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Modelling Dynamic Causal Relationship in Fuzzy Cognitive Maps

Yuan Miao
Centre of Applied Informatics, CES
Victoria University,
Melbourne, Australia
yuan.miao@vu.edu.au

***Abstract*— Most applications of Fuzzy Cognitive Maps (FCM) uses static causal links to connect different concepts. However, a causal impact may take effect immediately, or accumulate over a period of time. Consider two cognitive models with a same causal structure, state sets, decision functions and causal linkage strengths, if their causal links have different dynamics, they can have significantly different or even totally different hidden patterns. This paper proposes an easy to use model to represent the dynamics of causal relationships in fuzzy cognitive maps.**

Keywords—fuzzy cognitive map, dynamic, causal relationship, decision support

I. INTRODUCTION

A. *Fuzzy Cognitive Maps*

Fuzzy Cognitive Map (FCM) [1] is a visualised knowledge model representing human beings' causal knowledge of the external world. A fuzzy cognitive map represents factors as nodes/vertices and their causal relationships as links/arcs among nodes. As Fig. 1.1 shows, both factor A, *Diet Energy*, and factor B, *Physical Activity*, have impact on factor C, *Diabetes Risk*.

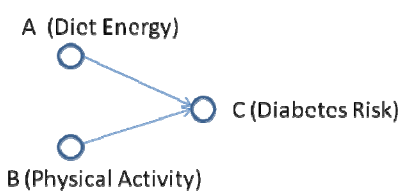

Fig. 1.1 Diabetes Risk factors

Fuzzy cognitive maps use signed links to indicate whether the impact is positive or negative.

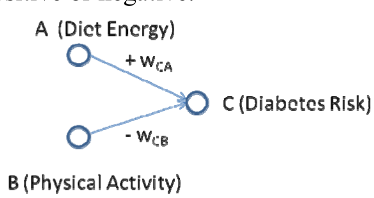

Fig. 1.2 Sign and Weight of FCMs

As shown in Fig.1.2, *Diet Energy* has positive impact on the *Diabetes Risk*, while *Physical Activity* has negative impact on the *Diabetes Risk*. To differentiate the strength of the causal relationship, causal links are modeled with a weight.

If $w_{CA} > w_{CB}$

Diet Energy has a stronger causal relationship with the *Diabetes Risk* than that of *Physical Activity*;

If $w_{CA} < w_{CB}$

Diet Energy has a weaker causal relationship with the *Diabetes Risk* than that of *Physical Activity*.

The factors in a fuzzy cognitive map can take binary/ternary states, or multi-value states [2]. When binary states are used, the *Diet Energy* can be either healthy (-1) or non-healthy (+1). When multi-value fuzzy cognitive map model is used, as exampled in Fig. 1.3, the state of factor A, *Diet*, can be

1	-	Very unhealthy,
0.5	-	Unhealthy ,
-1	-	Very healthy .

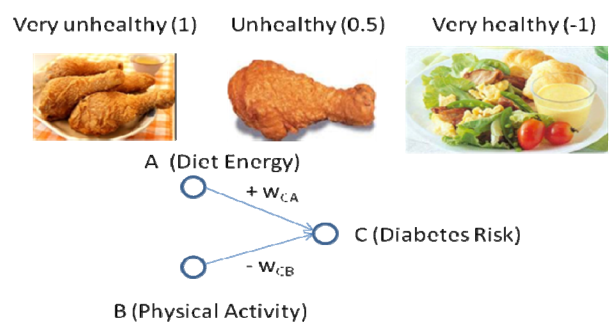

Fig. 1.3 Multi-value of factor states

Each node of a fuzzy cognitive map has a decision function to derive its next state based on the causal impacts it receives. The decision function can take any form but usually it is a threshold function when binary state is used, or a multi-threshold function when multiple states are used, or sigmoid function if continuous state is used.

The inference of fuzzy cognitive map is the calculation by each node of its next state based on the causal inputs. For example,

$$u_C(k) = x_A(k) \times w_{CA} + x_B(k) \times w_{CB} ;$$

$$x_C(k+1) = f_C(u_C(k)) ;$$

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where $x_A(k)$ is the state of node A at time k, $x_B(k)$ is the state of node B at time k, $u_C(k)$ is the total causal impact node C received at time k, f_C is node C's decision function, which decides its next state $x_C(k+1)$.

When there are many nodes causally inter-linked, the inference will be carried out by the nodes in parallel until a static state or a limit cycle is reached.

B. Applications of Fuzzy Cognitive Maps

In recent years, fuzzy cognitive maps are gaining popularity and have been applied in wide areas such as social studies (insurgency, anti-terrorism) [3][4][5], emotion modeling [6], healthcare [7][8][9][10][11], commerce [12][13] and management[14][15] [16]. Lacking proper models for domain experts to capture knowledge on dynamics of the causal relationships in the fuzzy cognitive map, majority of the applications use static linear causal links.

