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Triangular Fuzzy Number Representation of Relations in Fuzzy Cognitive Maps

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***Abstract*—In this paper, the conventional Fuzzy Cognitive Maps (FCMs), which has already achieved success in many fields, are extended by using triangular fuzzy numbers (TFNs). The advantage of FCMs is that they are relatively easy to construct and parameterize and are capable of handling the full range of system feedback structure, including density-dependent effects. However, it is a well-known fact that there are some limitations inherent in FCM, such as lack of adequate capability to handle uncertain information and lack of enough ability to aggregate the information from different sources. Triangular fuzzy numbers which are represented by a triplet has the capacity to represent the uncertain relations between the concepts. In this context, the weight matrix representing the causal relations are enhanced to a fuzzy weight matrix that has TFNs as element. As a result of this improvement, the dynamic reasoning algorithm of the conventional FCM is improved for the use of the proposed novel FCM. The proposed FCM is presented via four simulations and the results are discussed. The results of the simulation study shows how easily the uncertain information can be represented and interpreted by the proposed FCM design methodology.**

Keywords—Fuzzy Cognitive Maps, Triangular Fuzzy Numbers, Causal Links, Reasoning, Weight Matrix .

I. INTRODUCTION

Fuzzy Cognitive Maps (FCMs) are soft computing tools, which combine elements of fuzzy logic and neural networks. FCM theory is proposed as an extension of cognitive maps by applying fuzzy causal functions with real numbers in $[-1, 1]$ to the connections by Kosko [1]. Therefore, FCMs are signed directed graphs with feedbacks, and they model the world as a collection of concepts and causal relations between concepts. The nodes represent variable concepts, and edges represent the strength of the causal links among the concepts. FCMs have been initially used for planning and decision making in the fields of international relations, social systems modeling and the study of political developments in the context of such systems.

One of the most useful aspects of the FCM is its potential for use in decision support as a prediction tool. For a given initial state of a system, represented by a set of values of its constituent concepts, an FCM can simulate its evolution over time to predict its future behavior [2]. For example, it may infer that the system would converge to a point where a certain state of balance would exist, and no further changes would take place. This inference is arrived at through a process of forward chaining. While the prediction capability of FCMs can be useful in answering what-if questions in a decision support environment, little research has been done on the use of FCMs in goal-oriented analysis [3]. In such an application, the analysis starts with a desired goal, and aims to identify what initial state can lead to that state. One possible reason for the lack of reported investigation into goal-oriented FCM analysis is the difficulty in reversing the matrix multiplication and non-linear transformations involved in computing successive FCM states [4]. As mentioned in [5], there is a vast interest in FCMs and this interest on the part of researchers and industry is increasing, especially in the areas of control [4], [6], [7], business [8], [9], medicine [10]-[12], robotics [13], emotion modeling [14], environmental science [15], [16], education [17], information technology [16] and self-tuning controller design [19].

After the original FCM was proposed, several extensions have been researched [20]–[29]. The aim of these extensions is to bring more values to concepts including real-valued concepts, nonlinear weight, and time delays. Although FCM has achieved success in many fields, there are some limitations inherent in FCM, such as lack of adequate capability to handle uncertain information and lack of enough ability to aggregate the information from different sources.

Recently, three novel studies extended the theory and the application of FCMs. In [30], a FCM extension, called Fuzzy Grey Cognitive Maps (FGCMs) has been proposed as an extension of the FCMs for environments with high uncertainty, under discrete small and incomplete data sets. The relationships between nodes in FGCM are represented by directed edges, which model the grey causal influence of the causal variable on the effect variable. As a result, the weights between the nodes are not constants but intervals, which improve the representation of uncertainties in the knowledge.

In order to bring a better approximation of the human decision-making model, an extension of the FCM that involves the degree of hesitation, which the experts may have, to define the relations between the concepts of the FCM is proposed in [31]. This is achieved by representing the causal relations with intuitionistic instead of conventional fuzzy sets, and by a properly modified reasoning algorithm. The intuitionistic fuzzy cognitive maps (IFCMs) are extended in further studies [32], [33] for other medical applications.

Uncertain information fusion has been studied for many years, indicating that Dempster-Shafer theory (DS theory or

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evidence theory) is an effective framework to represent and fuse uncertain information. In [34], FCM and evidential theory is combined and the concept of evidential cognitive maps, which improves the ability to represent uncertainty and also the way of aggregating knowledge from different sources, is proposed.

