ and } b_3 < 0, \\ (a_2 / b_2 - \max((a_2 - a_1) / b_2, a_2 (1/b_2 - 1/b_1)) \\ , a_2 / b_2, a_2 / b_2 + \max((a_3 - a_2) / b_2, a_2 (1/b_3 - 1/b_2))), & \text{for } a_2 < 0,\ b_2 > 0 \text{ and } b_1 > 0. \end{cases} \tag{5}$$

III. NOVEL T_ω FUZZY DEMATEL METHOD

The operations of fuzzy DEMATEL [16] usually adopt traditional α-cut arithmetic (Appendix B). However, traditional fuzzy arithmetic has the accumulating phenomenon of fuzziness which will make the fuzzy model/method more complex [2]. This research develops the novel T_ω fuzzy DEMATEL to substitute for the fuzzy DEMATEL technology. The procedures of novel T_ω fuzzy DEMATEL can be illustrated in Figure 1. The ***Step*** 1 (average the assessment) and ***Step*** 2 (initial direct-relation fuzzy matrix) are the same traditional fuzzy DEMATEL which main arrange the expert's assessment to important factors. In following procedures the T_ω operations will be employed.

Step 3 (T_ω Normalizing) The T_ω normalizing direct-relation fuzzy matrix $\widetilde{X}^{T_\omega}$ can be obtained based on equation (6) as following:

$$\widetilde{X}^{T_\omega} = \begin{bmatrix} \widetilde{x_{11}} & \widetilde{x_{12}} & \cdots & \widetilde{x_{1n}} \\ \widetilde{x_{21}} & \widetilde{x_{22}} & \cdots & \widetilde{x_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{x_{n1}} & \widetilde{x_{n2}} & \cdots & \widetilde{x_{nn}} \end{bmatrix}^{T_\omega} \tag{6}$$

$$\begin{aligned} \widetilde{x_{ij}} &= \widetilde{z_{ij}} \div_{T\omega} \max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}} \\ &= (m_{ij} / \max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}} - \max((m_{ij}-l_{ij}) / \max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}}, \\ &\quad m_{ij}(1/\max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}} - 1/\max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}})), \\ &\quad m_{ij} / \max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}}, m_{ij} / \max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}} + \max((r_{ij}-m_{ij}) \\ &\quad / \max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}}, m_{ij}(1/\max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}} - 1/\max_{1\le i\le n}\sum_{j=1}^{n}\widetilde{z_{ij}}))) \end{aligned} \tag{7}$$

Step 4. (Total T_ω relation fuzzy matrix) The total T_ω relation fuzzy matrix can be defined and computedas following:

$$\widetilde{T}^{T_\omega} = \lim_{k\to\infty}(\widetilde{X_1} +_{T_\omega} \widetilde{X_2} +_{T_\omega} \cdots +_{T_\omega} \widetilde{X_k})$$

Then

$$\widetilde{T}^{T_\omega} = \begin{bmatrix} \widetilde{t_{11}} & \widetilde{t_{12}} & \cdots & \widetilde{t_{1n}} \\ \widetilde{t_{21}} & \widetilde{t_{22}} & \cdots & \widetilde{t_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{t_{n1}} & \widetilde{t_{n2}} & \cdots & \widetilde{t_{nn}} \end{bmatrix}^{T_\omega}$$

$$= \begin{bmatrix} \widetilde{x_{11}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{11}}^{-1}) & \widetilde{x_{12}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{12}}^{-1}) & \cdots & \widetilde{x_{1n}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{1n}}^{-1}) \\ \widetilde{x_{21}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{21}}^{-1}) & \widetilde{x_{22}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{22}}^{-1}) & \cdots & \widetilde{x_{2n}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{2n}}^{-1}) \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{x_{n1}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{n1}}^{-1}) & \widetilde{x_{n2}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{n2}}^{-1}) & \cdots & \widetilde{x_{nn}} \times_{T_\omega} (I -_{T_\omega} \widetilde{x_{nn}}^{-1}) \end{bmatrix}^{T_\omega} \tag{8}$$

Step 5. (Producing a T_ω causal diagram) The sum of rows and the sum of columns are respectively denotes as vectors $\widetilde{D}$ and $\widetilde{R}$ within the total T_ω relation fuzzy matrix.