C. Extensions to the Static Weights of Fuzzy Cognitive Maps

A number of researchers have realized that in many cases, the causal relationship has some dynamic characteristics that are essential in the modeling but cannot be represented by a static weight. Here are some important extensions that have been made to fuzzy cognitive map model:

(I) *Extended Fuzzy Cognitive Map* [17] has pointed out that fuzzy cognitive maps modeled with one-weight-links have three drawbacks:

1) could only model linear relationships between concepts/nodes ;
2) lack of concept of time; and
3) cannot deal with co-occurrence of multiple causes, such as conditions .

Extended fuzzy cognitive maps uses weight functions instead of weights for modeling causal relationships, which it *generally* takes a form of sigmoid function. Theoretically a weight function can be in any form thus can model all types of relationships. Though, domain experts have difficulties to use a general function. Sigmoid function based extended fuzzy cognitive maps have a number of issues:

1) it is much more difficult for domain experts in applications to map their familiar weights to sigmoid curves;
2) sigmoid function is monotonous increasing, but in many cases the nonlinearity of the causal links are not monotonous;
3) the co-occurrence of multiple causes shall be handled by the decision function rather than the causal link weight function;
4) when the weight value is a function of the state value of the cause, the weight shall be modeled as a concept and together with the cause having the impact on the target concept. It is much clearer than being modeled as a dependent weight.

(II) *Time modulated weight* [18] is another extension to fuzzy cognitive maps. In this extension, weights can be modulated by the time t:

$$\text{if } x_j(t) > 0 \qquad w_{ij} = w^{+}_{ij} \times (t^{\max} - t) / t^{\max},$$

$$\text{if } x_j(t) < 0 \qquad w_{ij} = w^{-}_{ij} \times (t^{\max} - t) / t^{\max},$$

where w^{+}_{ij} , w^{-}_{ij} are corresponding to the static weight from node j to node i of classic fuzzy cognitive map, $t^{\max}$ is a constant that related to the causal link, and t is the time.

Apparently, this extension has limited capability to model the dynamics of the causal relationship. It is also monotonous like sigmoid function.

(III) *Tangent weight* is used in an extension to fuzzy cognitive maps to model nonlinear weights [19].

$$w_{ji} = \tan(\phi)$$

$$F'_{ji} = \begin{cases} \frac{|w_{ji}|}{1+Qe^{-B|A_j|}}, & for \quad 0 \le w_{ji},\ A_j \le 1 \\ & \quad or \\ & \quad -1 \le w_{ji},\ A_j \le 0 \\ -\frac{|w_{ji}|}{1+Qe^{-B|A_j|}}, & for \quad 0 \le A_j \le 1,\ -1 \le w_{ji} \le 0 \\ & \quad or \\ & \quad -1 \le A_j \le 0,\ 0 \le w_{ji} \le 1 \end{cases}$$

where is the ϕ is a parameter adjusting the nonlinear function tangent angle, F'_{ji} is the impact from node j to node i; A_j is the state of the node j, Q, B are two free parameters to adjust the shape of the decision function curve.

Similarly, tangent weights are monotonous and cannot model nonlinearity of multiple segments or non monotonous. Also, domain experts may feel difficult to map commonly used weights to the tangent function parameters such like ϕ, Q and B.

(IV) *Dynamic casual links* have been proposed in DCN [20] to model dynamic causal relationship using differential functions, transfer functions or state space model. These are typical models used in system science to describe dynamics. Dynamic casual links of DCN can thus model a wide range of dynamics of causal relationships.

However, there are also a number of drawbacks of dynamic causal links in DCN. Firstly differential functions or transfer functions are not models widely used in application domains except those specialized in dynamics modeling such as control systems. It is unlikely that a doctor or a business planner would use differential equations or state space functions to model dynamics of causal relationships. Secondly, fuzzy cognitive maps are mainly for modeling human cognitive knowledge which could involves various decisions. The decision related dynamics can have strong nonlinearity. It is neither flexible nor easy to model strong nonlinear dynamics with DCN dynamic links. Therefore, this model has not been widely applied.

D. Organisation of the Paper

This paper will propose an easy to use dynamic causal links for domain experts to model dynamic causal relationships. It will allow domain experts to have direct mapping from their observations to link weights so that the modeling is not much more difficult than modeling the weight of classic fuzzy cognitive maps.

The rest of the paper is as organized follows: Section 2 proposes the dynamic causal relationship model. Section 3 provides the model of typical dynamic causal links and Section 4 concludes the paper.