Experts who are familiar with the system components and their relations can generate a related FCM which is a very convenient, simple, and powerful graphical tool to represent knowledge. This graphical form is represented in a mathematical form of a weight matrix, which contains all the connections, and also a state vector made of the current weights of the concepts in the system. Till now, the elements of the weight matrix and the values of the concepts are represented by crisp numbers (fuzzy singletons) or intervals. Consequently, this way of knowledge representation limited the description of uncertain information. In this paper, the causal relationships between the concepts are represented by triangular fuzzy numbers that has a better capacity to represent the uncertainty. For this purpose, the iterative reasoning algorithm is modified and formulated using this new approach. The efficiency of the proposed method is shown by various simulation examples. For simulations, Matlab environment is preferred and a small framework is developed.

In Section II a brief overview of conventional FCMs is presented, and in Section III after giving basic definitions and notations for fuzzy sets and the triangular fuzzy numbers the proposed novel FCM design is introduced. An illustrative example is presented, and then the results of four simulations are discussed in in Section IV. Finally, conclusions are presented in Section IV.

II. A Brief Overview of Conventional Fuzzy Cognitive Maps

A Fuzzy Cognitive Map is a fuzzy diagram that describes the behavior of an intelligent system in terms of quantifiable concepts; each concept represents a variable, a state, or a characteristic of the system. FCM nodes are named by such concepts forming the set of concepts $C = \{C_1, C2, \ldots, C_n\}$.Arcs (C_j, C_i) are oriented and represent causal links between concepts; that is how concept C_j causes concept C_i. Weights of arcs are associated with a weight value matrix W_{nxn}, where each element of the matrix w_{ji} taking values in $[-1, 1]$; thus, there are three types of weights: $w_{ij} = 0$ indicates no causality; $w_{ij} > 0$ indicates a causal increase (i.e., C_j increases as C_i increases, and C_j decreases as C_i decreases); $w_{ij} < 0$ indicates causal decrease [35]. Inference on FCM works in discrete steps, so, when a strong positive correlation exists between the current state of a concept and that of another concept in a preceding period, we say that the former positively influences the latter, indicated by a positively weighted arrow directed from the causing to the influenced concept. By contrast, when a strong negative correlation exists, it reveals the existence of a negative causal relationship indicated by an arrow charged with a negative weight. Two conceptual nodes without a direct link are, obviously, independent. The advantage of FCMs is that they are relatively easy to construct and parameterize and are capable of handling the full range of system feedback structure, including density-dependent effects.

An FCM can be described by a connection matrix and the activation levels of its nodes can be represented as a state vector, whereby simple vector-matrix operations allow extension to neural or dynamical systems techniques. Once constructed, a FCM is then solved numerically to find the equilibrium value of variables (C_i), given any fixed boundary conditions. FCMs can be subjected to an initial stimulus in the form of a state vector, representing the states of the system's variables. The outcome of the constructed map can be determined by using matrix algebra where the vector of initial states of variables (C) is multiplied with the adjacency matrix W of the FCM. The value of each concept is influenced by the values of the connected concepts with the corresponding causal weights and by its previous value. The concept values of nodes $C_1, C_2, \ldots, C_n$ together represent the state vector C. The calculation rule that was initially introduced to calculate the value of each concept is based only on the influence of the interconnected concepts

$$C_j(t+1) = f\left(\sum_{\substack{i=1 \\ i\neq j}}^{n} C_i(t) w_{ij}\right) \tag{1}$$

where n is the number of concepts, $C_j(t+1)$ is the value of concept C_j at time step $t+1$, $C_i(t)$ is the value of concept C_i at time step t, and w_{ij} is the weight of the causal interconnection from concept i^{th} toward concept j^{th}.

The transformation function, $f(.)$, is used to confine (clip) the weighted sum to a certain range, which is the set to $[0, 1]$ or $[-1\ 1]$. The normalization hinders quantitative analysis, but allows for comparisons between nodes, which can be defined as active, inactive, or active to a certain degree. Two most commonly used transformation functions, sigmoid and hyperbolic tangent, are as follows:

$$f(x) = \frac{1}{1+e^{-\lambda x}} \tag{2}$$

$$f(x) = \tanh(\lambda x) = \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\lambda x} + e^{-\lambda x}} \tag{3}$$

where λ is a parameter used to determine proper shape of the function [36]. Both of the transformation functions use λ as a constant for function slope (degree of fuzzification). The FCM designer has to specify the lambda value. In general, for large values of lambda the transformation function approximates a discrete function; for smaller values of lambda it approximates a linear function.

III. Representation of the Weight Matrix using Triangular Fuzzy Numbers

In this section, some basic definitions of fuzzy sets and fuzzy numbers will be briefly reviewed. These basic definitions and notations below will be used throughout the paper until otherwise stated.