$$\widetilde{D} = \widetilde{t_{1j}} +_{T_\omega} \widetilde{t_{2j}} +_{T_\omega} \cdots +_{T_\omega} \widetilde{t_{nj}} = \left[\sum_{i=1}^{n}\widetilde{t_{ij}}\right]_{1\times n}^{T_\omega} \tag{9}$$

$$\widetilde{R} = \widetilde{t_{i1}} +_{T_\omega} \widetilde{t_{i2}} +_{T_\omega} \cdots +_{T_\omega} \widetilde{t_{in}} = \left[\sum_{j=1}^{n}\widetilde{t_{ij}}\right]_{1\times n}^{T_\omega} \tag{10}$$

Then $\widetilde{D}_i +_{T_\omega} \widetilde{R_i}$ and $\widetilde{D}_i -_{T_\omega} \widetilde{R_i}$ are calculated by T_ω operations based on equations (9) and (10). To finalize the procedure, all calculated $\widetilde{D}_i +_{T_\omega} \widetilde{R_i}$ and $\widetilde{D}_i -_{T_\omega} \widetilde{R_i}$ are also defuzified through suitable defuzification method. $(\widetilde{D}_i +_{T_\omega} \widetilde{R_i})^{def}$ shows how the important the strategic objectives are, and $(\widetilde{D}_i -_{T_\omega} \widetilde{R_i})^{def}$ shows which strategic objectives is cause and which one is effect. If $(\widetilde{D}_i -_{T_\omega} \widetilde{R_i})^{def}$ is positive, the objectives belong to the cause group, otherwise; the objectives belong to the effect group. Moreover, the novel T_ω fuzzy DEMATEL can objectively analysis the fuzzy intervals of $\widetilde{D}_i +_{T_\omega} \widetilde{R_i}$ and $\widetilde{D}_i -_{T_\omega} \widetilde{R_i}$ while the fuzzy interval crosses different quadrants. This is traditional fuzzy DEMATEL could not achieve due to the accumulating phenomenon of fuzziness.

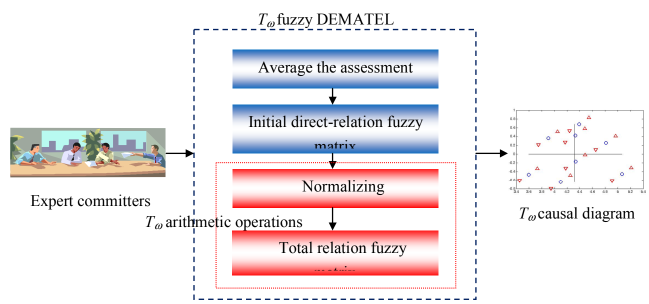


Fig. 1. Procedures of T_ω Fuzzy DEMATEL.

IV. CASE STUDY: GREEN SUPPLY CHAIN MANAGEMENT PRACTICES

Since environmental awareness affects almost all parts of our society and it is a special concern for our industry, nowadays, Companies and their decision makers must consider environment issues in all of their administrative activities [18]. It is critically considered as a new concept that integrates environment factors into supply chain management through product design, material purchasing, …, etc. ([6], [22], [24]). The influential factors of GSCM should be objectively analyzed by decision-making methods. Therefore, the GSCM case [15] is examined by novel T_ω fuzzy DEMATEL in this study. Table I shows the strategic objectives of GSCM which includes eight influence factors. By adopting a fuzzy triangular number, a fuzzy DEMATEL exertion will be in place by expressing different degrees of influences or causalities in crisp DEMATEL, with five linguistic terms as {NO, VL, L, H, VH} and their corresponding positive triangular fuzzy numbers. These linguistic terms are shown in Table II. Then, this GSCM case collects some Assessment Data Fuzzy Matrix in hand. As an example, initial-direct fuzzy matrix from the assessment data of GSCM's experts are depicted in Table III.