II. MODELING DYNAMIC CAUSAL RELATIONSHIPS

A causal impact may take effect immediately, or accumulate over a period of time. Consider two cognitive models with a same causal map structure, state sets, decision functions and causal linkage strengths, if their causal links have different dynamics, they can have significantly different hidden patterns. Therefore, it is important to model the dynamics of the causal relationship in fuzzy cognitive maps.

A dynamic causal link is not able to be modeled with one single weight. To make it easy for domain experts to easily map their observations and not increase much difficulty from the classic fuzzy cognitive map model, this section will propose a multiple weight causal link to model dynamic casual relationships.

A. Fuzzy Cognitive Maps with Dynamic Causal Links

A fuzzy cognitive map with dynamic causal links has the same definition of the map, nodes, state sets and decision functions. The only difference is that the links are modeled with multiple weights rather than a single weight. Each weight is corresponding to some typical characteristics of the observed dynamics so that domain experts can easily model it from his/her observations.

A *fuzzy cognitive map* is modeled as a tuple

$$\mathbf{M} = <\mathbf{V}, \mathbf{A}>, \tag{2.1}$$

where **V** is a set of vertices/nodes representing the concepts and **A** is a set of arcs representing the causal relationships among concepts.

$$\mathbf{V}=\{< v_1,\ f_{v_1},\ \mathrm{S}(v_1)>, <v_2,\ f_{v_2},\ \mathrm{S}(v_2)>,\ldots, <v_n,\ f_{v_n},\ \mathrm{S}(v_n)>\},\tag{2.2}$$

$$\mathbf{A} = \{< a(v_i, v_j),\ w(a(v_i, v_j)) > \mid v_i, v_j \in \mathbf{V}\}$$
$$= \{< a(v_i, v_j),\ (w_{ji}^1,\ w_{ji}^2,\ \ldots,\ w_{ji}^{n_{ji}}),\ y_{ji} > \mid v_i, v_j \in \mathbf{V}\}. \tag{2.3}$$

Vertices. Each vertex in a fuzzy cognitive map corresponds to an important factor (concept) in the real application being modeled. In the above definition, v_i ($i=1, 2, \ldots, n$) are the vertices (concepts); n is the number of vertices/concepts.

States. The state of vertex v_i at time (or step) k is denoted as x_i (k), i=1,2, … , n. The state spaces of concepts are finite value sets. S(v_i) denotes the finite state set of v_i :

$$\mathrm{S}(v_i) = \{ x^1_i, x^2_i, \ldots, x_i^{n_i} \},\ i=1, 2, \ldots, n \tag{2.1.4}$$

where n is the number of the concepts, n_i is the number of state values that concept v_i has. For example, a sequence of states of v_i can be

$$x_i(0)= x^2_i,\ x_i(1)= x_i^{n_i},\ x_i(2)= x^1_i,\ x_i(3)= x^2_i, \ldots .$$

Each concept can have its own value set. The definition of value sets depends on the needs of the system to be modeled. The state space of the fuzzy cognitive map M is defined as the production of the vertices' state space:

$$\mathrm{S}(M)= \prod_{i=1}^{n} S(v_i) \tag{2.5}$$

Causal Links. $a(v_i, v_j)$, or simply a_{ij} , is the arc from v_i to v_j representing that vertex v_i has causal impact on vertex v_j. The impact at time k is represented as $y_{ji}(k)$.

Dynamics of Causal Links. In classic CMs or FCMs, causal links are modeled with one weight, which cannot model the dynamics of the causal relationship. To model the dynamics of the causal relationships, a dynamic causal link $a(v_i, v_j)$ consists of a sequence of weights

$$w(a(v_i, v_j)) = \{ w_{ji}^1,\ w_{ji}^2,\ \ldots,\ w_{ji}^{n_{ji}} \},$$

where n_{ji} is the number of weights to represent the dynamics. The impact y_{ji} is modeled as

$$y_{ji}(k) = \sum_{m=1}^{n_{ji}} w_{ji}^m\ x_i(k-m);\quad i, j=1,2, \ldots, n \tag{2.6}$$

the dynamic impact reaches its stable full impact after n_{ji} time steps. If x_i (k-1)= x_i (k-2)= ... = x_i (k-m) =1, y_{ji} (k) is the normalized stable full impacts, which is termed as the Causal Link Strength of a_{ij} , or simply the Strength of a_{ij}. The strength of a_{ij} is denoted as $w(a_{ji})$ or simply w_{ji} . It can be proven that the w_{ji} is the same as the corresponding FCM weight w_{ji} .

A dynamic causal link is illustrated in Fig. 2.1, where v_1 has a causal impact on v_2, which is a dynamic causal link modeled with three weights: 1, 3 and -2.