A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0, 1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the degree of membership of x in $\tilde{A}$. A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all $x_1,\ x_2$ in X,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min\big(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big) \quad (4)$$

where $\lambda \in [0, 1]$. A fuzzy set $\tilde{A}$ of the universe of discourse X is called normal fuzzy set implying that $\exists x_i \in X, \mu_{\tilde{A}}(x_i) = 1$.

A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal. In accordance with the theory of fuzzy sets where classical sets are included in the superordinate class of fuzzy sets, crisp numbers can be considered as a special case of fuzzy numbers, for they possess all their properties. A crisp number $\bar{x}$ can be expressed by a fuzzy number $\tilde{p}$ defined through the membership function

$$\mu_{\tilde{p}}(x) = \begin{cases} 0, & x < \bar{x}, \\ 1, & x = \bar{x}, \\ 0, & x = \bar{x}. \end{cases} \quad (4)$$

When crisp numbers are considered as fuzzy numbers, they are usually referred to as fuzzy singletons as illustrated in Fig. 1.

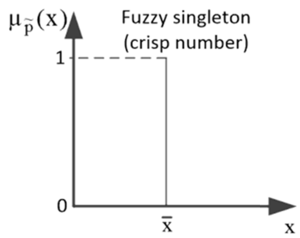


Fig. 1. Illustration of a fuzzy singleton.

A triangular fuzzy number (TFN) $\tilde{a}$ can be defined by a triplet (a^L, a^M, a^U) . The membership function $\mu_{\tilde{a}}(x)$ illustrated in Fig. 2 is defined as [37]:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a^L, \\ \dfrac{x - a^L}{a^M - a^L}, & a^L \leq x \leq a^M, \\ \dfrac{x - a^U}{a^M - a^U}, & a^M \leq x \leq a^U, \\ 0, & x \geq a^U. \end{cases} \quad (5)$$

where $a^L \leq a^M \leq a^U$, a^L and a^U stand for lower and upper values of the support of $\tilde{a}$, respectively, and a^M for the modal value.

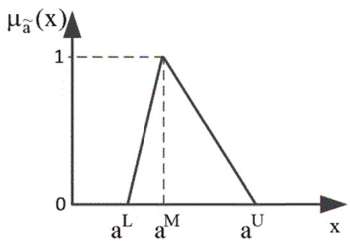


Fig. 2. Illustration of a triangular fuzzy number.

Basic operational laws related to positive triangular fuzzy numbers are given below:

$$\begin{aligned} \tilde{a} \oplus \tilde{b} &= (a^L, a^M, a^U) \oplus (b^L, b^M, b^U) \\ &= (a^L + b^L, a^M + b^M, a^U + b^U) \end{aligned} \quad (6)$$

$$\begin{aligned} \tilde{a} \otimes \tilde{b} &= (a^L, a^M, a^U) \otimes (b^L, b^M, b^U) \\ &= (a^L b^L, a^M b^M, a^U b^U) \end{aligned} \quad (7)$$

$$\lambda \otimes \tilde{a} = \lambda \otimes (a^L, a^M, a^U) = (\lambda a^L, \lambda a^M, \lambda a^U), \lambda > 0 \quad (8)$$

$$1/\tilde{a} = (1/a^U, 1/a^M, 1/a^L) \quad (9)$$

Moreover, $\widetilde{D}$ is a fuzzy matrix, if at least an element in $\widetilde{D}$ is a fuzzy number.

One of the basic concepts of the fuzzy set theory which is used to generalize crisp mathematical concepts to fuzzy sets is the extension principle. Let X and Y be two universes and discourse $f: X \to Y$ be a crisp function. The extension principle tells us how to induce a mapping $f: P(X) \to P(Y)$, where $P(X)$ and $P(Y)$ are the power sets of X and Y, respectively. The fuzzy extension principle is defined by Zadeh as follows [38]:

We have the mapping $f: X \to Y, y = f(x)$ which induce a function $f: \tilde{A} \to \tilde{B}$ such that

$$\widetilde{B} = f(\tilde{A}) = \{((y, \mu_{\tilde{B}}(y)) | y = f(x), x \in X\} \quad (10)$$

where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_{\tilde{A}}(x) & if\ f^{-1}(y) \neq \emptyset \\ 0 & otherwise. \end{cases} \quad (11)$$

In the original FCM design, the expert first determines the concepts, then the expert defines the causal links using linguistic terms such as weak medium or strong. In the initial FCM modeling these linguistic terms are mostly represented by triangular or trapezoidal membership functions, and the corresponding universe of discourse is either [0 1] or [-1 1] depending on the application. Afterwards, these linguistic terms are transformed to fuzzy singleton weights, which are values from a closed set of weights. In this final FCM modeling level, the linguistic terms are represented by fuzzy singletons that are crisp numbers within the interval of [-1 1]. Indeed, the uncertainty and ambiguity in the representation of causal links disappears because of the transformation to fuzzy singleton weights.