TABLE I. THE LIST OF STRATEGIC OBJECTIVES OF GSCM

Strategic objectives
C_1: Green purchasing
C_2: Green design
C_3: Supplier/customer collaboration
C_4: Recover and reuse of used products
C_5: Environmental performance
C_6: Economic performance
C_7: Recover and reuse of regulation
C_8: Stakeholders' pressures

TABLE II. CORRESPONDING RELATION BETWEEN LINGUISTIC VARIABLE AND FUZZY NUMBER.

Linguistic variable	Corresponding TFNs
No Influence (NO)	(0, 0.1, 0.3)
Very Low Influence (VL)	(0.1, 0.3, 0.5)
Low Influence (L)	(0.3, 0.5, 0.7)
High Influence (H)	(0.5, 0.7, 0.9)
Very High Influence (VH)	(0.7, 0.9, 1.0)

TABLE III. THE ASSESSMENT DATA FUZZY MATRIX OF A GENERAL MANAGER.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	–	L	VL	L	H	L	VL	VL
C_2	L	–	NO	NO	H	L	VL	VL
C_3	VL	NO	–	VL	H	H	NO	H
C_4	H	VL	VL	–	H	L	VL	H
C_5	VL	VL	L	L	–	L	L	NO
C_6	VL	L	VL	L	VL	–	VL	VL
C_7	H	L	L	L	H	L	–	L
C_8	L	L	VL	L	H	H	L	–

–: means triangular fuzzy number (0, 0, 0).

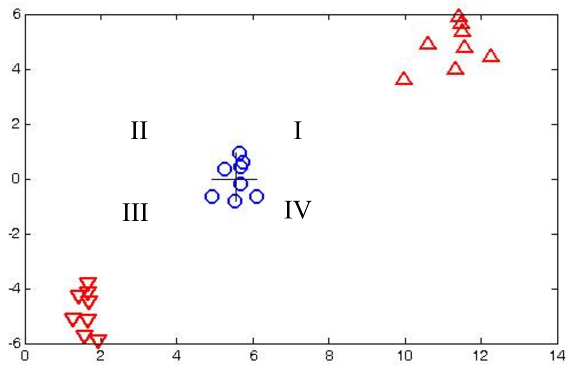
Following steps 2 to 4 of novel T_ω fuzzy DEMATEL, the total-relation fuzzy matrix $\widetilde{T}^{T_\omega}$ will be produced. Table IV shows the total-relation fuzzy matrix $\widetilde{T}^{T_\omega}$.

TABLE IV. THE TOTAL-RELATION FUZZY MATRIX $\widetilde{T}^{T_\omega}$.

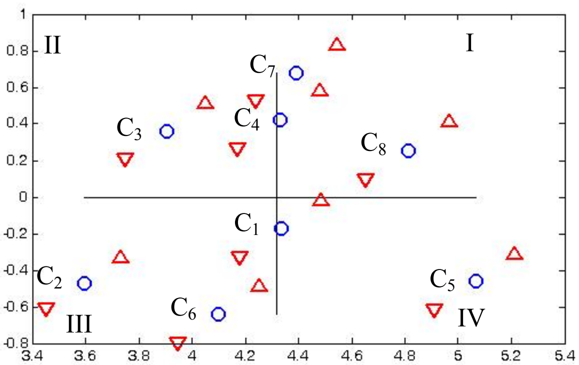
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	(0.16, 0.25, 0.34)	(0.14, 0.26, 0.38)	(0.12, 0.22, 0.31)	(0.14, 0.26, 0.38)	(0.20, 0.35, 0.49)	(0.17, 0.29, 0.41)	(0.13, 0.22, 0.31)	(0.15, 0.23, 0.28)
C_2	(0.12, 0.23, 0.35)	(0.11, 0.20, 0.28)	(0.09, 0.12, 0.16)	(0.11, 0.14, 0.17)	(0.15, 0.29, 0.43)	(0.13, 0.22, 0.30)	(0.10, 0.19, 0.27)	(0.11, 0.18, 0.23)
C_3	(0.17,0.26, 0.35)	(0.15, 0.21, 0.28)	(0.13, 0.22, 0.31)	(0.15, 0.24, 0.33)	(0.21, 0.36, 0.50)	(0.18, 0.30, 0.42)	(0.13, 0.23, 0.32)	(0.16, 0.18, 0.45)
C_4	(0.18, 0.34, 0.48)	(0.17, 0.26, 0.36)	(0.14, 0.24, 0.33)	(0.16, 026, 0.35)	(0.23, 0.38, 0.53)	(0.20, 0.32, 0.44)	(0.15, 0.25, 0.34)	(0.17, 0.34, 0.48)
C_5	(0.18, 0.28, 0.37)	(0.16, 0.26, 0.35)	(0.14, 0.26, 0.38)	(0.15, 0.28, 0.40)	(0.23, 0.32, 0.41)	(0.20, 0.32, 0.44)	(0.14, 0.27, 0.38)	(0.18, 0.33, 0.47)
C_6	(0.13, 0.22, 0.31)	(0.12, 0.24, 0.35)	(0.10, 0.19, 0.28)	(0.11, 0.20, 0.29)	(0.17, 0.26, 0.34)	(0.14, 0.23, 0.32)	(0.10, 0.17, 0.22)	(0.13, 0.22, 0.30)
C_7	(0.20, 0.35, 0.5)	(0.17, 0.30, 0.43)	(0.15, 0.27, 0.40)	(0.17, 0.29, 0.42)	(0.25, 0.40, 0.54)	(0.21, 0.34, 0.46)	(0.15, 0.26, 0.35)	(0.19, 0.33, 0.43)
C_8	(0.20, 0.32, 0.44)	(0.18, 0.30, 0.43)	(0.15, 0.25, 0.34)	(0.17, 0.29, 0.42)	(0.25, 0.40, 0.54)	(0.21, 0.36, 0.51)	(0.15, 0.28, 0.40)	(0.19, 0.33, 0.44)