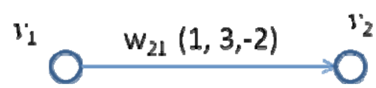


Fig. 2.1 Multiple weights model for dynamic causal link

Mapping domain experts' observation to the multiple weights is fairly straightforward. The three weights indicating that after the cause emerges, the first impact passed onto v2 was equivalent to a weight of 1, and then a stronger impact equivalent to a weight of 3 was added, and so on. More detailed mapping steps will be provided in the Section II- B.

Decision Functions. Each vertex or node can still have its own decision function. Based on the causal inputs, the decision function decides the next state of the concept. The decision function of v_i is denoted as f_{v_i} :

$$x_i(k) = f_{v_i}\left(y_{i1}(k) \quad y_{i2}(k) \quad \cdots \quad y_{in}(k)\right) \quad (2.7)$$

A typical decision function in the area of a cognitive map is a threshold function f:

$$f_v(u) = \begin{cases} x^1 & u < U_1 \\ x^2 & U_1 <= u < U_2 \\ \ldots & \ldots \\ x^R & U_{p1} <= u \end{cases}, \qquad (2.8)$$

where u is the total impact v receives; normally $u = y_{i1}(k) + y_{i2}(k) + \ldots + y_{in}(k)$. It specifies that if the impact falls in a certain threshold range, for example $U_1 <= u < U_2$, the corresponding state of the v is x^2. In some cases, the decision function is also related to the previous state of the vertex.

B. Mapping Observations to Weights- Dynamics Modeling

Domain experts have the knowledge on the characteristics of the dynamic causal relationships. They normally have observed the dynamics often in their work. Mapping this knowledge to multiple weights of dynamic causal link is not much difficult than that of fuzzy cognitive weight mapping. The following two examples illustrate how this mapping is constructed.

Step1: Mapping the causal structure (links)

If a causal relationship exists between two important factors, they can be modeled by the following map:

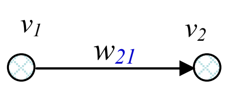


Fig. 2.2 (a) Mapping the causal relationship

This step is the same as the mapping of classic fuzzy cognitive maps. The state space and decision function of v_1 and v_2 can also be determined similarly. For the simplicity of presentation, assume that $S(v_1) = \{0,1\}$, $f_{v2}(u) = u$.

Step2: Mapping the observations to the weights

Suppose when factor v_1 state changes, e.g. x_1 turned from 0 to 1 at k =0, the following impacts on v_2 were observed:

(1) $k = 1$, the impact is Y_1;
(2) $k = 2$, the impact is Y_2;
(3) $k = 3$, the impact is Y_3;
(4) from time 3, the impact has been fully passed onto v_2; i.e. $k > 3$, the impact is kept as Y_3 .

The corresponding observations are shown in Fig. 2.2(b).

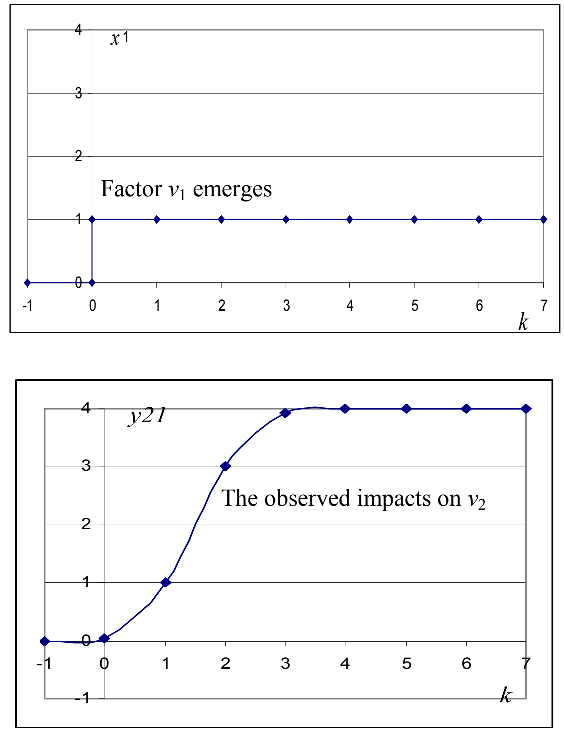


Fig. 2.2 (b) The factor v_1 emerges, and observed impacts on v_2 .

Firstly, let us review the mapping to FCM weights. As FCM has only one weight, the weight will reflect the final full impact. Therefore, $w_{21} = Y_3 = 4$.

Dynamic causal link applies multiple weights to describe the dynamics of the impact. Mapping the observation to the multiple weights is a similar easy modeling as that for classic fuzzy cognitive maps. In this example, the corresponding three gains observed are corresponding to the three weights:

$$w_{21}^1 = Y1, \quad w_{21}^2 = Y2 - Y1 \quad \text{and} \quad w_{21}^3 = Y3 - Y2 \ .$$

The corresponding mapping weights are shown in Fig. 2.2 (c).

