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Improvement on Fuzzy-Model-Based Stability Criteria of Nonlinear Networked Control Systems

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Abstract—**This paper is concerned with an improvement on fuzzy-model-based stability criteria of nonlinear networked control systems (NCSs) with time-varying transmission delays and transmission intervals. The real-time distribution of input delays is taken into account and modeled as a dependent and nonidentically distributed process, which leads to a randomly switched Takagi-Sugeno (T-S) fuzzy model with multiple input-delay subsystems for the nonlinear NCSs. Based on an improveed Lyapunov-Krasovskii method, which appropriately exploits the real-time distribution of input delays in estimating cross-product integral terms and the characteristics of T-S fuzzy model, new sufficient conditions are derived for the deterministic asymptotical stability of the overall systems. Numerical examples are presented to substantiate the effectiveness and advantage of our results.**

I. Introduction

NETWORKED control systems (NCSs) are systems in which control loops are closed over a communication network which may be share with other applications. NCSs have many advantages over point-to-point wired conventional control systems, including lower cost, simpler installation and maintenance, and better system flexibility. However, data exchange via network inevitably results in transmission delays, transmission intervals, packet losses, variable sampling and so on, which will degrade system performance and even lead to closed-loop instability [1], [2]

Recently, fuzzy-model-based stabilization of nonlinear NCSs with time-varying transmission delays and intervals has attracted much attention [3]-[15], where a nonlinear NCS is represented as a Takagi-Sugeno (T-S) fuzzy model which is a weighted sum of some simple linear subsystems, and then parallel distribution compensation scheme can be applied to design controller with this model. Input-delay approach was found to be widely adopted in existing results [3]-[15].To the best of our knowledge, it is still a real challenge to obtain effective and less conservative performance conditions based on T-S fuzzy model for a nonlinear NCS with time-varying transmission delays and intervals.

This paper is to give an improvement on fuzzy-model-based stabilization of a nonlinear NCS with time-varying transmission delays and intervals, and the resulting controller design method is formulated as a nonlinear convex optimization problem with LMI constraints. Our improvement comes from three aspects. Firstly, the real-time distribution of input delays is taken into account. It is worth noting that distribution of input delays can be exploited to reduce conservatism in the derived results [14], [16-[19]. Here, the real-time distribution of input delays is modeled as a dependent and nonidentically distributed (d.n.d.) process rather than i.i.d. processes in each transmission interval, which results in a randomly switched T-S fuzzy model of the nonlinear NCS. Secondly, the characteristics of T-S fuzzy model are more fully considered. We disclose an important characteristic through a new form of T-S fuzzy model, which is helpful to obtain a tighter estimation in Lyapunov-Krasovskii method. Thirdly, an improved Lyapunov-Krasovskii method is proposed to take advantage of the real-time distribution of input delays and the characteristics of T-S fuzzy model. Compared with the mean- square stability and performance conditions obtained by the existing average-dwell-time approaches [16] and direct Lyapunov-Krasovskii methods [14], [17]-[19], our derived conditions are of a deterministic sense, independent of average dwell time of the switched subsystems, and less conservative.

This paper is organized as follows. The problem formulation is presented in Section II. The main results are given in Section III, which include a random-delay T-S fuzzy model of the nonlinear NCS, stability analysis. Numerical examples are presented in Section IV to substantiate the effectiveness and advantage of our proposed method, and Section V concludes this paper.

Notations: Denote $x_t(\theta)=x(t+\theta)$ where $\theta\in[-\overline{\eta},0]$. The infinitesimal operator is denoted as

$$\mathcal{L}V(t,x_t,\dot{x}_t)=\lim_{\vartheta\to 0^+}\frac{1}{\vartheta}\mathbb{E}\{V(t+\vartheta,x_{t+\vartheta},\dot{x}_{t+\vartheta})-V(t,x_t,\dot{x}_t)\big|\mathrm{X}_t\}$$

where $X_t=\{x_t\}$. Γ (Γ^T) is an operator located at the left (right) side of a matrix for deleting the zero rows (columns) of the matrix.

II. Problem Formulation

Assume in the nonlinear NCS that: a) The sensor is clock- or event-driven while the fuzzy controller and the zero-order holder (ZOH) are only event-driven, and all of them are connected through communication network; b) Feedback and control signals are transmitted in single packet, respectively, and time-varying transmission delays and intervals exist in

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the closed loop; c) Only new packets are accepted by the controller and the ZOH, and $u(t)=0$ before the first updating instant or when input missing occurs.