To access the casual relationships between strategic objectives, the study will calculate $\widetilde{D}_i + \widetilde{R}_i$, $\widetilde{D}_i - \widetilde{R}_i$, $(\widetilde{D}_i + \widetilde{R}_i)^{def}$, and $(\widetilde{D}_i - \widetilde{R}_i)^{def}$ with different fuzzy operations (α-cut and T_ω) in which $\widetilde{D}_i$ and $\widetilde{R}_i$ are the sum of row and the sum of columns of total-relation fuzzy matrix respectively.

Fig. 2 shows the cause and effect diagram with different fuzzy arithmetic. In order to analyze the cause and effect, the four quadrants can be divided by average of $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ in x-axis and $(\widetilde{D}_i - \widetilde{R}_i)^{def}$ in y-axis. The I quadrant means the core factors while the factor $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ is larger than average of $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ and $(\widetilde{D}_i - \widetilde{R}_i)^{def}$ is positive. The core factors (C_4, C_7, C_8) should firstly consider or improve in GSCM practices. The II quadrant means driving factors while the factor $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ is smaller than average of $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ and $(\widetilde{D}_i - \widetilde{R}_i)^{def}$ is positive. The driving factor (C_3) only affect few others factors. The III quadrant means independent factors while the factor $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ is smaller than average of $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ and $(\widetilde{D}_i - \widetilde{R}_i)^{def}$ is negative. The independent factors (C_2, C_6) can be individually handled due the factors may not affect others factors. The IV quadrant means affected factors while the factor $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ is larger than average of $(\widetilde{D}_i + \widetilde{R}_i)^{def}$ and $(\widetilde{D}_i - \widetilde{R}_i)^{def}$ is negative. The affected factors (C_1, C_5) should be indirectly considered or improved. If manager can effectively control the factors of I and II quadrant, the affected factors also can be control well. Some phenomena can be observed: (1) both types of fuzzy arithmetic provide reliability analysis of the fuzzy DEMATEL through defuzification method (2) The fuzzy spreads of fuzzy DEMATEL method always lager than T_ω Fuzzy DEMATEL method due to excessive accumulating phenomenon of fuzziness. (3) Traditional fuzzy DEMATEL may not effectively analyze based on fuzzy bounds.



(a) Fuzzy DEMATEL method



(b) Novel T_{ω} fuzzy DEMATEL method

Fig. 2. Comparison of cause and effect diagram

V. Conclusion

The fuzzy DEMATEL has been proven to be an effective tool in evaluating cause and effected relationships. Thus, this paper has proposed a T_{ω} fuzzy DEMATEL technology for improving the fuzzy DEMATEL. The proposed T_{ω} fuzzy DEMATEL has been successfully applied to a GSCM with useful and more credit results obtained.