Step3: Conclude the impact model corresponding to equation (2.6) using the multiple weights

The corresponding impact of the causal link is

$$y_{21}(k) = w_{21}^1 \times x_1(k\text{-}1) + w_{21}^2 \times x_1(k\text{-}2) + w_{21}^3 \times x_1(k\text{-}3) \ ,$$

and v_2 decides its state based on the impact from v_1:

$$x_2(k) = f_2(y_{21}(k)) \ .$$

In general, v_2 decides its state based on all the impacts from other nodes, as defined in formula (2.6).

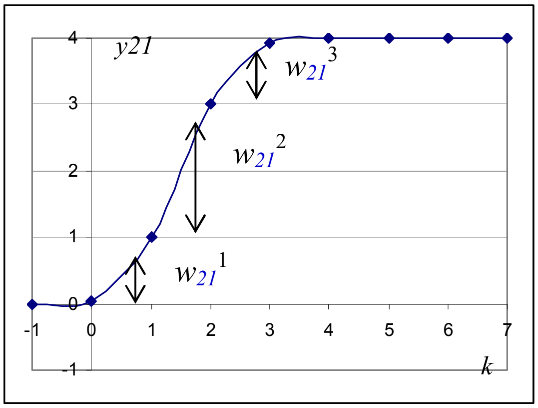


Fig 2.2 (c) Mapping of dynamic causal link weights

C. Comparison with Classic FCMs

The dynamics of causal relationship can play a critical role in causal inference. Consider two cognitive models with a same causal structure, state sets, decision functions and causal linkage strengths, if their causal links have different dynamics, they can have totally different inference outcomes, leading to different decision making. The following example illustrates the difference.

Fig. 2.3 (a) shows a FCM-a with dynamic link model and Fig. 2.3 (b) shows a classic FCM-b of the same knowledge. The only difference is that the classic FCM model does not model the dynamics of the causal relationship from v_1 to v_2 because it has only one weight.

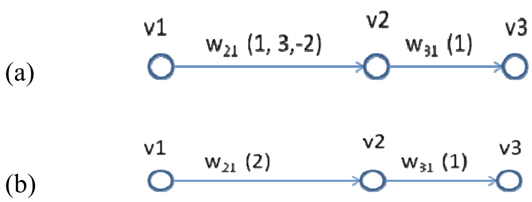


Fig 2.3 Comparing the FCM with and without dynamic link

Suppose both FCM-a and FCM-b have the same state sets:

state set of v_1 is

$S(v_1)=\{0, 1\}$,

state set of v_2 is

$S(v_2)=\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$, and

state set of v_3 is

$S(v_3)=\{-1, 0, 1\}$.

Suppose both FCM-a and FCM-b have the same decision functions:

$$f_{v2}(u) = u,$$

$$f_{v3}(u) \begin{cases} = 1 & \text{if } u >= 3, \\ = 0 & \text{if } 1 <= u < 3, \\ = -1 & \text{if } u < 1, \end{cases}$$

The only difference is that FCM-a have a dynamic causal link modeled by multiple weights

$$w_{21} = (1, 3, -2),$$

where FCM-b has only one weight so it could not model the dynamics but only the full impact which is 2:

$$w_{21} = 2.$$

Both FCM-a and FCM-b has the same weight for v_2 to v_3:

$$w_{32} = 1.$$

Suppose both FCM-a and FCM-b have the same initial values:

$$x_1(0) = 1, x_2(0) = 0, x_3(0) = 0 .$$

The causal inference of FCM –a and FCM-b can be carried out similarly except that the FCM-a needs to calculate the causal impact using the multiple weights (formula (2.6)).

For FCM-a

$$x_1(1) = 1, x_2(1) = 1, x_3(1) = 0$$

$$x_1(2) = 1, x_2(2) = 4, x_3(2) = 0$$

$$x_1(3) = 1, x_2(3) = 2, \boldsymbol{x_3(3) = 1}$$

$$x_1(4) = 1, x_2(4) = 2, \boldsymbol{x_3(4) = 1}$$

…….

while for FCM-b

$$x_1(1) = 1, x_2(1) = 1, x_3(1) = 0$$

$$x_1(2) = 1, x_2(2) = 2, x_3(2) = 0$$

$$x_1(3) = 1, x_2(3) = 2, \boldsymbol{x_3(3) = 0}$$

$$x_1(4) = 1, x_2(4) = 2, \boldsymbol{x_3(4) = 0}$$

…….

We can see that because FCM-b cannot model the dynamics of the causal relationship, the transit state of v_2 , which has shot up to 4, was not captured. Thus the state of v_3 remained at 0.