Consider a continuous-time nonlinear system which can be described as a T-S fuzzy model. The ith rule of the model is expressed in the following IF-THEN form:

R^i : IF $\theta_1(t)$ is W_1^i and, …, and $\theta_g(t)$ is W_g^i, THEN

$$\dot{x}(t)=A_i x(t)+B_i u(t) \quad (1)$$

where $i=1,2,\cdots,r$ and r is the number of IF-THEN rules; $\theta(t)=[\theta_1(t),\theta_2(t),\cdots,\theta_g(t)]$ is premise variable vector and W_j^i ($i=1,2,\cdots,r$, $j=1,2,\cdots,g$) are fuzzy sets; $x(t)\in\mathbb{R}^{n_x}$ and $u(t)\in\mathbb{R}^{n_u}$ are state and input vectors, respectively; A_i and B_i are system and input matrices, respectively. Then the global dynamics of T-S fuzzy model is given as

$$\dot{x}(t)=\sum_{i=1}^{r}\mu_i(\theta(t))[A_i x(t)+B_i u(t)] \quad (2)$$

where $\mu_i(\theta(t))$ denotes the normalized membership function satisfying

$$\mu_i(\theta(t))=v_i(\theta(t))\Big/\sum_{j=1}^{r}v_j(\theta(t)),\ v_i(\theta(t))=\prod_{j=1}^{g}v_{ij}(\theta_j(t))$$

with $v_{ij}(\theta_j(t))$ being the grade of membership of $\theta_j(t)$ in W_j^i. It is seen that $\mu_i(\theta(t))\ge 0$ ($i=1,\cdots,r$), $\sum_{i=1}^{r}\mu_i(\theta(t))=1$.

With the state feedback $x(s_k)$, where $k\in\mathbb{N}$ denotes the number of new data packets received by the ZOH and s_k the associated sampling instant, the jth rule for the fuzzy controller is given as follows:

R^j : IF $\theta_1(t)$ is W_1^j and, …, and $\theta_g(t)$ is W_g^j, THEN

$$u(t)=K_j x(s_k),\ t\in[t_k,t_{k+1}) \quad (3)$$

where $j=1,2,\cdots,r$, and t_k is the kth updating instant of the ZOH. Thus the defuzzified output of the fuzzy controller at the ZOH is given by

$$u(t)=\sum_{j=1}^{r}\mu_j(\theta(t))K_j x(s_k),\ t\in[t_k,t_{k+1}). \quad (4)$$

It follows from the T-S fuzzy model (2) and the fuzzy controller (4) that:

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta(t))\mu_j(\theta(t))[A_i x(t)+B_i K_j x(s_k)] \quad (5)$$

where $t\in[t_k,t_{k+1})$.

For $t\in[t_k,t_{k+1})$, let $\tilde{\tau}_k=t_k-s_k$ denote the associated time-varying transmission delays, $\eta_k(t)=t-s_k$ the input delays induced by time-varying transmission delays and intervals, and $\tilde{\eta}_k=t_{k+1}-s_k$ the maximum input delays. It is a standing assumption that there exist scalars $\overline{\tau}>0$, $\underline{\tau}\ge 0$ and $\overline{\eta}>0$ such that

$$\underline{\tau}\le\tilde{\tau}_k\le\overline{\tau},\ \underline{\tau}\le\tilde{\tau}_k\le\eta_k(t)<\tilde{\eta}_k<\overline{\eta}. \quad (6)$$

Then the T-S fuzzy model (5) is rewritten as follows

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta(t))\mu_j(\theta(t))[A_i x(t)+B_i K_j x(t-\eta_k(t))] \quad (7)$$

where $t\in[t_k,t_{k+1})$.

Before developing main results, we introduce the following lemmas.

Lemma 1 [20]: For any constant matrix $M>0$, scalars $\delta_2>\delta_1>0$, and vector function $\chi:[t-\delta_2,t-\delta_1]\to R^{n_x}$ such that the following integral is well defined, then

$$-(\delta_2-\delta_1)\int_{t-\delta_2}^{t-\delta_1}\dot{\chi}^T(s)M\dot{\chi}(s)ds \le \begin{bmatrix}\chi(t-\delta_1)\\ \chi(t-\delta_2)\end{bmatrix}^T\begin{bmatrix}-M & M\\ * & -M\end{bmatrix}\begin{bmatrix}\chi(t-\delta_1)\\ \chi(t-\delta_2)\end{bmatrix}.$$

Lemma 2 [20]: For any constant matrix $M>0$, scalars $\delta_2\ge\delta(t)\ge\delta_1>0$, and vector function $\chi:[t-\delta_2,t-\delta_1]\to R^{n_x}$ such that the following integral is well defined, then

$$-(\delta_2-\delta_1)\int_{t-\delta_2}^{t-\delta_1}\dot{\chi}^T(s)M\dot{\chi}(s)ds \le \begin{bmatrix}\chi(t-\delta_1)\\ \chi(t-\delta(t))\\ \chi(t-\delta_2)\end{bmatrix}^T\begin{bmatrix}\Xi_{11} & \Xi_{12} & 0\\ * & -3M & (1+\sigma)M\\ * & * & -(1+\sigma)M\end{bmatrix}\begin{bmatrix}\chi(t-\delta_1)\\ \chi(t-\delta(t))\\ \chi(t-\delta_2)\end{bmatrix}$$

where $\sigma=(\delta(t)-\delta_1)/(\delta_2-\delta_1)\in[0,1]$. $\Xi_{11}=-(2-\sigma)M$, $\Xi_{12}=(2-\sigma)M$.