Particularly, it is revealed that: (1) both types of fuzzy arithmetic provide reliability analysis of the fuzzy DEMATEL through suitable defuzification method, (2) fuzzy DEMATEL model has not really handled fuzzy cause and effect relationships while the fuzzy interval crosses different quadrants in GSCM practices (3) in the tradition fuzzy DEMATEL model the α-cut arithmetic realizes that the fuzziness of the model's complicated calculation was fuzzier than that of the T_{ω} fuzzy arithmetic due to the accumulating phenomenon of fuzziness of the α-cut arithmetic, and (4) T_{ω} arithmetic gives a smaller fuzziness/fuzzy spread because it takes account of only the maximal fuzziness encountered and calculates it in the operation. In this GSCM case, the T_{ω} arithmetic provides more credible (or conservative estimation) information/results to the amount of fuzziness, because the decision-maker usually wants to accurately grasp uncertain information/data in the GSCM.

Therefore, it can be concluded that due to the inclusion of fuzziness (uncertainties) and an arithmetic method, the T_{ω} fuzzy DEMATEL may change the decision-making process in that more accurate evaluations are possible. The T_{ω} fuzzy DEMATEL may provide more conservative information. Further research may investigate a number of issues remaining. This issue may extend its realizable application. T_{ω} fuzzy DEMATEL also can integrate others evaluation's methods such as Balance Scorecard (BSC) and analytic hierarchy process (AHP). Moreover, other industries also may consider using the T_{ω} fuzzy DEMATEL as a decision support to investigate cause and effect relation.

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Appendix

A. *Fuzzy Numbers*

Let $\widetilde{A}$ be a fuzzy set or a fuzzy number (FN) on $\Re$ and written as (a_1, a_2, a_3), where a_2 thus denotes the *mode* and a_1 and a_3 denote the *left and right bounds* of $\widetilde{A}$ respectively, with the *membership function* $\widetilde{A}(x)$ that defines the membership grades of elements $x \in \Re$ to $\widetilde{A}$:

$$\widetilde{A}(x)=\begin{cases}0, & x<a_1,\\ L((x-a_1)/(a_2-a_1)), & a_1\le x\le a_2,\\ R((a_3-x)/(a_3-a_2)), & a_2\le x\le a_3,\\ 0, & x>a_3,\end{cases}$$

where L and R respectively denote the *left and right shape functions* of $\widetilde{A}(x)$, and L and R are necessary for satisfying the below conditions:

(1) $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$ and for $x\in(0,1)$, $L(x)\in(0,1)$, $R(x)\in(0,1)$.

(2) L and R are non-increasing and continuous functions from [0, 1] to [0, 1].

In particular, fuzzy numbers with triangular membership functions or called triangular fuzzy numbers (TFNs) may be shown as

$$\widetilde{A}(x)=\begin{cases}0, & x<a_1,\\ (x-a_1)/(a_2-a_1), & a_1\le x\le a_2,\\ (a_3-x)/(a_3-a_2), & a_2\le x\le a_3,\\ 0, & x>a_3,\end{cases}$$

B. *α-cut Arithmetic Operations*

For an interval of confidence or α-cut at level $\alpha\in(0, 1)$, an ordinary subset A_{α} of $\tilde{A}$ may be defined:

$$A_{\alpha}=\left\{x\middle|\widetilde{A}(x)\ge\alpha\right\}, \tag{B1}$$

or given for a TFN

$$A_{\alpha}=[a_1^{(\alpha)},a_2^{(\alpha)}]=[a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)], \tag{B2}$$

where $a_1^{(\alpha)}$ and $a_2^{(\alpha)}$ respectively denote the *lower* and *upper bounds* of A_α.