Although FCM-a and FCM-b have the same causal structure, state set values, decision functions of all nodes, but they have totally different final decision point for v_3. Without being able to model the dynamics of causal relationship, classic fuzzy cognitive map can lead to wrong decision making.

III. TYPICAL CAUSAL RELATIONSHIP DYNAMICS

Section II has shown that modeling dynamics of causal relationship with multiple weights is straightforward and is not more difficult for domain experts than modeling with one static weight in classic fuzzy cognitive maps. An example has shown that without modeling the dynamics, the inference can give a wrong decision support, where v_3 had a wrong final state. In this section, a number of typical dynamics are given for presenting dynamic causal relationships.

A. Delayed Casual Links

Delayed causal links are a type of causal links where impacts are passed on after a certain period of delay from the emergence of the cause. For a delay impact from factor v_i to factor v_j, domain experts can simply model it with a number of zero weights followed by the final weight, which is equivalent to a weight and a delay time parameter. For example, if v_1 has a delayed causal link to v_2 with delay time k_{21}=4. The multiple weights are (0, 0, 0, w_{21}) then the impact from v_1 to v_2 is:

$$y_{21}(k) = w_{21} \times x_1(k-4) \ , \tag{3.1}$$

and the decision making of v_2 is :

$$x_2(k) = f_2(y_{21}(k)) \ . \tag{3.2}$$

If k_{21}=1, it becomes a typical FCM link (The impact is passed on in the immediately following step). Note: we will use f_i for f_{v_i} in the rest of the paper if no ambiguity is caused.

In general, if the delay time from v_i to v_j is k_{ji}. :

$$y_{ji}(k) = w_{ji} \times x_i(k - k_{ji}) \ , \tag{3.3}$$

$$x_j(k) = f_j\left(y_{j1}(k), y_{j2}(k), \ ..., y_{jn}(k) \right) \ . \tag{3.4}$$

Fig. 3.1 illustrates a causal impact from v_1 to v_2, with a time delay of 4, and w_{21}=1 .

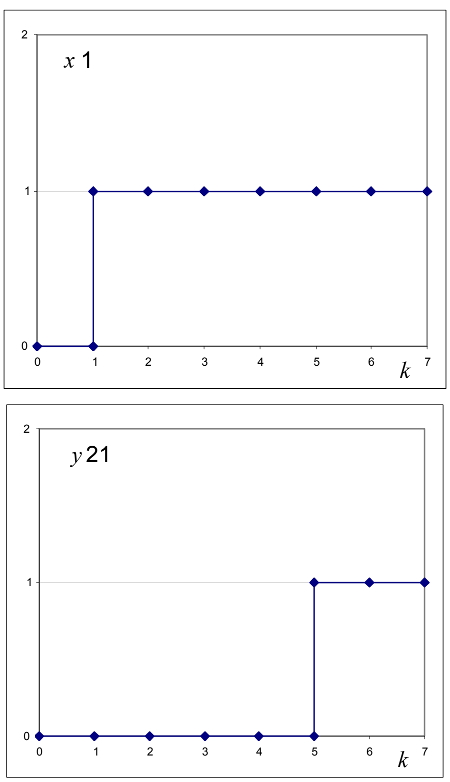


Fig. 3.1 A causal impact of time delay = 4

B. Gradually built up causal links-Linear links

In addition to delays, gradually built up causal links are the most common type of dynamic impact links. In fact, most of the impacts are gradually built up in the real world. If the dynamic transition is important in describing the characteristics of the causal link, the dynamics should be modeled. Using multiple weights to model the dynamics is easy and straightforward for domain experts. If the full impact can be established in N time slots, then the domain expert simply needs N weights to describe the impact gain or loss in each time slot. There are four typical gradually built up causal links, covered in Section II B, C, D and E.

Linear dynamic causal links model the dynamics that the causal impact builds up at a constant speed. Fig. 3.2 shows the transition of a linear dynamic link of total time slots N=3. If the total impact weight is w (which is also the FCM weight), the mapping of observation to the multiple weights are illustrated in Fig. 3.2 (a). The three weights are all equal to 1/3 w:

$$w_{ji}^1 = \ 1/3\ w\ , \ \ w_{ji}^2 = \ 1/3\ w\ , w_{ji}^3 = \ 1/3\ w\ .$$

The impact model is

$$y_{ji}(k) = \frac{w}{3}\ x_i(k-1) + \frac{w}{3}\ x_i(k-2) + \frac{w}{3}\ x_i(k-3)\ .$$