III. MAIN RESULTS

A. A Random-Delay T-S Fuzzy Model of the Nonlinear NCS

For $t\in[t_k,t_{k+1})$, note that the time series $\{\eta_k(s),s\in[t_k,t]\}$ can be deemed as the realization of a random variable $h(t)$ with a uniform probability density function (PDF) $f_{h(t)}(\bullet)$ on $[\tilde{\tau}_k,\eta_k(t)]$ given as

$$f_{h(t)}(d)=1/(t-t_k),\ d\in[\tilde{\tau}_k,\eta_k(t)].$$

Then a d.n.d. stochastic process $\{h(t),t\in[t_k,t_{k+1})\}$ is obtained for every transmission interval $[t_k,t_{k+1})$. It is noted that the associated uniform PDFs $f_{h(t)}$ are satisfied in real time with the evolution of $\eta_k(t)$ along $t\in[t_k,t_{k+1})$. With the stochastic process $\{h(t),t\in[t_k,t_{k+1})\}$, the T-S fuzzy model (7) can be rewritten into a more general case as

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta(t))\mu_j(\theta(t))[A_i x(t)+B_i K_j x(t-h(t))] \quad (8)$$

where $t\in[t_k,t_{k+1})$.

Remark 1: For the i.i.d. models of input delays in [14], [16]-[19], it is obvious that the identical-distribution assumption can not be met in real time with the evolution of $\eta_k(t)$ along $t\in[t_1,+\infty)$, and the independence assumption is also inappropriate with respect to the time series $\{\eta_k(s),s\in[t_k,$

$t]\}$ where $t \in [t_k, t_{k+1})$.

Suppose that:

$$[\underline{\tau}, \overline{\tau}) = \bigcup_{i=1}^{m_1} [\overline{\eta}_{i-1}, \overline{\eta}_i), \ [\overline{\tau}, \overline{\eta}) = \bigcup_{i=1}^{m_2} [\overline{\eta}_{m_1+i-1}, \overline{\eta}_{m_1+i}). \quad (9)$$

To exploit the real-time distribution of input delays in theoretical development, we introduce $m = m_1 + m_2$ indicator functions $\alpha_i(t) \in \{1, 0\}$, $i = 1, 2, \cdots, m$, of the form

$$\alpha_i(t) = \begin{cases} 1, & h(t) \in [\overline{\eta}_{i-1}, \overline{\eta}_i) \\ 0, & h(t) \notin [\overline{\eta}_{i-1}, \overline{\eta}_i) \end{cases}, \ t \in [t_k, t_{k+1}) \quad (10a)$$

to denote the occurrences of $h(t) \in [\overline{\eta}_{i-1}, \overline{\eta}_i)$. For $t \in [t_k, t_{k+1})$, the probability mass functions (PMFs) of $\alpha_i(t) = 1$, $i = 1, \cdots, m$, are given as

$$\overline{\alpha}_i(t) = \int_{t_k}^{t} \frac{1}{t - t_k} \alpha_i(s) ds. \quad (10b)$$

It is seen that $\overline{\alpha}_i(t)$ are time-varying functions and satisfy $\sum_{i=1}^{m} \overline{\alpha}_i(t) = 1$.

Then based on (10), the model (8) is generalized into a randomly switched T-S fuzzy model of the following form for $t \in [t_k, t_{k+1})$:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\theta(t)) \mu_j(\theta(t)) \\ &\quad \times \sum_{l=1}^{m} \alpha_l(t) [A_i x(t) + B_i K_j x(t - h_l(t))] \\ &= \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\theta(t)) \mu_j(\theta(t)) 2^{-sign(|j-i|)} \\ &\quad \times \sum_{l=1}^{m} \alpha_l(t) [\overline{A}_{ij} x(t) + \overline{B}_{ij} x(t - h_l(t))] \end{aligned} \quad (11)$$

where $\overline{A}_{ij} = A_i + sign(j-i) A_j$, $\overline{B}_{ij} = B_i K_j + sign(j-i) B_j K_i$, and $h_l(t) \in [\overline{\eta}_{l-1}, \overline{\eta}_l)$, $l = 1, 2, \cdots, m$, are uncertain bounded time-varying variables without distribution information. So $h_l(t) \in [\overline{\eta}_{l-1}, \overline{\eta}_l)$ is a more general variable than $h(t) \in [\overline{\eta}_{l-1}, \overline{\eta}_l)$.

Remark 2: The randomly switched T-S fuzzy model in (11) integrates the real-time distribution of input delays, and is ready to be analyzed by an appropriate Lyapunov-Krasovskii method. With the integration of the real-time distribution of input delays, the model (11) is more special than the ones in (7) and [3]-[7], which helps to reduce the conservatism in the derived results. Moreover, the T-S fuzzy model is rewritten in a new form in the second equality of (11), which is useful to obtain a tighter estimation in Lyapunov-Krasovskii method.