In fuzzy arithmetic approaches, Zadeh's sup-min operator can be stated as

$$(\widetilde{A} \circ \widetilde{B})(z) = \sup_{x \circ y = z} \min(\widetilde{A}(x), \widetilde{B}(y)), \tag{B3}$$

where $\circ$ denotes any arithmetic operation. Equation (1) may be shown to be performed in the equivalent manner by using the α-cuts of fuzzy numbers and interval arithmetic (e.g. see [21]). The resulting fuzzy arithmetic may be called the α-cut (fuzzy) arithmetic. In α-cut arithmetic, addition/subtraction, multiplication/division, and others may be performed at each α on the intervals of confidence by interval arithmetic (e.g. also see [14]). The α-cuts of fuzzy numbers and the interval arithmetic can be seen as follows:

Addition of α-cut arithmetic: Following the interval operation let $\tilde{A}$ and $\tilde{B}$ be two FNs and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$, $\alpha \in$ (0, 1]. $\forall \tilde{A}, \tilde{B} \in \Re$, we can write

$$\begin{aligned}&A_\alpha + B_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] + [b_1^{(\alpha)}, b_2^{(\alpha)}] = \\ &[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}], \forall \alpha \in (0,1].\end{aligned} \tag{B4}$$

Subtraction of α-cut arithmetic: It can be defined as: $\forall\ \tilde{A}, \tilde{B}$ and $\alpha \in$ (0, 1],

$$\begin{aligned}&A_\alpha - B_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] - [b_1^{(\alpha)}, b_2^{(\alpha)}] \\ &= [a_1^{(\alpha)} - b_2^{(\alpha)}, a_2^{(\alpha)} - b_1^{(\alpha)}],\ \forall \alpha \in (0,1].\end{aligned} \tag{B5}$$

Multiplication of α-cut arithmetic: It can be defined as: $\forall\ \tilde{A}, \tilde{B}$ and $\alpha \in$ (0, 1],

$$\begin{aligned}&A_\alpha \times B_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] \times [b_1^{(\alpha)}, b_2^{(\alpha)}] \\ &= [\min(a_1^{(\alpha)} b_1^{(\alpha)}, a_1^{(\alpha)} b_2^{(\alpha)}, a_2^{(\alpha)} b_1^{(\alpha)}, a_2^{(\alpha)} b_2^{(\alpha)}), \\ &\max(a_1^{(\alpha)} b_1^{(\alpha)}, a_1^{(\alpha)} b_2^{(\alpha)}, a_2^{(\alpha)} b_1^{(\alpha)}, a_2^{(\alpha)} b_2^{(\alpha)})],\ \forall \alpha \in (0,1].\end{aligned} \tag{B6}$$

Division of α-cut arithmetic: It can be defined as: $\forall\ \tilde{A}, \tilde{B}$ and $\alpha \in$ (0, 1],

$$\begin{aligned}&A_\alpha \div B_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] \div [b_1^{(\alpha)}, b_2^{(\alpha)}] \\ &= [\min(a_1^{(\alpha)} / b_1^{(\alpha)}, a_1^{(\alpha)} / b_2^{(\alpha)}, a_2^{(\alpha)} / b_1^{(\alpha)}, a_2^{(\alpha)} / b_2^{(\alpha)}), \\ &\max(a_1^{(\alpha)} / b_1^{(\alpha)}, a_1^{(\alpha)} / b_2^{(\alpha)}, a_2^{(\alpha)} / b_1^{(\alpha)}, a_2^{(\alpha)} / b_2^{(\alpha)})], \\ &\forall \alpha \in (0,1].\end{aligned} \tag{B7}$$

REFERENCES

[1] B. Chang, C.W. Chang, and C.H. Wu, "Fuzzy DEMATEL method for developing supplier selection criteria," *Expert Systems with Applications*, vol. 38, pp. 1850–1858, 2011,.

[2] P.-T. Chang, P.-F. Pai, K.-P. Lin, and M.-S. Wu, "Applying fuzzy arithmetic to system dynamics for the customer-producer-employment model," International Journal of Systems Science, vol. 37, pp. 673–698, 2006.

[3] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, 1980.

[4] E. Fontela and A. Gabus, *The DEMATEL Observer*, DEMATEL 1976 Report, Switzerland, Geneva, Battelle Geneva Research Center, 1976.

[5] A. Gabus and E. Fontela, *Perceptions of the World Problematique: Communication Procedure, Communicating with Those Bearing Collective Responsibility*, DEMATEL Report No. 1, Battelle Geneva Research Center, Geneva, Switzerland, 1973.