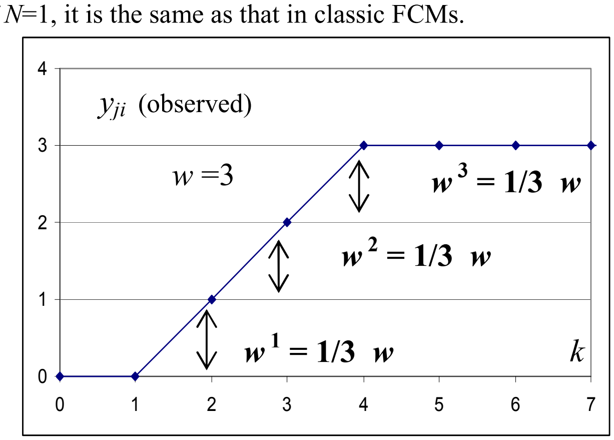
When the weights are negative, the impacts are also negatively linear. Fig. 3.2 (b) shows a case of

$$y_{ji}(k) = -\frac{w}{3}\ x_i(k-1) - \frac{w}{3}\ x_i(k-2) - \frac{w}{3}\ x_i(k-3).$$

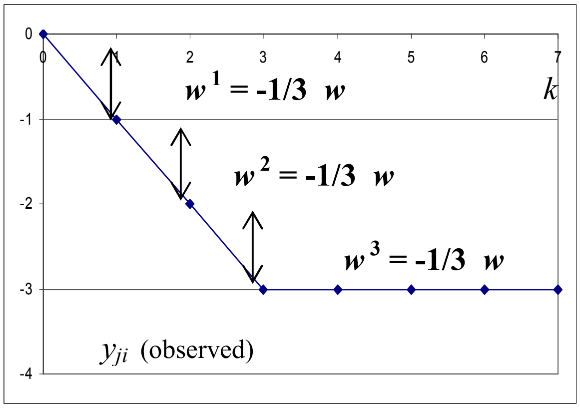
Without losing generality, the rest of the gradually built up dynamics will only be illustrated with positive impact cases. In general, if the full impact is w_{ji} , the total transition period is N slots, then each weight is $1/N \times w_{ji}$:

$$y_{ji}(k) = \frac{1}{N} w_{ji} \times x_i(k-1) + \frac{1}{N} w_{ji} \times x_i(k-2) + \ ... + \frac{1}{N} w_{ji} \times x_i(k-N)\ . \tag{3.5}$$

if N=1, it is the same as that in classic FCMs.



(a) positive impacts



(b) negative impacts

Fig. 3.2 Linear dynamic causal impact of 3 slots

C. Convex nonlinear dynamic causal links

A convex nonlinear dynamic causal link is a gradually built up causal link. It represents the dynamic impacts that build up rapidly in the initial stage. Fig. 3.3 shows the transition of a convex nonlinear dynamic link of total time slots N=3.

The mapping of the observation to multiple weights are marked on Fig. 3.3 , where the three weights are:

$$w_{ji}^1=4,\quad w_{ji}^2=1.5,\quad \text{and}\quad w_{ji}^3=0.5\ .$$

The impact is modeled as

$$y_{ji}(k) = 4 \times x_i(k-1) + 1.5 \times x_i(k-2) + 0.5 \times x_i(k-3)\ . \tag{3.6}$$

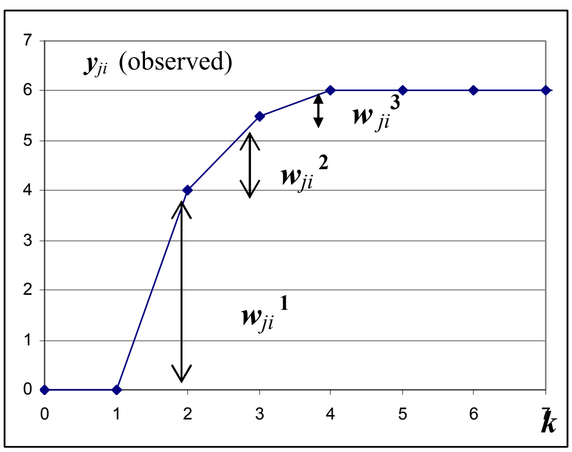


Fig. 3.3 Convex non-linear dynamic causal impact of 3 slots

In general, a convex nonlinear dynamic causal link can be modeled as:

$$y_{ji}(k) = w_{ji}^1 \times x_i(k-1) + w_{ji}^2 \times x_i(k-2) + \dots + w_{ji}^N \times x_i(k-N)\ . \tag{3.7}$$

D. Concave nonlinear dynamic causal links

A concave dynamic link represents the dynamic impacts that build up slowly in the initial stage and speed up later. Fig.3.4 shows a transition of a concave nonlinear dynamic link of total time slots N=3.