B. Stability Analysis

Suppose that $[0, \underline{\tau})$ is divided as follows:

$$[0, \underline{\tau}) = \bigcup_{i=1}^{m_0} [\underline{\tau}_{i-1}, \underline{\tau}_i). \quad (12)$$

Based on (9) and (12), a Lyapunov-Krasovskii functional is constructed using delay decomposition approach for $t \in [t_k, t_{k+1})$ as follows:

$$\begin{aligned} V(t, x_t, \dot{x}_t) &= V_1(t, x_t) + V_2(t, x_t) + V_3(t, \dot{x}_t) \\ V_1(t, x_t) &= x^T(t) P x(t) \\ V_2(t, x_t) &= \sum_{i=1}^{m_0} \int_{t-\underline{\tau}_i}^{t-\underline{\tau}_{i-1}} x^T(s) Q_{0i} x(s) ds \\ &\quad + \sum_{i=1}^{m} \int_{t-\overline{\eta}_i}^{t-\overline{\eta}_{i-1}} x^T(s) Q_i x(s) ds \\ V_3(t, \dot{x}_t) &= \sum_{i=1}^{m_0} \Delta_{0i} \int_{-\underline{\tau}_i}^{-\underline{\tau}_{i-1}} \int_{t+\theta}^{t} \dot{x}^T(s) R_0 \dot{x}(s) ds d\theta \\ &\quad + \sum_{i=1}^{m} \Delta_i \int_{-\overline{\eta}_i}^{-\overline{\eta}_{i-1}} \int_{t+\theta}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds d\theta \end{aligned} \quad (13)$$

where $P > 0$, $Q_{0i} > 0$, $i = 1, 2, \cdots, m_0$, $Q_i > 0$, $i = 1, \cdots, m$, and $R_i > 0$, $i = 0, 1, \ 2, \cdots, m$, are of appropriate dimensions, and $\Delta_{0i} = \underline{\tau}_i - \underline{\tau}_{i-1}$, $\Delta_i = \overline{\eta}_i - \overline{\eta}_{i-1}$. Based on (13), we have the following stability conditions:

Theorem 1: Given (6), (9), (10), (12) and controller gain matrices K_j, $j = 1, 2, \cdots, r$, the T-S fuzzy model (7) is asymptotically stable, if there exist matrices $P > 0$, $Q_{0i} > 0$, $i = 1, 2, \cdots, m_0$, $Q_i > 0$, $i = 1, 2, \cdots, m$, $R_i > 0$, $i = 0, 1, \cdots, m$, of appropriate dimensions and a scalar $\varepsilon > 0$ such that:

$$\Gamma \left(\Omega^{ij} \Big|_{\overline{\alpha}_{l_1}(t)=1, \overline{\alpha}_{l_2}(t)=0, l_2=1,2,\cdots,m, l_2 \neq l_1} \right) \Gamma^T < 0,$$

$$l_1 = 1, 2, \cdots, m_1, \ i, j = 1, 2, \cdots, r, \ i \leq j \quad (14a)$$

$$\Gamma \left(\Omega^{ij} \Big|_{\substack{\overline{\alpha}_{l_1}(t)=0, l_1=1,2,\cdots,m_1, \\ \overline{\alpha}_{l_2}(t)=(\overline{\eta}_{l_2} - \overline{\eta}_{l_2-1})/(\overline{\eta}_m - \overline{\eta}_{m_1}), l_2=m_1+1,\cdots,m}} \right) \Gamma^T < 0,$$

$$i, j = 1, 2, \cdots, r, \ i \leq j. \quad (14b)$$

for all $\sigma_l = 0, 1$, $l = 1, 2, \cdots, m$, where

$$\Omega^{ij} = \Omega_1^{ij} + \Omega_2 + \Omega_3 + \Omega_4^{ij} + \Omega_5^{ij} + \Omega_6$$

$$\Omega_1^{ij} = \begin{bmatrix} \Omega_{11}^{ij} & 0_{n_x \times m_0 n_x} & \Omega_{12}^{ij} \\ * & & \\ * & 0_{(m_0+2m)n_x \times (m_0+2m)n_x} & \end{bmatrix}$$

$$\Omega_{11}^{ij} = P\overline{A}_{ij} + \overline{A}_{ij}^{\ T} P, \Omega_{12}^{ij} = P\overline{B}_{ij} [col\{\overline{\alpha}_1 I, 0, \overline{\alpha}_2 I, 0, \cdots, \overline{\alpha}_m I, 0\}]^T$$

$$\begin{aligned} \Omega_2 &= 2^{sign(j-i)} diag\{Q_{01}, -Q_{01} + Q_{02}, \cdots, -Q_{0,m_0-1} + Q_{0m_0}, \\ &\quad -Q_{0m_0} + Q_1, 0, -Q_1 + Q_2, \cdots, 0, -Q_{m-1} + Q_m, 0, -Q_m\} \end{aligned}$$

$$\Omega_3 = 2^{sign(j-i)} [\Omega_{31} + \hat{\Omega}_{32} + \hat{\Omega}_{32}^T + \hat{\Omega}_{33} + \hat{\Omega}_{33}^T]$$

$$\hat{\Omega}_{32} = \left[\begin{array}{cc} 0_{(m_0+2m)n_x \times n_x} & \Omega_{32} \\ \hline \multicolumn{2}{c}{0_{n_x \times (m_0+2m+1)n_x}} \end{array} \right], \hat{\Omega}_{33} = \left[\begin{array}{cc} 0_{(m_0+2m-1)n_x \times 2n_x} & \Omega_{33} \\ \hline \multicolumn{2}{c}{0_{2n_x \times (m_0+2m+1)n_x}} \end{array} \right]$$