In the proposed FCM design, the fuzzy singletons used for the representation of the weights of the causal interconnections are replaced by TFNs. Therefore, the experts can easily represent their uncertain and vague knowledge via TFNs without choosing a predetermined linguistic term, which will be transformed afterwards to a fuzzy singleton weight, from a closed set. Accordingly; the above mentioned definitions and notations are needed to design the proposed FCMs. With the use of the TFNs, the reasoning (inference) algorithm of the FCM has to be revised. For this reason, the following new reasoning approach is described step by step:

Step 1:

Check the concept initial state vector. If it is describe using TFNs, then use a defuzzification method to obtain a fuzzy singleton.

In this study, center of gravity (COG) defuzzification method defined as follows is preferred:

$$C(t) = \frac{\int \mu_{\tilde{C}(t)}(x)\, x dx}{\int \mu_{\tilde{C}(t)}(x)\, dx} \quad (12)$$

Here $C(t)$ is the defuzzified output of fuzzy set $\tilde{C}(t)$, and x is the output variable.

Depending on the meaning of the concepts $C(t)$ may be defined in the interval of $[0, 1]$ or $[-1, 1]$.

Step 2:

The concept values are calculated as follows:

$$\underline{\tilde{C}}_j(t+1) = \sum_{\substack{i=1 \\ i \neq j}}^{n} C_i(t) \otimes \widetilde{w}_{ij} \tag{13}$$

where $C_i(t)$ represents the fuzzy concept singletons, and $\widetilde{w}_{ij}$ is the weight of the causal interconnection represented by TFNs. It is clear that the value of $\underline{\tilde{C}}_j(t+1)$ is still a TFN, but the values of the triple (a^L, a^M, a^U) may not be in the predefined universe of discourse.

Step 3:

The value of $\underline{\tilde{C}}_j(t+1)$ is mapped to its universe of discourse using the transformation function. For this mapping fuzzy extension principle has to be used. As mentioned in the second Section, the sigmoid and the hyperbolic tangent functions are mostly preferred as transformation functions.

$$\tilde{C}_j(t+1) = f\left(\underline{\tilde{C}}_j(t+1)\right) \tag{14}$$

Step 4:

The value of $\tilde{C}_j(t+1)$ is still represented via a TFN. This TFN can be used for the experts to see the uncertainty boundary defined with the upper and lower parameters of the TFN as $[a^L, a^U]$, or it can be defuzzified using COG defined in Eq. (12) to continue the iterations. After the defuzzification the uncertainty boundary is reduced to zero, which is therefore a crisp number.

IV. Illustrative Examples

In this section, a synthetic FCM with six concepts is randomly generated. The graphical representation of the FCM used in this study is illustrated in Fig. 3. Concept 1 ($C1$) and Concept 3 ($C3$) are the input concepts since these nodes influence but are not influenced by other nodes; oppositely Concept 6 ($C6$) is the output concept since it is only influenced by other nodes. Eventually, Concept 2 ($C2$), Concept 4 ($C4$), and Concept 5 ($C5$) are the intermediate concepts since these nodes are influenced by input concepts and/or by the other intermediate concepts and also they influence the other concepts but not input nodes. In addition, input concept $C1$ only linked to three intermediate concepts, but the second input concept $C3$ is linked to an intermediate concept C4 and to the output concept. For all simulations, hyperbolic tangent transformation functions is used, and the value of λ is chosen as 1.2. As mentioned in the previous section, λ is the parameter that determines shape of the function. When λ is chosen much larger than 1.2 the transformation function approximates a discrete function so FCM may lead to limit cycles, conversely for much smaller values of lambda, transformation function approximates a linear function, and consequently the FCM will quickly converge to a fix point.

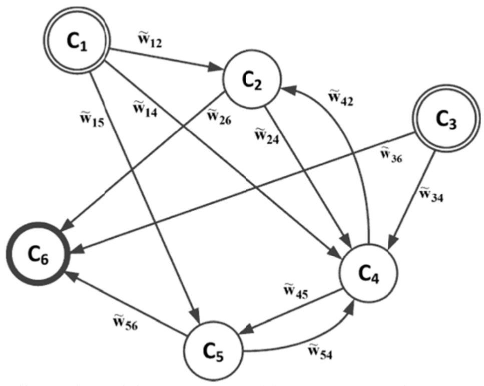


Fig. 3. Illustration of the Fuzzy Cognitive Map example.