[6] K. Green, B. Morton, and S. New, "Purchasing and environment management: interaction, policies, and opportunities," *Business Strategy and the Environment*, vol. 5, pp. 188–197, 1996.

[7] D.H. Hong, "Some results on the addition of fuzzy intervals," *Fuzzy Sets and Systems*, vol. 122, pp. 349–352, 2001.

[8] D.H. Hong, "Shape preserving multiplications of fuzzy numbers," *Fuzzy Sets and Systems*, vol. 123, pp. 81–84, 2001.

[9] D.H. Hong, "Fuzzy measures for a correlation coefficient of fuzzy numbers under T_W (the weakest t-norm)-based fuzzy arithmetic operations," *Information Sciences*, vol. 176, pp. 150–160, 2006.

[10] S. Hori and Y. Shimizu, "Designing methods of human interface for supervisory control systems," Control Eng. Pract., vol. 7, pp. 1413–1419, 1999.

[11] C. Y. Hsu, W. H. Lee, K. T. Chen, and G. H. Tzeng, "FMCDM with fuzzy DEMATEL approach for customers' choice behavior model," *International Journal of Fuzzy Systems*, vol. 9, pp. 236–246, 2007.

[12] S. Jiunn-I, W. Hsin-Hung, and H. Kuan-Kai, "A DEMATEL method in identifying key success factors of hospital service quality," *Knowl-Based Syst. Arch.*, vol. 23, pp. 277–282, 2010.

[13] A. Kolesárová, "Additive preserving the linearity of fuzzy intervals," Tatra Mountains Mathematical Publications, vol. 6, pp. 75–81, 1995.

[14] A. Kaufmann and M.M. Gupta, *Fuzzy Mathematical Models in Engineering and Management Science*, North-Holland, Amsterdam, 1988.

[15] R.-J. Lin, "Using fuzzy DEMATEL to evaluate the green supply chain management practices," *Journal of Cleaner production*, vol. 40, pp. 32–39, 2013.

[16] C.J. Lin and W.W. Wu, "A causal analytical method for group decision-making under fuzzy environment," *Expert Systems with Applications*, vol. 34, pp. 205–213, 2008.

[17] J.J.H. Liou, G.H. Tzeng, and H.C. Chang, "Airline safety measurement using a hybrid model," *J. Air Transp. Manag.*, vol. 13, pp. 243–249, 2007.

[18] A. Marcus and A. Fremeth, "Green management matters regardless," *The Academy of Management Perspectives*, vol. 23, pp.17–26, 2009.

[19] J. Michnik, "Weighted Influence Non-linear Gauge System (WINGS) – An analysis method for the systems of interrelated components," *European Journal of Operational Research*, vol. 3, pp. 536–544, 2013.

[20] R. Mesiar, "Shape preserving additions of fuzzy intervals," *Fuzzy Sets and System*, vol. 86, pp. 73–78, 1997.

[21] M. Mizumoto and K. Tanaka, The four operations of arithmetic on fuzzy numbers, S*ystems-Computers-Controls*, vol. 7, pp. 73–81, 1976.

[22] J. Sarkis, "Performance measurement for green supply chain management," *Benchmarking: An International Journal of*, vol. 12, pp. 330–353, 2005.

[23] C. Shih-Chi, C.C. Sun, and A. Herchan, "The DEMATEL approach applied to solar cell industry material selection process in Taiwan," *Session Interd. Manage. Sem.*, vol. 15, pp. 253–267, 2011.

[24] S.K. Srivastava, "Green supply-chain management: a state-of-the-art literature review." *International Journal of Management Review*, vol. 9, pp. 53–80, 2007.

[25] T. Wei, "The causal relationship between technologies attributes," International Journal of Management Science, vol. 23, pp. 782–793, 2006.

[26] W.W. Wu and Y.T. Lee, "Developing global managers' competencies using the fuzzy DEMATEL method," *Expert Systems with Applications*, vol. 32, pp. 499–507, 2007.

[27] L.A. Zadeh, Fuzzy sets, *Information and Control*, vol. 8, pp. 338–353, 1965.