$$\begin{aligned} \Omega_{31} &= diag\{-R_0, -R_0 - R_0, \cdots, -R_0 - R_0, -R_0 - (\overline{\alpha}_1 \rho_1 - \sigma_1 \overline{\alpha}_1 \rho_1 \\ &\quad + 1) R_1, -3\overline{\alpha}_1 \rho_1 R_1, -(\sigma_1 \overline{\alpha}_1 \rho_1 + 1) R_1 - (\overline{\alpha}_2 \rho_2 - \sigma_2 \overline{\alpha}_2 \rho_2 \\ &\quad + 1) R_2, -3\overline{\alpha}_2 \rho_2 R_2, \cdots, -(\sigma_{m-1} \overline{\alpha}_{m-1} \rho_{m-1} + 1) R_{m-1} - (\overline{\alpha}_m \rho_m \\ &\quad - \sigma_m \overline{\alpha}_m \rho_m + 1) R_m, -3\overline{\alpha}_m \rho_m R_m, -(\sigma_m \overline{\alpha}_m \rho_m + 1) R_m\} \end{aligned}$$

$$\begin{aligned} \Omega_{32} &= diag\{R_0, \cdots, R_0, (2 - \sigma_1) \overline{\alpha}_1 \rho_1 R_1, (1 + \sigma_1) \overline{\alpha}_1 \rho_1 R_1, \\ &\quad \cdots, (2 - \sigma_m) \overline{\alpha}_m \rho_m R_m, (1 + \sigma_m) \overline{\alpha}_m \rho_m R_m\} \end{aligned}$$

$$\Omega_{33}=diag\{\underbrace{0,\cdots,0}_{m_0},(1-\bar{\alpha}_1\rho_1)R_1,0,\cdots,(1-\bar{\alpha}_{m-1}\rho_{m-1})R_{m-1},0,(1-\bar{\alpha}_m\rho_m)R_m\}$$

$$\Omega_4^{ij}=2^{-sign(j-i)}\sum_{l=1}^{m}\bar{\alpha}_l\Omega_{4l}^{ij}\Lambda(\Omega_{4l}^{ij})^{\mathrm{T}}$$

$$\Omega_{4l}^{ij}=col\{\bar{A}_{ij}^T,0_{(m_0+2(l-1))n_x\times n_x},\bar{B}_{ij}^T,0_{(2m-2l+1)n_x\times n_x}\}$$

$$\Lambda=\sum_{i=1}^{m_0}(\underline{\tau}_i-\underline{\tau}_{i-1})^2R_0+\sum_{i=1}^{m}(\bar{\eta}_i-\bar{\eta}_{i-1})^2R_i$$

$$\rho_l=\bar{\alpha}_l(t)\Big/\sum\nolimits_{i=l}^{m}\bar{\alpha}_i(t),l=1,2,\cdots,m.$$

Proof: Along the trajectories of the model (11), it follows that for $t\in[t_k,t_{k+1})$

$$\mathcal{L}V(t,x_t,\dot{x}_t)=\mathcal{L}V_1(t,x_t)+\mathcal{L}V_2(t,x_t)+\mathcal{L}V_3(t,\dot{x}_t) \tag{15a}$$

$$\begin{aligned}\mathcal{L}V_1(t,x_t)&=2x^T(t)P\sum_{i=1}^{r}\sum_{j=i}^{r}\mu_i(\theta(t))\mu_j(\theta(t))\\&\quad\times\sum_{l=1}^{m}\bar{\alpha}_l(t)[\bar{A}_{ij}x(t)+\bar{B}_{ij}x(t-h_l(t))]\end{aligned} \tag{15b}$$

$$\begin{aligned}\mathcal{L}V_2(t,x_t)&=\sum_{i=1}^{m_0}\zeta_{\tau_i}^T(t)\begin{bmatrix}Q_{0i}&0\\ *&-Q_{0i}\end{bmatrix}\zeta_{\tau_i}(t)\\&\quad+\sum_{i=1}^{m}\zeta_{\eta_i}^T(t)\begin{bmatrix}Q_i&0\\ *&-Q_i\end{bmatrix}\zeta_{\eta_i}(t)\end{aligned} \tag{15c}$$

$$\begin{aligned}\mathcal{L}V_3(t,\dot{x}_t)&\le\mathbb{E}\{\dot{x}^T(t)\Lambda\dot{x}(t)\\&\quad-\sum_{i=1}^{m_0}\Delta_{0i}\int_{t-\underline{\tau}_i}^{t-\underline{\tau}_{i-1}}\dot{x}^T(s)R_0\dot{x}(s)ds\\&\quad-\sum_{i=1}^{m}\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}}\dot{x}^T(s)R_i\dot{x}(s)ds\big|\mathrm{X}_t\}\end{aligned} \tag{15d}$$

where $\zeta_{\tau_i}(t)=col\{x(t-\underline{\tau}_{i-1}),x(t-\underline{\tau}_i)\}$, $\zeta_{\eta_i}(t)=col\{x(t-\bar{\eta}_{i-1}),x(t-\bar{\eta}_i)\}$.