To show the practicality and effectiveness of the proposed FCM design, four simulation examples are presented using the FCM given in Fig. 3.The codes needed for the simulations are written in Matlab, and a small framework is developed for FCM design using TFNs, then all of the simulations are perform in Matlab using this framework.

Only for the first simulation the conventional FCM design using fuzzy singletons (crisp numbers) is studied. The randomly generated weight matrix representing the causal interconnections is given in Table 1.

TABLE I

Weight Matrix of the Conventional Fuzzy Cognitive Map

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	−0.65	0	−0.60	0.60	0
C_2	0	0	0	0.65	0	−0.85
C_3	0	0	0	−0.85	0	0.10
C_4	0	0.80	0	0	−0.20	0
C_5	0	0	0	−0.95	0	−0.75
C_6	0	0	0	0	0	0

For the last three simulations the same FCM with six concepts is used but the weight matrix is enhanced to fuzzy weight matrix having elements TFNs in order to represent the uncertainties of causal interconnections. The weight matrix used for the last three simulations is tabulated in Table 2. As it is seen, the weights indicating no causality are still kept as zero using a triple as $(0, 0, 0)$. In addition, the nonzero fuzzy singleton weights given in Table 1 are kept as the modal value, a^M, to obtain the corresponding TFN, and upper and lower values are randomly generated so that $a^L \leq a^M \leq a^U$ condition is hold.

TABLE II

Weight Matrix of the Fuzzy Cognitive Map Designed using Triangular Fuzzy Numbers

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(0.00 0.00 0.00)	(−0.85 − 0.65 − 0.10)	(0.00 0.00 0.00)	(−0.70 − 0.60 − 0.40)	(0.10 0.60 0.70)	(0.00 0.00 0.00)
C_2	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.60 0.65 0.70)	(0.00 0.00 0.00)	(−0.95 − 0.85 − 0.20)
C_3	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(−1.00 − 0.85 − 0.70)	(0.00 0.00 0.00)	(0.00 0.10 0.25)
C_4	(0.00 0.00 0.00)	(0.35 0.80 1.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(−0.80 − 0.20 − 0.10)	(0.00 0.00 0.00)
C_5	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(−1.00 − 0.95 − 0.10)	(0.00 0.00 0.00)	(−0.95 − 0.75 − 0.70)
C_6	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)	(0.00 0.00 0.00)

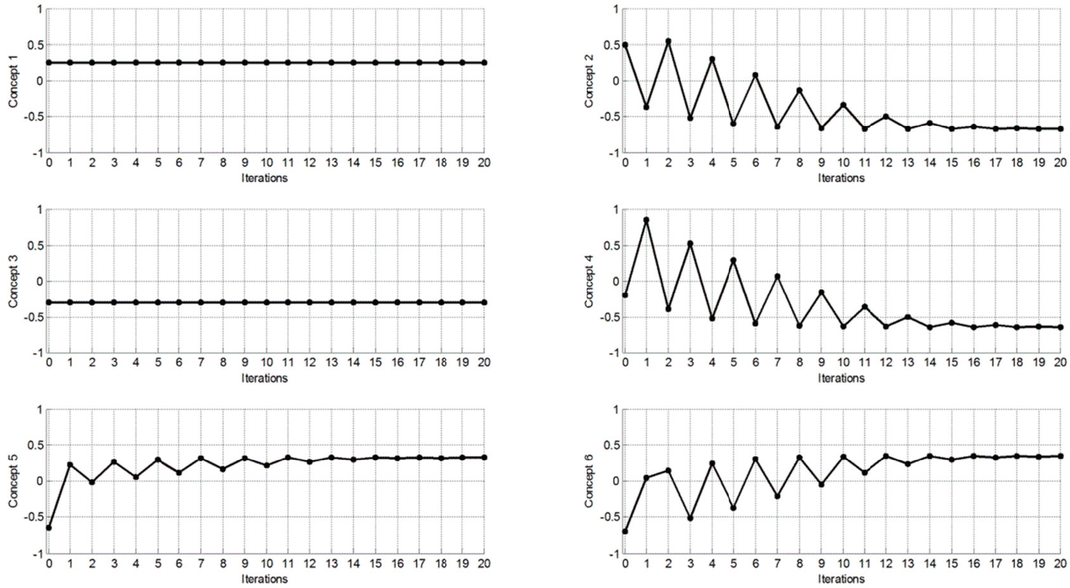


Fig. 4. The evaluation of the concepts for the first simulation.