The mapping of the observation to multiple weights is marked on Fig. 3.4, where the three weights are:

$$w_{ji}^1=0.5,\quad w_{ji}^2=1.5,\quad \text{and}\quad w_{ji}^3=4\ .$$

The impact model is

$$y_{ji}(k) = 0.5 \times x_i(k-1) + 1.5 \times x_i(k-2) + 4 \times x_i(k-3)\ . \tag{3.8}$$

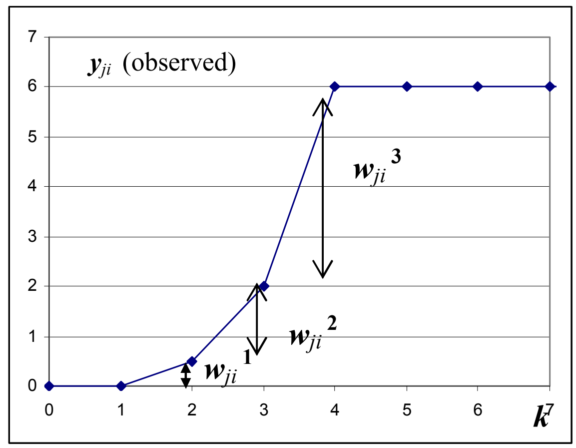


Fig. 3.4 Concave non-linear dynamic causal impact of 3 slots

In general, a concave nonlinear dynamic causal link can be modeled as:

$$y_{ji}(k) = w_{ji}^1 \times x_i(k-1) + w_{ji}^2 \times x_i(k-2) + \dots + w_{ji}^N \times x_i(k-N)\ . \tag{3.9}$$

E. Oscillation dynamic causal links

An oscillation dynamic causal link represents the dynamic impacts which have oscillations in the build-up transition. Fig.3.5 shows the transition of an oscillation dynamic causal link with total time slots N=6.

The mapping of the observation to multiple weights is marked on Fig. 3.5, where the six weights are:

$$w_{ji}^1= 2,\ w_{ji}^2=2,\ w_{ji}^3= 2,\ w_{ji}^4= -1,\ w_{ji}^5= -2,\ \text{and}\ w_{ji}^6= 1.$$

The impact model is

$$y_{ji}(k) = 6 \times x_i(k-1) + 2 \times x_i(k-2) + 2 \times x_i(k-3) - 1 \times x_i(k-4) - 2 \times x_i(k-5) + 1 \times x_i(k-6). \tag{3.10}$$

k

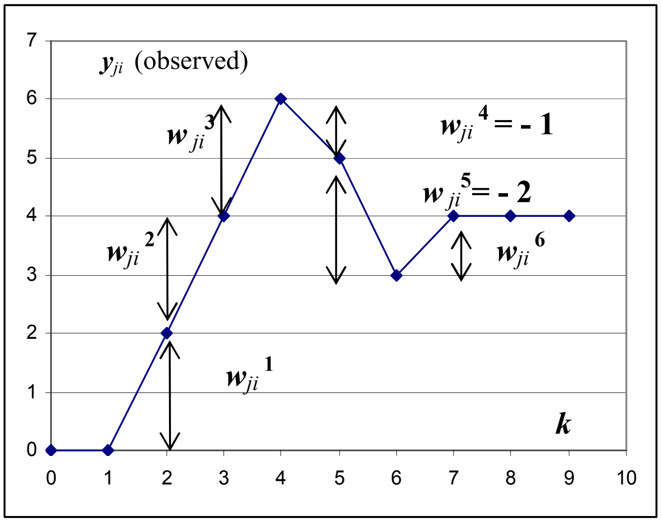


Fig.3.5 A oscillation non-linear dynamic causal impact lasts 6 slots

In general, an oscillation dynamic causal link can be modeled as:

$$y_{ji}(k) = w_{ji}^{1} \times x_i(k-1) + w_{ji}^{2} \times x_i(k-2) + \ldots + w_{ji}^{N} \times x_i(k-N)\,. \quad (3.11)$$

A typical property of an oscillation dynamic causal link is that some of the weights are negative and some are positive. The oscillation-type of dynamics widely exists in causal impact build up transitions. They can lead to complex inference dynamics of the causal system.

IV. CONCLUSIONS

Different causal impacts have very different dynamics. The dynamics can lead to significantly different causal outcomes. Two cognitive models, even with same map graph, same node state space, same decision functions and same full impact weights, a difference in the causal relationship dynamics can lead to totally different inference outcome, which means totally different decisions. Classic fuzzy cognitive maps have only one weight for causal links thus cannot model the dynamic causal relationships. This paper provides an easy to use model by using multiple weights to represent the dynamics. The weights have a direct mapping with domain experts' knowledge and observations. The new model adds little difficulties than modeling in classic fuzzy cognitive maps.

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