With the model (11), it follows that for $t\in[t_k,t_{k+1})$

$$\begin{aligned}&\mathbb{E}\{\dot{x}^T(t)\Lambda\dot{x}(t)\big|\mathrm{X}_t\}\\&=\mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\tilde{i}=1}^{r}\sum_{\tilde{j}=1}^{r}\mu_i(\theta(t))\mu_j(\theta(t))\mu_{\tilde{i}}(\theta(t))\mu_{\tilde{j}}(\theta(t))\\&\quad\times\{\nu\sum_{l=1}^{m}\alpha_l(t)(\zeta_{h_l}^{ij}(t))^T\Phi^T\}\Lambda\{\tilde{\nu}\sum_{l=1}^{m}\alpha_l(t)\Phi\zeta_{h_l}^{\tilde{i}\tilde{j}}(t)\}\big|\mathrm{X}_t\}\\&\le\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta(t))\mu_j(\theta(t))2^{-2sign(|j-i|)}\\&\quad\times\sum_{l=1}^{m}(\zeta_{h_l}^{ij}(t))^T\Phi^T\bar{\alpha}_l(t)\Lambda\Phi\zeta_{h_l}^{ij}(t)\\&=\sum_{i=1}^{r}\sum_{j=i}^{r}\mu_i(\theta(t))\mu_j(\theta(t))2^{-sign(j-i)}\\&\quad\times\sum_{l=1}^{m}(\zeta_{h_l}^{ij}(t))^T\Phi^T\bar{\alpha}_l(t)\Lambda\Phi\zeta_{h_l}^{ij}(t)\end{aligned} \tag{16}$$

where $\zeta_{h_l}^{ij}(t)=col\{x(t),x(t-h_\mu(t))\}$, $\Phi=\begin{bmatrix}\bar{A}_{ij}&\bar{B}_{ij}\end{bmatrix}$, $\nu=2^{-sign(|j-i|)}$, $\tilde{\nu}=2^{-sign(|\tilde{j}-\tilde{i}|)}$.

For $t\in[t_k,\ t_{k+1})$, it follows by Lemmas 1 and 2 that for $i=1,2,\cdots,m_0$

$$\mathbb{E}\{-\Delta_{0i}\int_{t-\underline{\tau}_i}^{t-\underline{\tau}_{i-1}}\dot{x}^T(s)R_0\dot{x}(s)ds\big|\mathrm{X}_t\}\le\zeta_{\tau_i}^T(t)\begin{bmatrix}-R_0&R_0\\ *&-R_0\end{bmatrix}\zeta_{\tau_i}(t) \tag{17}$$

and for $i=1,2,\cdots,m$

$$\begin{aligned}&\mathbb{E}\{-\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}}\dot{x}^T(s)R_i\dot{x}(s)ds\big|\mathrm{X}_t\}\\&=\rho_i\mathbb{E}\{-(\alpha_i(t)+1-\alpha_i(t))\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}}\dot{x}^T(s)R_i\dot{x}(s)ds\big|\mathrm{X}_t\}\\&\quad+(1-\rho_i)\mathbb{E}\{-\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}}\dot{x}^T(s)R_i\dot{x}(s)ds\big|\mathrm{X}_t\}\\&\le\rho_i\xi_i^T(t)\mathbb{E}\{\alpha_i(t)\Psi+(1-\alpha_i(t))\Theta\big|\mathrm{X}_t\}\xi_i(t)\\&\quad+(1-\rho_i)\xi_i^T(t)\Theta\xi_i(t)\\&=\xi_i^T(t)\Pi\xi_i(t).\end{aligned} \tag{18}$$

where $\xi_i(t)=col\{x(t-\bar{\eta}_{i-1}),x(t-h_i(t)),x(t-\bar{\eta}_i)\}$, $\sigma_i=[(h_i(t)-\bar{\eta}_{i-1})/\Delta_i]\in[0,1]$.

$$\Pi=\begin{bmatrix}\tilde{\Pi}&(2-\sigma_i)\bar{\alpha}_i\rho_iR_i&(1-\bar{\alpha}_i\rho_i)R_i\\ *&-3\bar{\alpha}_i\rho_iR_i&(1+\sigma_i)\bar{\alpha}_i\rho_iR_i\\ *&*&-(\sigma_i\bar{\alpha}_i\rho_i+1)R_i\end{bmatrix}$$

$$\tilde{\Pi}=-(\bar{\alpha}_i\rho_i-\sigma_i\bar{\alpha}_i\rho_i+1)R_i$$

$$\Psi=\begin{bmatrix}-(2-\sigma_i)R_i&(2-\sigma_i)R_i&0\\ *&-3R_i&(1+\sigma_i)R_i\\ *&*&-(1+\sigma_i)R_i\end{bmatrix}$$

$$\Theta=\begin{bmatrix}-R_i&0&R_i\\ *&0&0\\ *&*&-R_i\end{bmatrix}.$$

Combining (15)-(18) yields that for $t\in[t_k,t_{k+1})$

$$\mathcal{L}V(t,x_t,\dot{x}_t)\le\sum_{i=1}^{r}\sum_{j=i}^{r}\mu_i(\theta(t))\mu_j(\theta(t))\xi_{ij}^T(t)\Omega^{ij}\xi_{ij}(t) \tag{19}$$