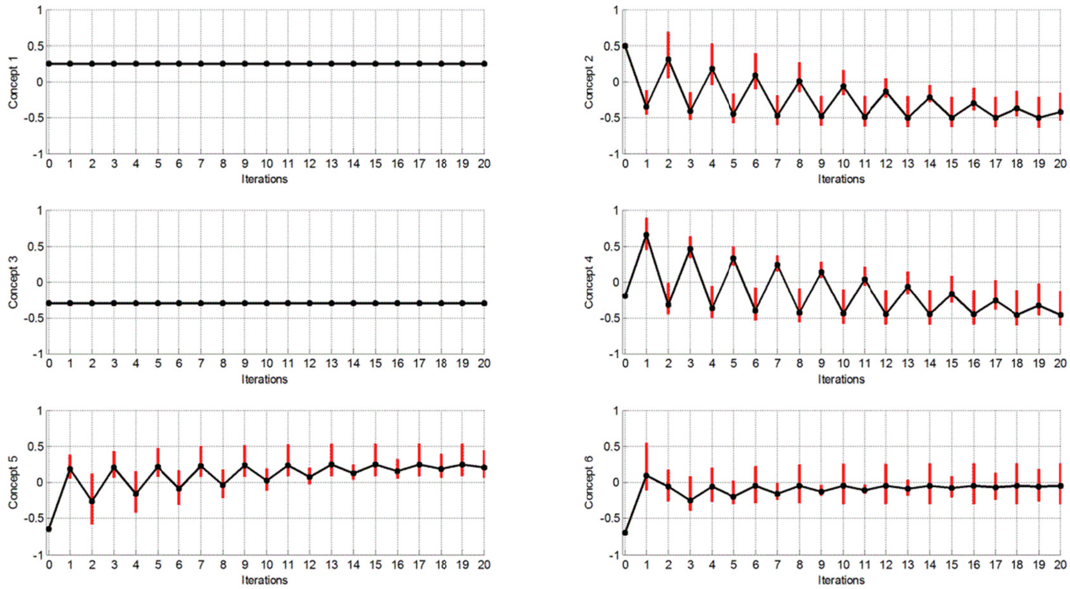


Fig. 5. The evaluation of the concepts for the second simulation.

A. Simulation 1

For the first simulation, the conventional FCM given in Table 1 is used. When the FCM is simulated using the initial concept state vector

$$C(0) = [0.25, 0.50, -0.30, -0.20, -0.65, -0.70]$$

for 20 iterations, the evaluation of the concepts illustrated in Fig. 4 is obtained. After 20 iterations, the Concept 6 (C_6) that is the output concept converges to a fix point. The initial value of output concept assigned as -0.70 increases by iterations to 0.3439 which is a crisp number. As seen, there is no uncertainty or fuzziness, but it is possible to interpret 0.3439 using linguistic terms for example as positive small.

B. Simulation 2

As the second simulation, the FCM modelled with the weight matrix given in Table 2 is used but the simulation is achieved for the initial state vector

$$C(0) = [0.25, 0.50, -0.30, -0.20, -0.65, -0.70]$$

which is also used for the first simulation. Therefore, the initial values of all concepts are kept as fuzzy singletons, while the causal links are represented by TFNs. Since the initial state vector is already crisp the first step of the reasoning algorithm including COG defuzzification should be skipped. The results of the simulations are illustrated in Fig. 5. The uncertainty boundary of the concepts evaluated by iteration is represented by red lined. Also, the defuzzified value of the concepts using COG is marked as dots for 20 iterations. Because of the TFNs in the definition of the weight matrix, the final TFN value of the output concept C_6 is (-0.3094, 0.1594, 0.2630) and the defuzzified value is -0.0491. This fix point is much less than the result obtained in the first simulation.

C. Simulation 3

For the third simulation, the weight matrix given in Table 2 is used, but this time, TFNs are used for the initial concepts values of the intermediate and output concepts. It is assumed that the input concepts are measured accurately and have no vagueness so the C1 and C3 are kept their crisp values constant during 20 iterations as defined in the previous two simulations. The simulation results are given in Fig. 6 for the following initial state vector:

$$\mathrm{C}(0) = [0.25, (-0.1, 0.50, 0.60), -0.30, (-0.30, -0.20, 0), (-0.70, -0.65, 0), (-0.90, -0.70, 0.15)].$$

As seen from Fig. 6, the uncertainty boundaries for the concepts are changes by iterations. For example, the uncertainty boundary of the output concept C_6 increase or decreases in different simulation steps depending on the value of the other concepts. At the end of the 20th iteration, the output concept converges to (-0.3093, 0.1593, 0.2628) and its defuzzified value is calculated as -0.0492. Remarkably, this is very close to the result obtained from the second simulation. As a special result of this simulation, the uncertainty in the definition of the initial state vector did not change the final value of the output concept much.