where $\xi_{ij}(t)=col\{x(t),x(t-\underline{\tau}_1),\cdots,x(t-\underline{\tau}_{m_0}),x(t-h_1(t)),\ x(t-\bar{\eta}_1),\cdots,x(t-h_m(t)),x(t-\bar{\eta}_m)\}$. For all possible pairs of s_k and τ_k under (6): LMIs (14a) guarantee by linear combination that $\Omega^{ij}<0$ for $t\in[t_k,(s_k+\bar{\tau})\wedge\eta_k)$, and LMI (15b) guarantees that $\Omega^{ij}<0$ for $t\in[s_k+\bar{\tau},t_{k+1})$ with $\bar{\alpha}_i(t)=0$, $i=1,2,3,\cdots,\ m_1$, and $\sum\nolimits_{i=m_1+1}^{m}\bar{\alpha}_i(t)=1$. So LMIs (17) under (8) are sufficient to guarantee that $\Omega^{ij}<0$ for $t\in[t_k,t_{k+1})$. Then we obtain from (19) and the real-time PMFs $\bar{\alpha}_i(t)$, $i=1,2,\cdots,m$, of input delays that

$$\mathcal{L}V(t,x_t,\dot{x}_t)=\dot{V}(t,x_t,\dot{x}_t)<0$$

for every $t\in[t_k,t_{k+1})$ along the trajectories of (11), which means that $\mathcal{L}V(t,x_t,\dot{x}_t)=\dot{V}(t,x_t,\dot{x}_t)<-\underline{\lambda}\|x(t)\|^2$ with $\underline{\lambda}=\min\{\lambda(-\Omega^{ij});i,j=1,2,\cdots,r;i\le j\}>0$ and guarantees that the T-S fuzzy model (7) is asymptotically stable in a deterministic sense. The proof of Theorem 1 is completed.

Remark 3: Compared with the model (7) and the ones in [3]-[7] which ignore the real-time distribution of input delays, applying the model (11) in $\mathcal{L}V(t, \mathrm{X}_t, \dot{\mathrm{X}}_t)$ helps to reduce the conservatism of the derived results for the concerned NCS for the presence of the resulting PMFs $\bar{\alpha}_i(t)$, $i = 1, 2, \cdots, m$ in (15b) and (16).

Remark 4: In (16), the new form of the T-S fuzzy model in the second equality of (11) is exploited to obtain a tighter bound in estimating $\mathbb{E}\{\dot{x}^T(t)\Lambda\dot{x}(t)|\mathrm{X}_t\}$, which is less conservative than the existing techniques applied in [3]-[15].

Remark 5: In (18), a new bounding technique is proposed to exploit the real-time distribution of input delays in estimating cross-product integral terms $\mathbb{E}\{-\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}} \dot{x}^T(v)R_i\dot{x}(v)\times dv|\mathrm{X}_t\}$ for $i = 1, 2, \cdots, m$, which is important to ensure the effectiveness of the derived results under the model (11). In the first equality of (18), $h_i(t)$ for $i = 1, 2, \cdots, m$ are similarly treated as multiple delays by proportionally dividing the integral terms $\mathbb{E}\{-\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}} e^{2\lambda(v-t)}\dot{x}^T(v)R_i\dot{x}(v)dv|\mathrm{X}_t\}$ according to ρ_i. That is, the first integral term $\rho_i\mathbb{E}\{-(\alpha_i(t)+1-\alpha_i(t))\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}} e^{2\lambda(v-t)}\dot{x}^T(v)R_i\dot{x}(v)dv|\mathrm{X}_t\}$ at the right side of the equality is related to $h_i(t)$ while the second one $(1-\rho_i)\times \mathbb{E}\{-\Delta_i\int_{t-\bar{\eta}_i}^{t-\bar{\eta}_{i-1}} e^{2\lambda(v-t)}\dot{x}^T(v)R_i\dot{x}(v)dv|\mathrm{X}_t\}$ is related to $h_j(t)$ for $j = i+1, \cdots, m$. To appropriately bound the first integral term, Lemmas 2 and 1 are applied with respect to $\alpha_i(t) = 1$ ($h(t) \in [\bar{\eta}_{i-1}, \bar{\eta}_i)$) and $\alpha_i(t) = 0$ ($h(t) \notin [\bar{\eta}_{i-1}, \bar{\eta}_i)$), respectively. For more about the two bounding techniques, please refer to [20]. Here the essential difference of the bounding technique in (18) from those in [17]-[20] is that the probabilities of $h(t) \in [\bar{\eta}_{i-1}, \bar{\eta}_i)$ and $h(t) \notin [\bar{\eta}_{i-1}, \bar{\eta}_i)$ are further considered in estimating cross-product integral terms.

IV. Numerical Example

Consider the balancing and swing-up problem of an inverted pendulum on a cart [11]. The dynamics of the pendulum motion are given as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{g\sin(x_1(t)) - amlx_2^2(t)\sin(2x_1(t))/2 - a\varpi(t)u(t)}{4l/3 - aml\varpi^2(t)} \end{cases}$$

where $x_1(t)$ denotes the angle (in radians) of pendulum from the vertical, and $x_2(t)$ is the angular velocity. $g = 9.8\mathrm{m}\cdot\mathrm{s}^2$ is the gravity constant, $\varpi(t) = \cos(x_1(t))$, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and $u(t)$ is the force applied to the cart (in Newtons). $a = 1/(m+M)$. Choose $m = 2.0\mathrm{kg}$, $M = 8\mathrm{kg}$, $2l = 1\mathrm{m}$.

The control objective here is to balance the inverted pendulum for the appropriate range $x_1(t) \in (-\pi/2, \pi/2)$ in an NCS with time-varying transmission delays and intervals. The nonlinear system is represented by a two-rule T-S fuzzy model:

R^1: IF $x_1(t)$ is about 0, THEN $\dot{x}(t) = A_1x(t) + B_1u(t)$

R^2: IF $x_1(t)$ is about $\pm\frac{\pi}{2}(|x_1(t)| < \frac{\pi}{2})$, THEN

$$\dot{x}(t) = A_2x(t) + B_2u(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{4l/3 - aml} & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi(4l/3 - aml\phi^2)} & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ \dfrac{-a}{4l/3 - aml} \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ \dfrac{-a\phi}{4l/3 - aml\phi^2} \end{bmatrix}$$

and $\phi = \cos(88^o)$. Membership functions for Rules 1 and 2 are $\mu_1(x_1(t)) = 1 - \frac{1}{0.5\pi}|x_1(t)|$, and $\mu_2(x_1(t)) = 1 - \mu_1(x_1(t))$, respectively.

Given that $\lambda = 0$, $\underline{\tau} = 0$, $m_0 = 0$, $\bar{\tau} = 0$, $m_1 = 0$ $K_1 = [235.3335 \quad 66.0171]$ and $K_2 = [1839.9 \quad 599.3]$. Assume that the interval $[\bar{\tau}, \bar{\eta})$ is uniformly decomposed into m_2 subintervals. It is obtained by Theorem 1 that the maximal values of $\bar{\eta}$ are 0.0185, 0.0219, and 0.0223 for m_2 being 1, 2, and 4, respectively. Based on Theorem 2 in [11], the maximum allowable transmission interval is 0.019 and the associated controller gains

$K_1 = [790.7897 \quad 249.0322]$, $K_2 = [1812.1815 \quad 585.0441]$.

Then it is seen that our numerical results with $m_2 \geq 2$ are better than those of [11]. So our proposed approach is advantageous.

Fig. 1 plots the membership functions, and Fig. 2 plots the state responses of the closed-loop NCS under the three cases with $m_2 = 1, 2, 4$, and the initial state $x(t_0) = [\pi/3, 0]^T$. It is seen that all trajectories are convergent. So it is concluded that our derived results are effective.

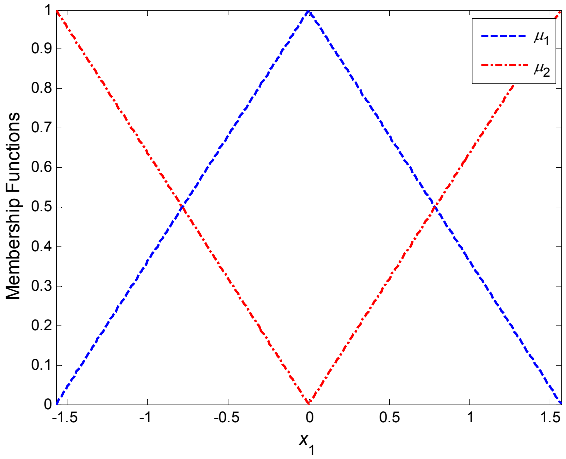

Fig. 1. Membership functions.

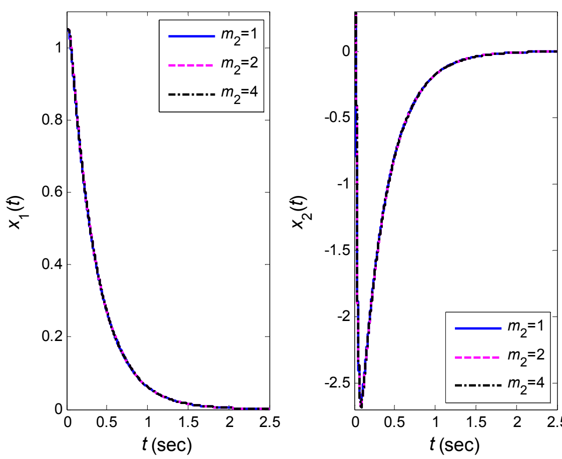

Fig. 2. State responses of the closed-loop system.

V. Conclusions

This paper develops an improvement on fuzzy-model-based stability criteria of nonlinear NCSs with time-varying transmission delays and intervals based on a random delay approach. The real-time distribution of input delays is taken into account and modeled as a d.n.d. process. A randomly switched T-S fuzzy system with multiple input-delay systems is proposed as the closed-loop model of the nonlinear NCSs. An improved Lyapunov-Krasovskii method is proposed to appropriately exploit the real-time distribution of input delays in estimating cross-product integral terms and the characteristics of T-S fuzzy model. New delay-distribution-dependent sufficient conditions are derived for the deterministic asymptotical stability of the nonlinear NCS. Numerical examples are presented to substantiate the effectiveness and advantage of our derived results. The controller design problem would be our future work.

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