D. Simulation 4

As the last simulation, the initial state vector is taken as

$$\mathrm{C}(0) = [(0.15, 0.25, 0.80), (-0.1, 0.50, 0.60), (-0.80, -0.30, -0.25), (-0.30, -0.20, 0), (0.70, 0.65, 0), (-0.90, -0.70, 0.15)]$$

for the FCM defined in Table 2. As seen from the initial state vector, for this example, the initial values of the input concepts are also represented by TFNs. This change effects the convergence of the output concept and reduces the small oscillations occurred in the third simulation. At the end of the 20 iterations, output concept converges to $(-0.4098, 0.1544, 0.2884)$ and its defuzzified value is -0.0942. From Fig. 7a, we can conclude that the uncertainty boundary of C_6 increases and its defuzzified crisp value decreases compared to third simulation example. In addition, the values of concepts represented by TFNs before defuzzification are illustrated in Fig. 7b. This 3D representation will help the designer understanding what-if scenarios in a better way since the uncertainties represented by TFNs are also included.

V. Conclusions

In this paper, the uncertain information representation capacity of FCM is increased using triangular fuzzy numbers (TFNs) for the weights. The fuzzy singletons (crisp numbers) used in the original FCM design is enhanced to TFNs; therefore, the experts can express their uncertain knowledge in the causal links using TFNs. The FCM reasoning method is enhanced for TFNs and a systematic procedure is presented. The proposed FCM design is simulated for various initial states and the results are compared with the conventional FCM. It is shown via simulations that the initial values of the concepts can be also represented by TFNs when the proposed FCM design is used. In addition, when there is more than one expert for the design of FCM the proposed design using TFNs can be used and the knowledge of the experts can be aggregated and expressed in one FCM.

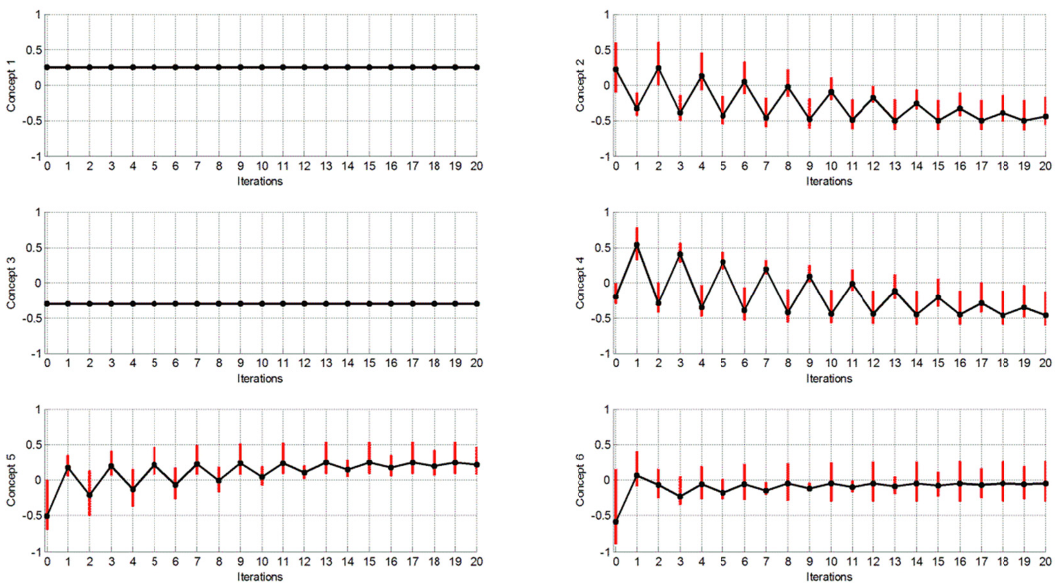
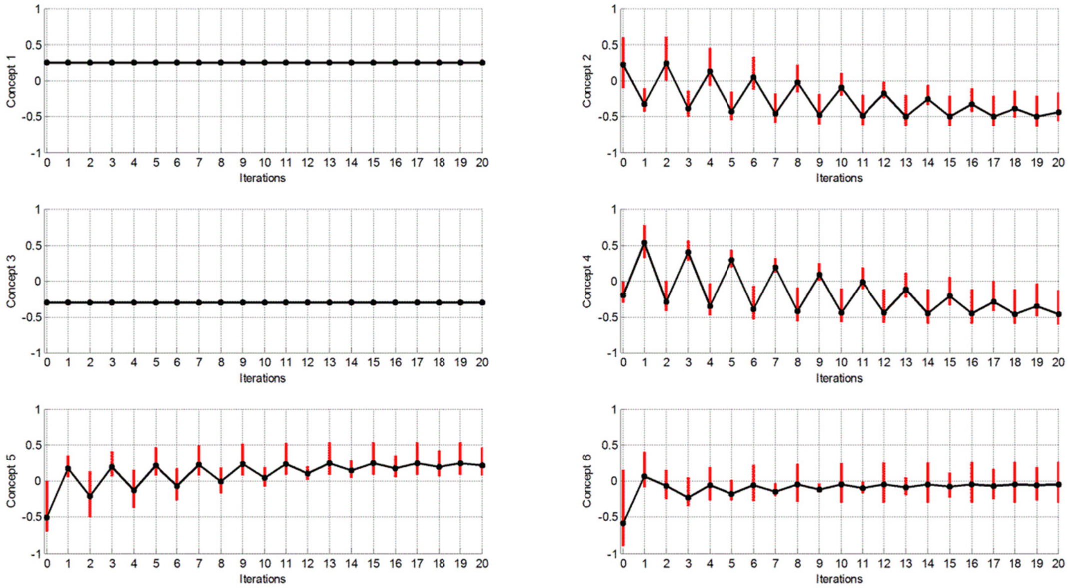
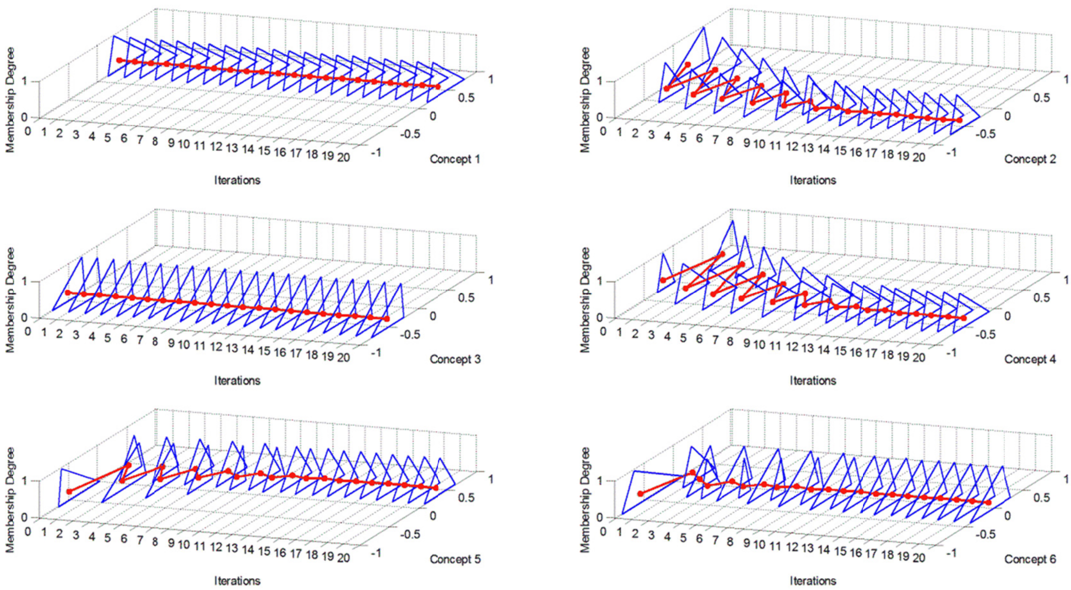


Fig. 6. The evaluation of the concepts for the third simulation.



(a)



(b)

Fig. 7. The evaluation of the concepts for the fourth simulation.

In the design step a linear transformation function may also be preferred; also in such a case if a causal link is defined as a symmetrical TFN, that is the modal value is the mean of lower and upper value of the support, the COG defuzzification will give the modal value again. Therefore, the FCM will be evaluated as a conventional FCM. Also, instead of COG method, the designer may choose any plausible defuzzification method but not height method. When height method is used for TFNs, again the modal value, which has the maximum membership value, is obtain; therefore the conventional FCM is obtained.

In this study, the uncertainty in information is represented only via triangular fuzzy numbers. As a future work, this representation will extended to general fuzzy numbers. In addition, the aggregation of multi-expert information will be considered for a real-life example.

The outcomes of this simulation study shows that the weight matrix can be represented easily by TFNs to build a fuzzy weight matrix to represent the relations between the concepts to include vague information of the experts.

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